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Numerical analysis of necking in stretch-bending based on modified maximum force criteria

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Abstract: The necking criteria based on the maximum force principle, such as the Swift and Hill necking models, have been extensively used in the past to study sheet formability. Many extensions or modifications of these criteria have been proposed to improve necking predictions under only stretching conditions. Recently, the authors proposed two approaches to predict necking under stretch-bending conditions: a generalisation of classical maximum force criteria to stretch-bending; and a necking criterion based on critical distance concepts, suggesting that necking is controlled by the damage of a critical material volume located at the inner side of the sheet. Both approaches were evaluated with analytical models and the failure was successfully predicted in different materials, such as steel, brass and aluminium. This work presents a numerical study of the proposed necking criteria in stretch-bending processes with different punch radii. The simulations are validated with experimental tests over H240LA steel sheets.

Keywords: Sheet metal forming, Stretch-bending, Necking, Maximum force criterion, Bending effect.

1. Introduction

The maximum force criterion (MFC) states that diffuse necking is initiated in a tensile test when the maximum force is reached. Many extensions and modifications of the MFC have been used for the evaluation of necking in metal sheets subjected to different stretching conditions in the sheet plane. The classical models of Hill [1] and Marciniak-Kuczynski (M-K) [2] predict the onset of localised necking in the left and right sides, respectively, of the Forming Limit Diagram (FLD).

Recently, Aretz [3] assumed that localised necking is initiated when the axial tension reaches a critical value, rather than its maximum value. The author aimed to scale the prediction of the Hill's model to fit the major strain at necking under plane-strain conditions predicted by the M-K model, thus obtaining an analytical Forming Limit Curve (FLC) for necking without discontinuities. The idea of Aretz can be also used to scale the Hill's curve to fit the experimental major strain at necking under plane-strain conditions, usually referred to as FLD_0.

The above MFC-based criteria were developed to predict necking in stretching operations. In a previous work [4], the authors generalise the maximum force principle to stretch-bending operations, taking into account the *beneficial* effect of the stress/strain gradient through the sheet thickness on formability. Two types of failure criteria were proposed, the first based on the generalisation of the maximum force principle to stretch-bending (named as MFC-SB, Maximum Force Criterion for Stretch-

Bending) and the other based on the concept of damage in a critical material volume (named as CDR, Critical Distance Rule). The CDR criterion suggests that the effect of stress/strain gradients on sheet thickness may be assessed by applying the maximum force principle locally at a given material's critical distance from the concave sheet surface. Both necking models were successfully applied to describe the failure by necking in sheets under stretch-bending conditions in different materials, such as 1008 AK steel, 70/30 brass, 6010 aluminium alloy and 7075-O aluminium alloy.

The concept of damaged material volume has been widely used to account for the effect of stress/strain gradients on different failure mechanisms, such as fatigue and fracture. In this sense, the Critical Distance Methods (CDM) [5] have been used to analyse the fatigue behaviour of notched members, predict brittle and ductile static fracture in metals, plastics, ceramics and even bones. In previous research works [4,6-8], the authors explored the ideas behind the CDM to evaluate sheet formability in stretch-bending conditions.

This work presents a numerical analysis of the proposed MFC-SB and CSR criteria to predict the onset of necking of high-strength H240LA steel sheets under stress/strain gradients through the sheet thickness. A finite element model in Abaqus/Standard© of Nakazima and stretch-bending tests with different punch radii has been developed to evaluate both criteria. The predictive capabilities of both criteria are analysed and discussed in the light of experimental results.

2. Maximum force criterion for stretch-bending

The Hill's localised necking criterion states that the localisation is initiated when the major tension T_1 (force per unit of width in principal direction 1) reaches its maximum [1]:

$$dT_1 = d(\sigma_1 t) = 0 \qquad \Rightarrow \qquad \frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_1} = 1 + \beta \tag{1}$$

where t is the sheet thickness, σ_1 and ε_1 are the true stress and true strain in direction 1, and $\beta = d\varepsilon_2/d\varepsilon_1$ defines the strain path in the FLD. The application of the Hill criterion is restricted to the left-hand side of the FLD ($\beta \leq 0$), that is, uniaxial-stretching.

The Hill model of plastic instability in equation (1) can be conveniently expressed in terms of the equivalent (or effective) variables σ_{eq} and ε_{eq} as:

$$\frac{1}{\sigma_{eq}}\frac{d\sigma_{eq}}{d\varepsilon_{eq}} = \frac{1+\beta}{\rho} \tag{2}$$

where $\rho = d\varepsilon_{eq}/d\varepsilon_1$. The left-hand side of equation (2) depends on the material stress-strain relation, whereas the right-hand side depends on the strain path and the yield surface.

Most of the works in literature assume in their analysis a potential law $\sigma_y = K\varepsilon_{eq}^n$, a Mises yield surface and a proportional strain path. Thus, both terms of equation (2) are transformed to

$$\frac{1}{\sigma_{eq}}\frac{d\sigma_{eq}}{d\varepsilon_{eq}} = \frac{nK\varepsilon_{eq}^{n-1}}{K\varepsilon_{eq}^n} = \frac{n}{\varepsilon_{eq}} \qquad , \qquad \frac{1+\beta}{\rho} = \frac{1+\beta}{2/\sqrt{3}\cdot\sqrt{1+\beta+\beta^2}} \tag{3}$$

where the expression $\rho = 2/\sqrt{3}\cdot\sqrt{1+\beta+\beta^2}$ is found from Mises plasticity in plane stress (see Appendix in [4]). Thus, the limit necking strains can be obtained from equations (2) and (3) as

$$\varepsilon_{eq}^{neck} = n\frac{2}{\sqrt{3}}\frac{\sqrt{1+\beta+\beta^2}}{1+\beta} \qquad , \qquad \varepsilon_1^{neck} = \frac{\varepsilon_{eq}^{neck}}{\rho} = \frac{n}{1+\beta} \tag{4}$$

Note that for plane strain conditions ($\beta = 0$), the limit major strain reduces to $\varepsilon_1^{neck} = n$.

Aretz assumed that necking is initiated when the axial tension reaches a critical value, rather than its maximum value [3]. This model can be used to scale the Hill's curve in the FLD to fit the experimental major strain at necking under plane-strain conditions (FLD_0). Accordingly, equation (2) is turned into

$$\frac{1}{\sigma_{eq}}\frac{d\sigma_{eq}}{d\varepsilon_{eq}} = \frac{1+\beta}{\rho}\frac{n}{FLD_0} \tag{5}$$

which leads to $\varepsilon_1^{neck} = FLD_0$ in plane-strain.

The above models of plastic instability lead to the formulation of the maximum force principle in a single and unified expression:

$$\frac{1}{\sigma_{eq}} \frac{d\sigma_{eq}}{d\varepsilon_{eq}} = \frac{1}{Z} \tag{6}$$

where the left-hand side is a material property, sometimes referred to as the non-dimensional strain-hardening characteristic, whereas Z is the critical value for the subtangent of the stress-strain curve [9].

Under stretch-bending conditions, strain and stress gradients through the sheet thickness are induced in the metal sheet. In this situation, the maximum force principle was reformulated by the authors to take into account these gradients [4] as

$$\frac{1}{\overline{\sigma}_{eq}} \frac{d\overline{\sigma}_{eq}}{d\overline{\varepsilon}_{eq}} = \frac{1}{Z} \tag{7}$$

where $\overline{\sigma}_{eq}$ and $\overline{\varepsilon}_{eq}$ are the average values of the equivalent stress and strain on the sheet thickness, respectively. This model, which was referred as to MFC-SB, is numerically evaluated in section 4.

3. Critical Distance Rule for necking

The CDR criterion was introduced by the authors in a recent research work [4]. It assumes that necking is controlled by the development of damage in a certain material volume located at the concave surface of the sheet. The size of the critical volume is assumed to be a material constant, which can be related with the microstructure of the material. The CDR can be formulated using equation (6) as

$$\left.\frac{1}{\sigma_{eq}} \frac{d\sigma_{eq}}{d\varepsilon_{eq}}\right|_{d_{cr}} = \frac{1}{Z} \tag{8}$$

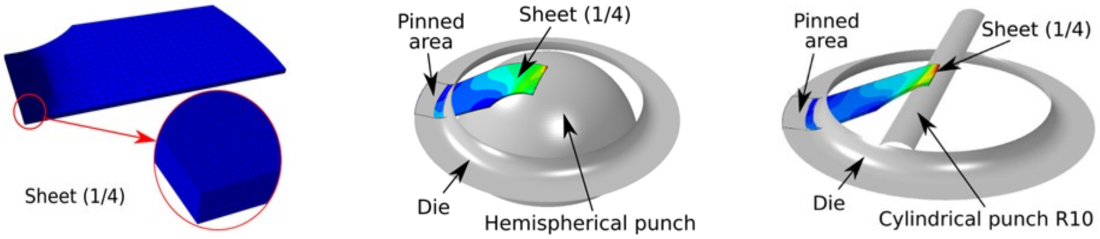
where d_{cr} is the critical distance measured from the concave surface that defines the critical material volume.

The CDR can be rationalised as follows. Under stretch-bending conditions, a gradient of the material characteristic $\left(d\sigma_{eq}/d\varepsilon_{eq}\right)/\sigma_{eq}$ through the sheet thickness is induced. The more strained material in the convex surface first reaches the limit value $1/Z$ and this condition is propagated inward through the thickness during the forming process. The critical material volume in the concave surface is assumed to resist the plastic instability of the entire sheet thickness until the failure condition reaches it. This criterion is numerically evaluated in next section.

4. Numerical simulation

In previous works [10,11], the authors developed a numerical model to analyse the formability of H240LA sheets of 1.2-mm thickness in stretch-bending. This model very precisely described the deformation of a series of stretch-bending tests under plane-strain conditions using cylindrical punches with radii 10, 5, 2.5, 1.5 and 0.5 mm. In this work, a new 3D finite element model was developed for the simulation of Nakazima and stretch-bending tests in Abaqus/Standard© that is based on the previous one.

Figure 1 depicts the modelled sheet and punch for both types of tests. The die effect was simulated by pinning the sheet external edge. The punch was modelled using rigid-shell elements. 3D solid deformable elements were chosen to model the metal sheet in order to precisely capture the effects of bending through the sheet thickness. A combination of linear wedge elements with hybrid formulation (C3D6H) in the central area of the specimen, and linear brick elements with reduced integration (C3D8R) in the rest of the sheet, was able to reproduce accurately the deformation process.



(a) (b) (c)

Figure 1. (a) Finite element mesh and virtual setup in **(b)** Nakazima **(c)** stretch-bending simulations.

Table 1. Mechanical properties of H240LA steel sheets.

Direction	*YS* (MPa)	*UTS* (MPa)	Elongation	*r*
Rolling (0°)	253.0	380.8	0.290	1.18
Diagonal (45°)	254.2	373.7	0.267	1.10
Transverse (90°)	250.8	364.8	0.286	1.43

Table 1 shows the mechanical properties of the material (yield stress, tensile strength, elongation and Lankford anisotropy coefficient) obtained from tensile tests in three in-plane directions (rolling, diagonal and transverse). The material behaviour was modelled using a strain rate-independent elastoplastic model with non-linear hardening. A Hooke's law was assumed for the elastic behaviour. The plastic hardening was modelled using a Voce-type law:

$$\sigma_{eq} = 252.41 + 279.27(1 - e^{-8.20\varepsilon_{eq}}) \quad (9)$$

The non-linear hardening model by Chaboche [12] and the Barlat'91 anisotropic yielding criterion [13] were assumed. Table 2 shows the anisotropy parameters C_i and the exponent m of the Barlat'91 model. A friction coefficient of 0.08 was used for the lubricated sheet-punch interface.

Table 2. Anisotropy coefficients of Barlat'91 yielding criterion for H240LA steel sheets.

C_1	C_2	C_3	C_4	C_5	C_6	m
0.868	0.881	1.114	1	1	1.047	2

To evaluate the proposed necking criteria, a path of 20 points through the sheet thickness was defined in the central point of the sheet plane, where the onset of necking is assumed to occur. Thus, the MFC-SB criterion in equation (7) was numerically evaluated as

$$\frac{1}{\overline{\sigma}_{eq}^{i+1}}\frac{\overline{\sigma}_{eq}^{i+1}-\overline{\sigma}_{eq}^{i}}{\overline{\varepsilon}_{eq}^{i+1}-\overline{\varepsilon}_{eq}^{i}} = \frac{1}{Z_{MFC}} \quad (10)$$

where the average value of σ_{eq} and ε_{eq} at every increment of time *i* was found by using the Riemann integral through every point *j* of the path:

$$X = \{\sigma_{eq}, \varepsilon_{eq}\} \qquad , \qquad \overline{X} = \frac{1}{20}\sum_{j=1}^{20} X_j \quad (11)$$

Similarly, the CDR criterion in equation (8) was evaluated as

$$\frac{1}{\sigma_{eq,k}^{i+1}}\frac{\sigma_{eq,k}^{i+1}-\sigma_{eq,k}^{i}}{\varepsilon_{eq,k}^{i+1}-\overline{\varepsilon}_{eq,k}^{i}} = \frac{1}{Z_{CDR}} \quad (12)$$

where *k* is the point of the path nearest to the critical distance d_{cr}. Based on previous works [4,6], a value of $d_{cr} = 0.3t_0$ was assumed for H240AL sheets.

The material constants Z_{MFC} and Z_{CDR} were found from the simulation of the Nakazima test under plane-strain conditions by evaluating the left-hand side of equations (10) and (12), respectively, when the major strain at the outer sheet surface reached the measured experimental value ($\varepsilon_{1,out} = FLD_0$).

5. Results and discussion

This section evaluates the capability of the failure criteria described above to predict localised necking under stretch-bending conditions. The experimental results analysed were provided in previous works [10,11].

The simulation of the Nakazima test with $FLD_0 = 0.473$ provided the following values for the constants of the proposed necking criteria: $Z_{MFC} = 0.0460$ and $Z_{CDR} = 0.0458$.

The simulation of the stretch-bending tests with different punch radii (R) provided the value of the strains at the outer sheet surface ($\varepsilon_{1,out}$) predicted by MFC-SB and CDR criteria. The experimental and predicted strains are represented in figure 2 for the different initial thickness to punch radius ratio (t_0/R).

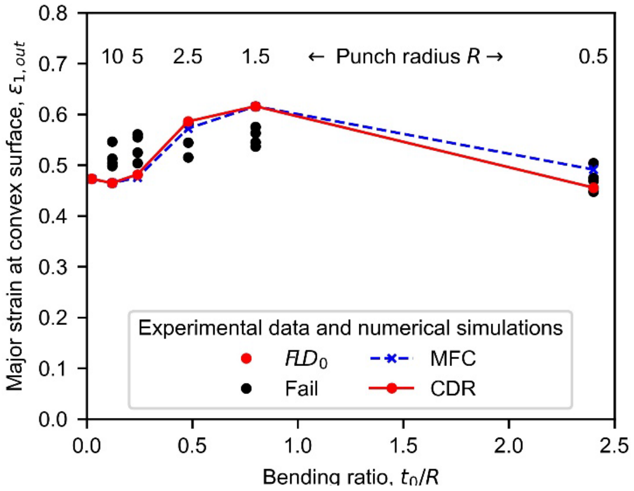


Figure 2. Necking predictions of the proposed maximum force criterion in stretch-bending (MFC-SB) and critical distance rule (CDR) for stretch-bent H240LA steel sheets.

As can be seen in figure 2, predictions of both necking criteria are very similar. This is due to that the value of the material characteristic $\left(d\sigma_{eq}/d\varepsilon_{eq}\right)/\sigma_{eq}$ at a certain distance on thickness from the sheet surface is representative of the average value on thickness. In this sense, it seems that both criteria are equivalent. The advantage of the MFC-SB criterion over the CDR is that it does not require computing the integral through the thickness.

Both necking criteria slightly underestimated the experimental results using punch radii of 10 and 5 mm, whereas they slightly overestimated the experimental results using a 1.5-mm punch radius, as can be observed in figure 2. However, the predictions are very good considering the scatter of the experimental data.

6. Conclusions

Two types of failure criteria based on the maximum force principle have been evaluated to predict necking in stretch-bending conditions using finite element analysis. One based on the generalization of the traditional maximum force principle to stretch-bending and the other based on the concept of damage in a critical material volume. The following conclusions can be drawn from this study:

- The strain-hardening characteristic $\left(d\sigma_{eq}/d\varepsilon_{eq}\right)/\sigma_{eq}$ used to propose both types of failure criteria seems to control necking in both stretching and stretch-bending processes.
- Both criteria predict similar results and therefore seem to be equivalent. This corroborates the use of critical distance methods to evaluate the formability in sheet metal.
- The necking predictions fit well with the experimental data. To characterise the failure with the CDR criterion, a critical distance of $0.3t_0$ has been proposed for H240AL sheets. This value is based on previous works on different materials such as steel, aluminium and brass sheets. However, more extensive research is currently underway to relate critical distance to material properties.

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References

[1] Hill R 1952 On discontinuous plastic states, with special reference to localized necking in thin sheets *Journal of the Mechanics and Physics of Solids* **1** (1) pp 19–30

[2] Marciniak Z and Kuczyński K 1967 Limit strains in the processes of stretch-forming sheet metal *International Journal of Mechanical Sciences* **9** pp 609–620

[3] Aretz H 2010 An extension of Hill's localized necking model *International Journal of Engineering Science* **56** pp 609–618

[4] Morales-Palma D, Martínez-Donaire A J and Vallellano C 2017 On the use of maximum force criteria to predict localised necking in metal sheets under stretch-bending *Metals* **7** p 469

[5] Taylor D 2007 *The Theory of Critical Distances* (Oxford: Elsevier Science Ltd)

[6] Morales-Palma D, Vallellano C and García-Lomas F J 2013 Assessment of the effect of the through-thickness strain/stress gradient on the formability of stretch-bend metal sheets *Materials and Design* **50** pp 798–809

[7] Vallellano C, Morales D, Martínez-Donaire A J and García-Lomas F J 2010 On the use of concave-side rule and critical-distance methods to predict the influence of bending on sheet-metal formability *International Journal of Material Forming* **3** (1) pp 1167–1170

[8] Morales D, Martínez-Donaire A J, Vallellano C and García-Lomas F J 2009 Bending effect in the failure of stretch-bend metal sheets *International Journal of Material Forming* **2** (1) pp 813–816

[9] Marciniak Z, Duncan J L and Hu S J 2002 *Mechanics of sheet metal forming* (Butterworth-Heinemann)

[10] Martínez-Palmeth L, Martínez-Donaire A and Vallellano C 2019 Formability limits of high-strength H240LA steel sheets under stress/strain gradients *Mechanics of Materials* **132** pp 47–56

[11] Martínez-Donaire A, Martínez-Palmeth L, Borrego M, Morales-Palma D and Vallellano C 2019 Analysis of the failure of H240LA steel sheets subjected to stretch-bending conditions *Procedia Manufacturing* **41** pp 626–633

[12] Chaboche J 1986 Time-independent constitutive theories for cyclic plasticity *International Journal of Plasticity* **2** (2) pp 149–188

[13] Barlat F, Lege D J and Brem J C 1991 A six-component yield function for anisotropic materials *International Journal of Plasticity* **7** (7) pp 693–712