Correct denition of the Poynting vector in electrically and magnetically polarizable medium reveals that negative refraction is impossible: comment

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Abstract: It is shown that, when all macroscopic currents associated with the electric and magnetic polarizability are properly accounted for, the standard expression for the Poynting vector and the average work exerted by the electric field on the electric charges provide exactly the same value for the heating rate. Therefore, there is no contradiction between negative refraction and thermodynamics.

OCIS codes: (160.1245) Artificially engineered materials; (350.3618) Left-handed materials

References and links

1. V. A. Markel "Correct denition of the Poynting vector in electrically and magnetically polarizable medium reveals that negative refraction is impossible." Opt. Express. **16**, 19152-19168 (2008).
2. J. D.Jackson *Classical Electrodynamics* Wiley 1999 (3rd Ed.).
3. In some textbooks (e.g. [4]) this equation is formulated only for the entire body, and the surface integral is taken in free space, outide the body. In this case the surface integral formally disappear (however its contribution is still present, due to the infinite derivatives of **M** just on the surface of the body). However, when the surface integral is placed inside the body (or just on the surface of the body), this contribution is necessary in order to recover the magnetization.
4. L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii *Electrodynamisc of continuous media*. Pergamon, 1984.

In the above paper [1] the author claims that the standard formula for the heating rate q

$$q = \frac{1}{4\pi}\left\langle \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} + \mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t}\right\rangle \tag{1}$$

is inconsistent with the averaged heating rate computed as the average work exerted by the electric field on the electric charges

$$q = \langle \mathbf{J}\cdot\mathbf{E}\rangle \ . \tag{2}$$

From this claim he concludes that the conventional definition of the Poynting vector is incorrect and, moreover, that negative refraction violates the second law of thermodynamics.

However, as it will be shown in this comment, (1) is fully consistent with (2) if all the currents associated with a continuous distribution of magnetic moment are properly taken into account. In fact, let us consider a body with volume polarization **P** and magnetization **M**, and let us

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assume that these polarization and magnetization are consequence of a periodic excitation, and therefore are itself periodic in time. After substitution of the definitions of **D** and **H** (i.e. $\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$ and $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$) in (1), and also using $c\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t$, it is found that

$$q=\left\langle \mathbf{E}\cdot\frac{\partial\mathbf{P}}{\partial t}+c\mathbf{M}\cdot\nabla\times\mathbf{E}\right\rangle . \tag{3}$$

Let us now integrate Eq.3 over the entire body. Using the well known vectorial identity $\nabla\cdot(\mathbf{E}\times\mathbf{M})=\mathbf{M}\cdot(\nabla\times\mathbf{E})-\mathbf{E}\cdot(\nabla\times\mathbf{M})$, it is found that

$$\int\int\int q\,dv=\int\int\int\mathbf{E}\cdot\left(\frac{\partial\mathbf{P}}{\partial t}+c\nabla\times\mathbf{M}\right)dv+c\int\int\mathbf{E}\cdot(\mathbf{M}\times\mathbf{n})\,ds \tag{4}$$

where integrals are done over the entire volume and surface of the body, and **n** is the unit vector normal to the suface. It can be easily recognized in Eq.4 the integral of Eq.2 if we take into account that the body supports not only a volumetric current density

$$\mathbf{J}_v=\frac{\partial\mathbf{P}}{\partial t}+c\nabla\times\mathbf{M} \tag{5}$$

but also a surface current density [2]

$$\mathbf{J}_s=c\mathbf{M}\times\mathbf{n}\,. \tag{6}$$

Therefore, (1) and (2) are fully consistent in order to compute the total rate of heating in the entire body.

It may seem, however, that Eq.1 does not give correctly the heating ratio in a differential volume of the body, since the surface current density Eq.6 is not present inside the body. However,this interpretation is also incorrect, because the volume current density $\mathbf{J}_v=c\nabla\times\mathbf{M}$ is not enough for properly characterizing a *physical* portion of the body. In fact, let us consider a small (but still macroscopic) region δV inside the body, limited by a surface $\delta\Sigma$. The total magnetic moment $\delta\mathbf{m}$ of this small portion of the body must be the integral of the magnetization **M** over the entire volume, and it can be shown that

$$\delta\mathbf{m}\equiv\int\int\int_{\delta V}\mathbf{M}\,dv=\frac{1}{2}\int\int\int_{\delta V}\mathbf{r}\times(\nabla\times\mathbf{M})\,dv+\frac{1}{2}\int\int_{\delta\Sigma}\mathbf{r}\times(\mathbf{M}\times\mathbf{n})\,ds \tag{7}$$

where **r** is the vector of position with regard to an arbitrary point in space [3]. Therefore, the surface current Eq.6 can not be ignored in order to properly describe any small portion of the magnetized body: if it is ignored, the magnetic moment of this portion is not recovered. Of course, these surface currrents cancel with the surface currents of the neighboring portions of the body (unless they are placed just on the surface of the body), and can be ignored for most practical applications. However, they must be taken into account when the power dissipated at each small (but macroscopic) portion of the body is computed. Otherwise there will be a ”part“ of these atoms and mulecules which will not be considered. Therefore, Eq.1 and Eq.2 both gives the same result when all the currents associated with the magnetization of any small but macroscopic portion of the body are computed, provided the contribution of all currents associated to the magnetization of this portion are properly accounted for.

As a last remark: let us assume that we do not impose that the aforementioned small region of the body is a *physical* region, i.e. a region actually made of a collection of magnetic dipoles. In such case we can of course assume that this region is simply characterized by the volume density of curent $\mathbf{J}_v=c\nabla\times\mathbf{M}$. In such case Markel’s theory arise. However, from this point of view, considering only this kind of regions is arbitrary, as arbitrary as considering in addition a

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surface current on each region given by $\mathbf{J}_s = c\mathbf{M} \times \mathbf{n}$, or any other hypothesis giving the same total power dissipated in the entire body. Therefore, there will be no reason to prefer Markel's theory to the standard one, or to any other giving the same dissipated power for the entire body, because we have done an arbitrary partition of the body. Only when we impose that each region of the body must be *physical* (in the sense developped above), we can choose the correct theory. And, as it was shown above, this choice must be the standard theory.

Finally, since all claims on the failure of the conventional Poynting vector expressions, or on the violation of the second law of thermodynamics by negative refraction, reported in [1] are heavily based on the inconsistency between Eq.1 and Eq.2, and we have shown that both equations are fully consistent, we must conclude that all the aforementioned claims are not supported by any consistent argument. Note that the above demonstrations are not based on any specific assumption on the specific constitutive relations of the considered body, therefore they remain valid for any kind of consitutive relations, linear or not, provided the body can be described as a continuum.

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