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RESEARCH

Proportions and Deformations in the Mosque-Cathedral of Cordoba

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Abstract
This research presents a dimensional analysis of the arcades of one of the most emblematic spaces of the Mosque-Cathedral of Cordoba: its central nave, located besides the mihrab in the ambitious extension carried out by Al-Hakam II. In order to accomplish this survey, a digital model was created from the point cloud obtained with a 3D laser scanner. Subsequently, earlier theories of architectural proportions related to this monument have been reviewed. The comparison of these earlier drawings with the corresponding digital models allows the precision of their different hypotheses to be assessed. Architectonic deformations not considered by the scientific literature are also quantified here. Finally, this research indicates certain partial order relations close to the proportion 1.3, though it should be noted that not all the arcades share the same dimensions and thus no general theory applies to their proportions.

Keywords Proportion · Geometry · Measurement · Virtual architecture

Introduction

The Mosque-Cathedral of Cordoba, its monumental core and its surroundings, included in the World Heritage List in 1984,[1] has undergone different transformations throughout history in order to be adapted to new times, but preserving the essential

[1] Historic Centre of Cordoba [World Heritage List]. Available online: https://whc.unesco.org/en/list/313 (accessed on 9 January 2022).

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features of its architectural identity (Moneo-Vallés 1985). This history comprises three different defined stages described hereunder (Nieto-Cumplido 2001).

The first Umayyad Mosque was built in the eighth century by Abd al-Rahman I. Its structure of parallel arcades and naves enabled a first extension towards the south by Abd al-Rahman II, and a subsequent one by Al-Hakam II. In the tenth century Almanzor ordered the last extension of the monument towards the east, completing an astonishing interior composed of columns and double arches. The second stage was started after the conquest of Ferdinand III in 1236, when the edifice was consecrated as a Christian church. Towards 1313 the Royal chapel was built to host the tomb of Ferdinand IV of Castile, beside the Villaviciosa Chapel. Between 1523 and 1607 a large, central volume was erected, and consequently the monument was greatly altered; the inner perimeter was occupied by chapels, and in the eighteenth century new skylights were opened and plaster vaults were added in several naves. The third stage, starting in the early nineteenth century, saw numerous reformations, juxtaposing architectonic fragments of miscellaneous historical periods (Herrero-Romero 2016).

It is helpful to highlight the extension commissioned by Al-Hakam II (Chueca-Goitia 1965) planned around a central nave located between the Royal Chapel and the mihrab, whose arcades are the subject of this study (Fig. 1).

The aim of this research is to analyze the dimensions of the arcades of the central nave in the extension built by Al-Hakam II in order to assess the accuracy of three notable theories about arcade proportions in the monument. The purpose is also to quantify their architectonic deformations, considering that the building has undergone numerous transformations and restorations for centuries.

To illustrate this, a novel graphic construction was created from the point cloud obtained in situ with a 3D scanner. Likewise, thorough graphic transcriptions of the

Fig. 1 Floor plan and frontal perspective of the mihrab area. Al-Hakam II extension

descriptive drawings of the three proportion theories were elaborated to superimpose them upon the digital model of the monument's current state and thus evaluate their precision. Finally, several charts and graphic diagrams were developed to display the main data of the deformations analyzed.

Model Construction from a 3D Laser Scanner

The major plans of the Mosque-Cathedral of Cordoba, elaborated throughout the history with traditional drawing techniques and compiled in an influential work by Nieto-Cumplido and Luca-de-Tena in (1992), were reviewed together with other varied selections (Gámiz-Gordo 2019). The publications of computerized plans based on measurements manually obtained were also reviewed (Ruiz-Cabrero 2009; Almagro-Gorbea 2015; Messina 2017).

The digitization of the architectural heritage has experienced a major breakthrough in recent years thanks to the development of new technologies for capturing metric data. Late research in the Mosque-Cathedral has applied photogrammetry techniques (Fuentes-González 2019) or 3D laser scanners (Gámiz-Gordo et al. 2021) as the basis for precise graphic documentation. In any case, the survey precision must be adjusted according to the scale used and the dimensions of the model, considering the objectives proposed here.

In this study, a point cloud from the Mosque-Cathedral area enlarged by Al-Hakam II was obtained using two laser scanners, the Leica C10 and the Leica BLK360, with the technical features detailed hereafter. The Leica C10 possesses a dual axis compensator for levelling with an accuracy of 1″, and 6 mm in the 3D positioning of a point. It is able to station on a specific point thanks to its laser plumb line. Its scanning field is 360º horizontally and 270º vertically. The density of points can be set to different values; for example, a medium density captures points every 10 cm on a plane located at 100 m. The Leica BLK360 is not able to station at a specific point nor be levelled; its scanning field is 360º. Medium quality capture density is 10 mm at 10 m.

By stationing at mq1, the C10 scanner was used to create the reference system and level the point cloud (see Fig. 2). Using mq1 as the origin and the mihrab nave as the X axis, the reference scan was performed stationing in mq1. Also, mq7 was observed from mq1. Four different trajectories were performed with the BLK, using the C10 scan over mq1 as the origin to record each trajectory. Figure 2a shows the scans from the C10 (yellow) and the four trajectories followed with the BLK (white); the number 1 corresponding to the central nave is focused on in the present study. Additionally, Fig. 2b shows the scanner positions symbolized by spheres and the corresponding registered point cloud.

All scans were accomplished using a medium resolution, which implies a separation between captured points of 1 cm at 10 m distance. In the quality of the registers of the scans the following variables were considered: overlap (quantity of volume shared between two registered scans, measured in %), balance (measure of the homogeneity in the points distribution through the 3 perpendicular directions

Fig. 2 Reference system and scanner trajectories: **a** plan view, **b** perspective view of the scanner positions represented by spheres

of the space) and the points percentage with an error less than 6 mm in each of the registers.

The geometric model of the current state of the building was produced by importing the point cloud to CAD and using it as a reference. The sections were extracted from the point cloud and projected orthogonally to obtain the corresponding orthoimages. To define the geometry, some commands were used to obtain lines or edges, as well as planes and intersections from the orthoimages. Each element was defined according to the degree of precision and definition of the analysed model (Fig. 3).

Theories on Proportions in the Mosque-Cathedral of Cordoba

There are several studies devoted to the field of proportions of the Mosque-Cathedral of Cordoba. Some of them focus on the general floor plan (Fernández-Puertas 2000; Hoz-Arderius 1991; Hoz-Arderius 1991), and others discuss elements such as the Villaviciosa Chapel (Fuentes-González 2019), Puerta de San Esteban (Hoz-Arderius 1991; Fernández-Puertas 1999; Fernández-Puertas 2008b), or the Puerta de los Deanes (Fernández-Puertas 2009).

The main theories about the proportions of the arcades, analyzed and graphically transcribed here, are the following three: one by Emilio Camps-Cazorla[2] [Camps] which analyzes the relation between the arch modules in the area of the original construction commissioned by Abd al-Rahman I (Camps Cazorla 1953); another

[2] Emilio Camps-Cazorla, born in Fuensanta de Martos (Jaén) on October 31st, 1903, and deceased in Madrid on January 28th, 1952. PhD in History of Art by the University of Oviedo in 1949, he never practiced as Professor, as he was appointed Director of the National Archaeological Museum. He is the author of numerous publications about Romanesque and Islamic art (Gentil-Baldrich 2021: 315-316).

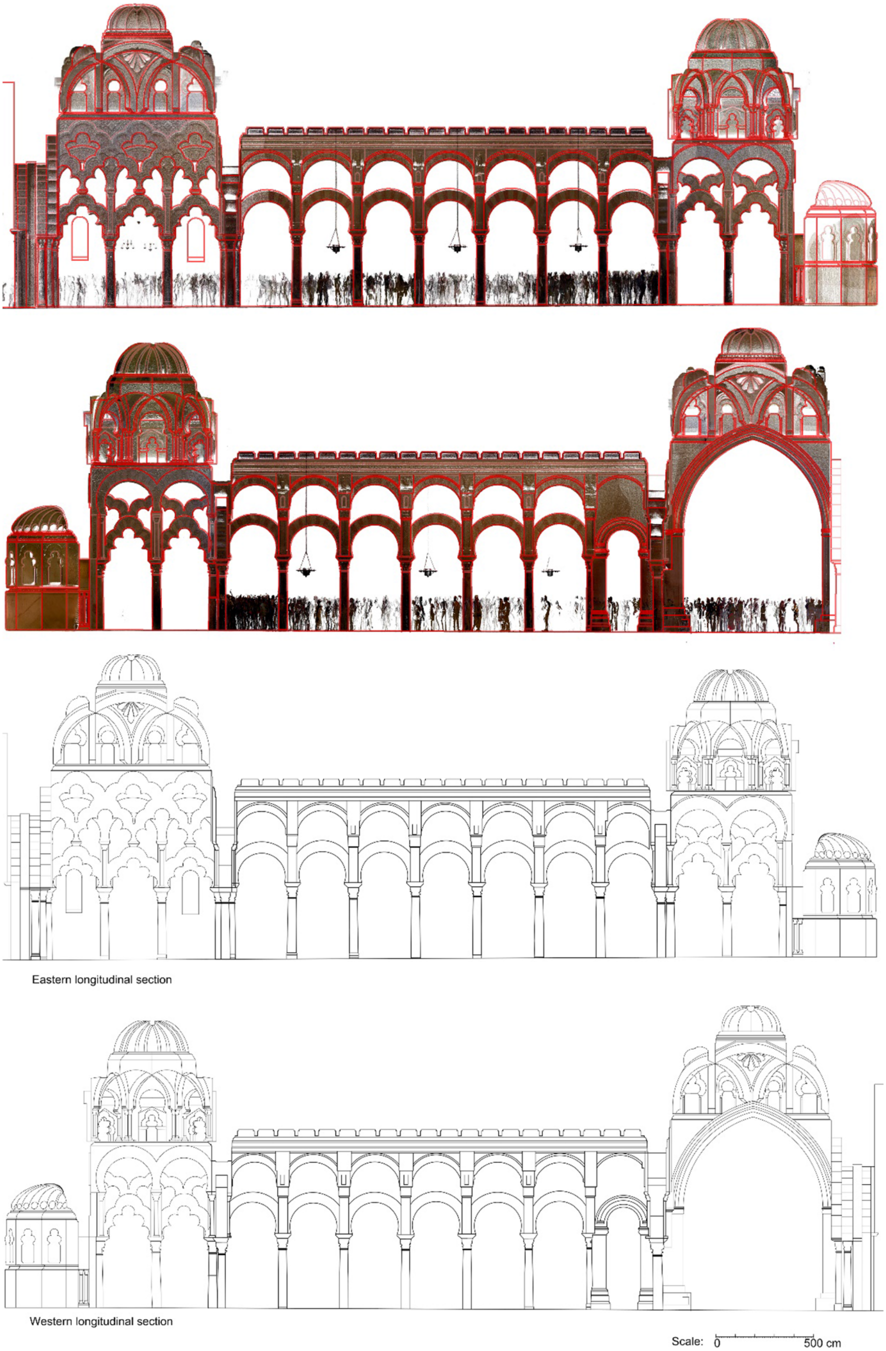

Fig. 3 Digital model on the point cloud and actual geometry. Central nave sections

by Rafael de la Hoz-Arderius[3] [Hoz], based on the relation 1.300, known as the *proporción cordobesa* (Doblado-González 2008; Hoz-Arderius 1991); and finally one developed by Antonio Fernández-Puertas[4] [Fernández], also around the area built by Abd al-Rahman I, which proposes that the monument was conceived and drawn using Pythagorean proportions (Fernández-Puertas 2008a).

Each of these theories was graphically transcribed using a geometric reconstruction, taking the intercolumn found in the area ordered by Al-Hakam II as a reference. The relations between the architectural elements in the theory by Camps were represented with a Cartesian modulated grid, in the theory by Hoz with slanted lines and numbered nodes to identify their relations, and in the one by Fernández with a sequence of geometric forms, a square that serves as the base for two equilateral triangles and an upper rectangle. The main aspects of each theory are detailed below.

Theory by Emilio Camps-Cazorla [Camps]

The book *Módulo, proporciones y composición en la arquitectura califal cordobesa* (Camps-Cazorla 1953) poses the existence of an arithmetic modular. It focuses on the primitive Mosque built by Abd al-Rahman I, where the author claims that "the arches of the primitive arcades, though following the Hispano-Visigothic tradition, possess however such a simple structure which does not allow any evolution, so they remain essentially unaltered in their subsequent repetitions."[5] The arcades are comprised of two levels of arches, the lower having horseshoe shaped buttress arches with a stilt height equal to 1/3 of the radius. This research considers stilt as the vertical displacement of the horseshoe arch center. This typology is also seen in some earlier Hispano-Visigoth arches. The upper level has semicircular arches of extra thickness, with modillions and mouldings.

Regarding the question of proportions, Camps considers two modules with no common divisor (see Fig. 4). The semicircular arches conform to the division of the total width in six equal parts, "module b", and the horseshoe arches in five, "module a", their radii being five and four times their respective modules (5b and 4a). The keystone width is in each case equal to these modules, and the front width of the columns three-fifths of "a" (3/5 a). Heights are related to the semicircular arc

[3] Rafael de la Hoz-Arderius, PhD Architect was born in Madrid on October 9th, 1924, he graduated in 1951 and died in the same city on June 13th, 2000. President of the International Union of Architects (UIA) from 1981 to 1985, he was also Fellow Member of the San Fernando Royal Academy of Fine Arts of Madrid since 1990. Besides a relevant work on architecture which he started to produce in Cordoba, his adopted homeland, he explained the so-called *Proporción cordobesa* theory (Gentil-Baldrich 2021: 896-897).

[4] Antonio Fernández-Puertas, PhD in History of Art, was born in Granada on August 2nd, 1950, and died in London on August 21st 2016. In 1981 he became Professor of Islamic Art History at the University of Granada and Director-Curator of the National Museum of the Alhambra from 1978 to 1992 (Gentil-Baldrich 2021: 651).

[5] *… los arcos de las arquerías primitivas, si por una parte están muy dentro de la tradición hispano-visigoda, por otra parte son de estructura tan sencilla que casi no admite evoluciones, y así se repiten sustancialmente inalterados en sus repeticiones posteriores…* (Camps-Cazorla 1953: 28, our Eng. trans.).

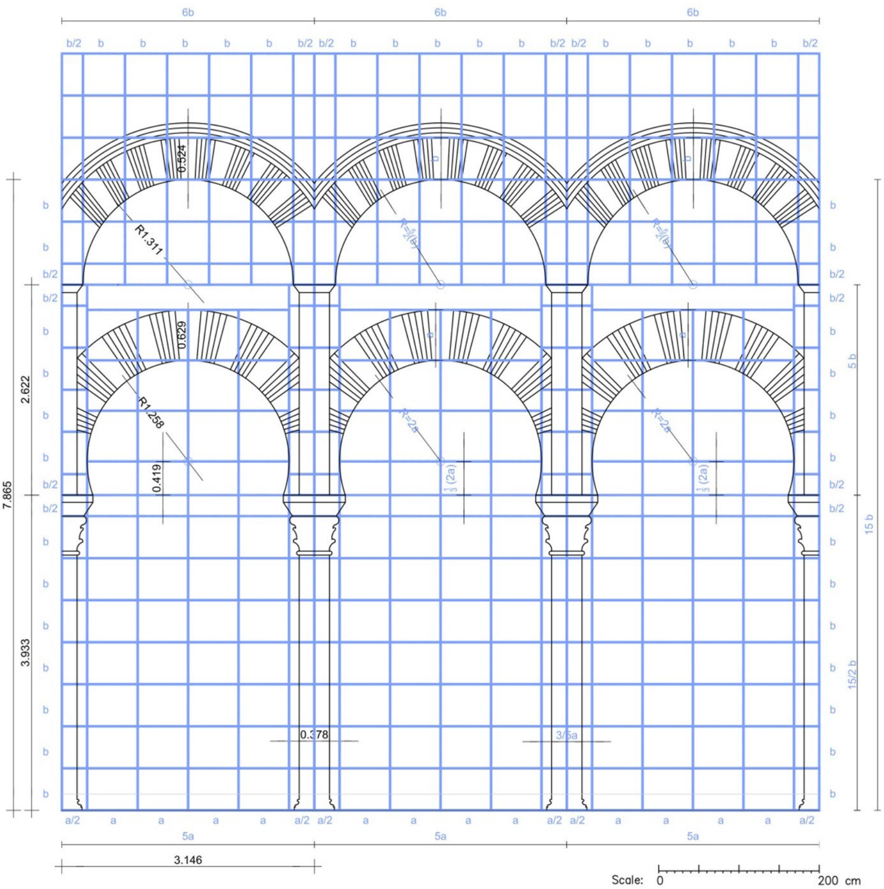

Fig. 4 Graphic transcription of the theory by Camps

diameter (5b) so that, taking as a reference the floor level, the total height is three times the latter (15b); between the imposts of the two levels of arches, just once (5b); and half of the total height up to the impost of the horseshoe arches (15/2 b).

Theory by Rafael de la Hoz-Arderius [Hoz]

This author's theory, raised in 1973 under the name of *proporción cordobesa*, considers of special interest the rectangle with relation $\sqrt{1+\sqrt{2}/2} \approx 1.307$, derived from the ratio between the side and circumradius of the octagon. Hoz claims that this pattern is present not only in the Mosque-Cathedral of Cordoba, but also in other contrasting buildings such as the Aqueduct of Segovia, Puerta de Alcala in

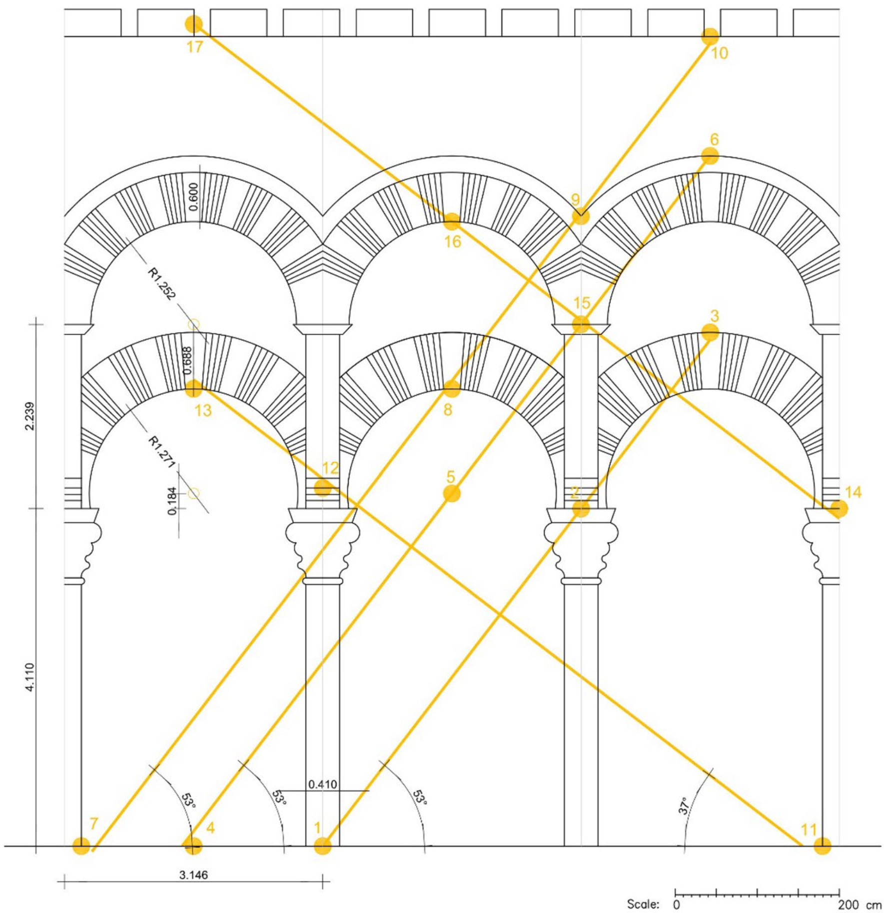

Fig. 5 Graphic transcription of the theory by Hoz

Madrid, the Pantheon of Agrippa in Rome, the Egyptian pyramid, or the Pyramid of the Moon in Teotihuacan, Mexico (Doblado-González 2008; Hoz-Arderius 1991).

Hoz represents the diagrams of proportions by "…two systems of tilted lines, perpendicular to each other symbolizing the diagonals of both families of rectangles…",[6] that is, vertically and horizontally. He refers to the arches of the Mosque-Cathedral in a general way without mentioning the study area: "… the arches also rise following an identical grid"[7] (Hoz-Arderius 1991). Their drawings reveal inaccuracy in their relations, as the lines represented do not exactly coincide with the nodes' centers (see Fig. 5). There is consistency between relations 1-2-3 and 4-5-15-6. Notwithstanding, it should be noted

[6] *…dos sistemas de rectas inclinadas, perpendiculares entre sí y correspondientes a las diagonales de ambas familias de rectángulos…* (Hoz-Arderius 1991, our Eng. trans.).

[7] *… las arquerías también se alzan dentro de una trama idéntica* (Hoz-Arderius 1991, our Eng. trans.).

that the floor level was modified in the restorations conducted in the 20th century and, furthermore, the relation between the horseshoe arch radius and stilt is not considered. It seems that there is a possible excess of relations, so that the satisfaction of some of them restricts others.

Theory by Antonio Fernández-Puertas [Fernández]

Fernández has accomplished numerous studies on proportions. In the case of the Mosque general floor plan, he considers "… the system of proportions under the whole Mosque of Cordoba was conceived and traced, it is the Pythagorean model of Ancient Greek and Rome classical era, which reached the Umayyad emirate in the 8th and 9th centuries, and then the Caliphate art, prevailing in the taifas, Almoravid, Almohad and Nasrid dynasties"[8] (Fernández-Puertas 2000). This system would be characterized by the use of set squares with 45º and 90º angles and with 30º, 60º, and 90º angles. Starting from the square, it is very simple to obtain a rectangle with a longer side equal to its diagonal, with a dimension $\sqrt{2}(\approx 1.414)$. By successively repeating this operation it results in relations of $\sqrt{3}(\approx 1.732)$, $\sqrt{4}(= 2)$, $\sqrt{5}(\approx 2.236)$, etc.

According to Fernández the floor plan in the area of Abd al-Rahman I extension in the Mosque comprises two squares. Its central nave is defined by the intersection of two arches with radii of the same length of that of the side of the square, and the aisles dividing the rest in five equal parts. The rectangular grid, supporting the column axes, is traced either through divisions in five parts or creating a grid drawn using rhombuses with a 60-degree set square (Fernández-Puertas 2008a). The extension floor plan commissioned by Al-Hakam II also shows a Pythagorean layout using a 30-degree set square (Fernández-Puertas 2000).

Fernández considers that the arcade follows the same system of proportions as the one applied to the floor plan (Fernández-Puertas 2008a), with a square resting on the floor level, with a side equal to that of the intercolumn (see Fig. 6). The height of the capital, which may vary, is obtained by drawing an arc with a radius half the diagonal (R1) and then by the vertical folding of the midpoint (R2). To define the impost line and stilt, an arc is placed on the latter, centered on the middle of the lower side of the square (R3) and the middle of the length of the diagonal (R4), respectively.

The interior of the horseshoe arch and the exterior of the rounded arch are determined by drawing two equilateral triangles with reversed vertices forming a rectangle with side $\sqrt{3}$ and diagonal $\sqrt{4}$. The height of the ceiling correlates to the intersection of two lines at 45°, whose height equals half the distance between the axes.

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[8] Fernández-Puertas considers that …*el sistema de proporciones con el que la mezquita de Córdoba entera fue concebida y trazada, y que es el pitagórico heredado de la Antigüedad Clásica grecorromana que llegó al emirato omeya de Córdoba de los siglos VIII y IX, y luego pasó al arte del califato, y pervivió bajo los taifas, almorávides, almohades y nazaríes* (Fernández Puertas 2000: 217, our Eng. trans.).

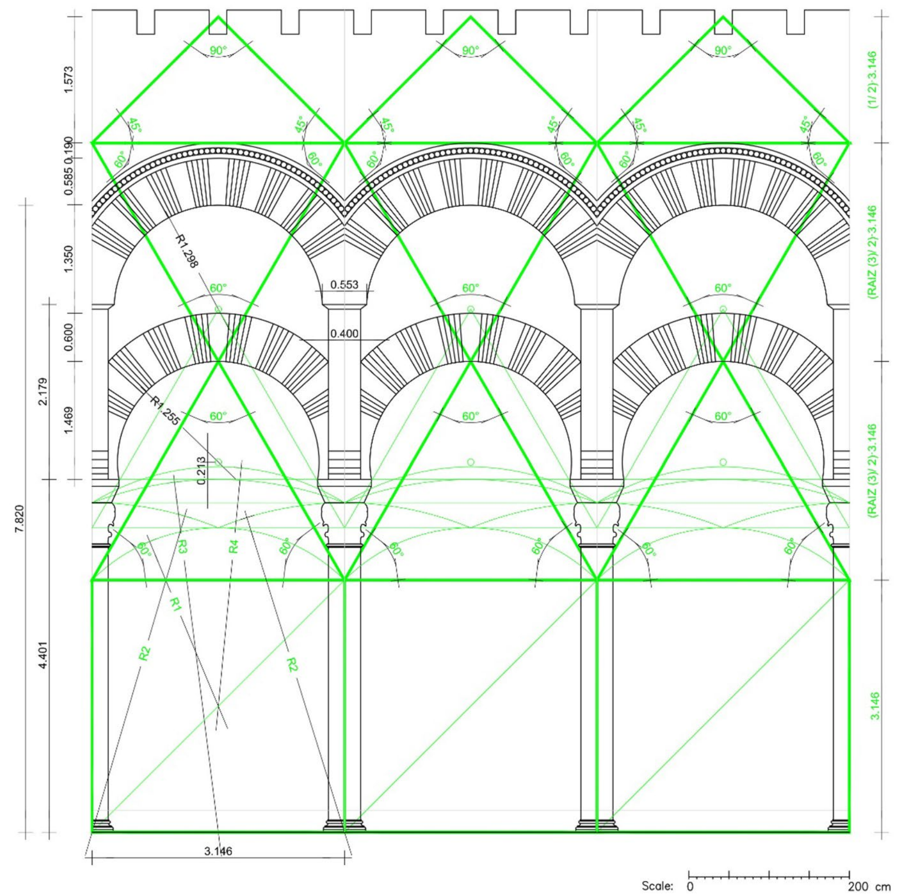


Fig. 6 Graphic transcription of the theory by Fernández

It should be noted that the proposed relationships are linked to the floor level, but as mentioned before, this surface has been subject to modifications through all the different stages of the monument's history. Likewise, an absence of relations between major composition and construction elements is observed: the radius and stilt of the horseshoe arch, and, above all, the relation between the intercolumn and the distance between the imposts of the two levels of arches.

Analysis of the Degree of Precision of the Different Theories

The arcade analyzed here has been graphically superimposed with the module of each of the three theories on proportions, for the purpose of assessing their precision (see Fig. 7). If the upper face of the cymatia and the distance between the columns

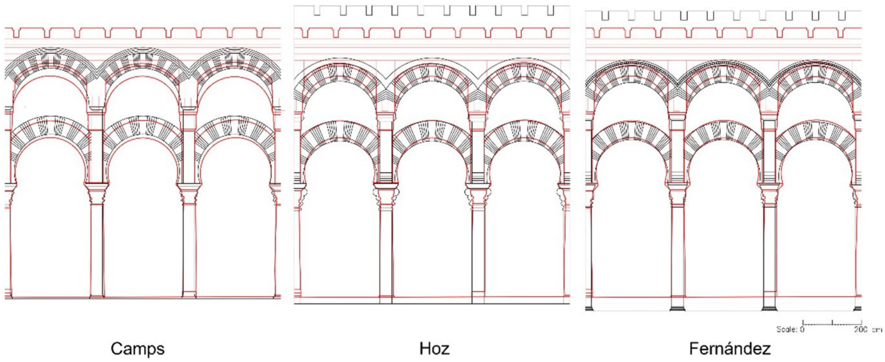


Fig. 7 Superposition of the transcription of the theories by Camps, Hoz and Fernández with a section of the mihrab nave

are made to coincide, significant misalignments can be observed. Additionally, there are differences between the floor and ceiling levels – which determine the total height – in Hoz and Fernández and the springing line in the horseshoe arches. The same occurs with the distance between the imposts of the two levels of arches, whose difference is notable, above in Hoz and Fernández, and below in Camps.

It should be noted that for the analysis of the three theories the reference used has been the intercolumn of the arcades in the Al-Hakam II extension, which feature a mean width of 315 cm. This has made it possible to establish a common basis of comparison, with specific dimensions for each element. The definition of a mean value of the geometric model, obtained from the precise digital model, facilitated the elaboration of a table which allowed the degree of dimensional precision to be quantified (Table 1).

According to the data, the resulting space from the theories raised by Camps and Fernández should be of a slenderer nature, even considering that in the second case the lower part of the columns is now partially buried. When the height between the imposts of both levels of arches is analyzed, all the theories present differences due to excess or fault. Their proportions, such as the relation between the distance of the intercolumns, do not coincide to the following degree; in Camps (6/5 = 1.200), Mean Value (1.294), and in Hoz and Fernández (1.405 and 1.443, respectively).

The same applies to the springing height of the horseshoe arch, which shows quite significant variations due to excess in Hoz and Fernández (23.5 cm and 52.5 cm, respectively). These differences can also be observed in the relation with the intercolumn width: Median Value (1.231); Camps (1.250); Hoz (1.306); and Fernández (1.399).

The drafting of both horseshoe and semicircular arches is quite similar in all the cases. Notwithstanding this, the average stilt, which accordingly to several authors is 1/3, as in the Visigoth era (Chueca-Goitia 1965; Camps-Cazorla 1953; Nieto-Cumplido 2001), is quite dissimilar in the theories by Hoz and Fernández (1/7 and 1/6, respectively).

Table 1 Dimensional analysis: mean value and theories of proportions

Mihrab Nave	Distance between axes	Horseshoe arch					Semicircular arch		Heights		
		Intrados radius (r)	Stilt (s)	Relation (s/r)	Keystone height	Column diameter	Intrados radius	Keystone height	Total height	Height between imposts	Impost height horseshoe arch
Mean value (3D Geometric Model)	315	120	39	1/3	56	44	130	47	753	243	387
Camps	315	126	42	1/3	63	38	131	52	787	262	393
	5a = 6b	2a	1/3(2a)	1/3	a	3/5(a)	5b/2	b	15b	5b	15/2 b
Hoz	315	127	18	1/7	69	41	125	60	760	224	411
Fernández	315	126	21	1/6	60	40	130	59	782	218	440

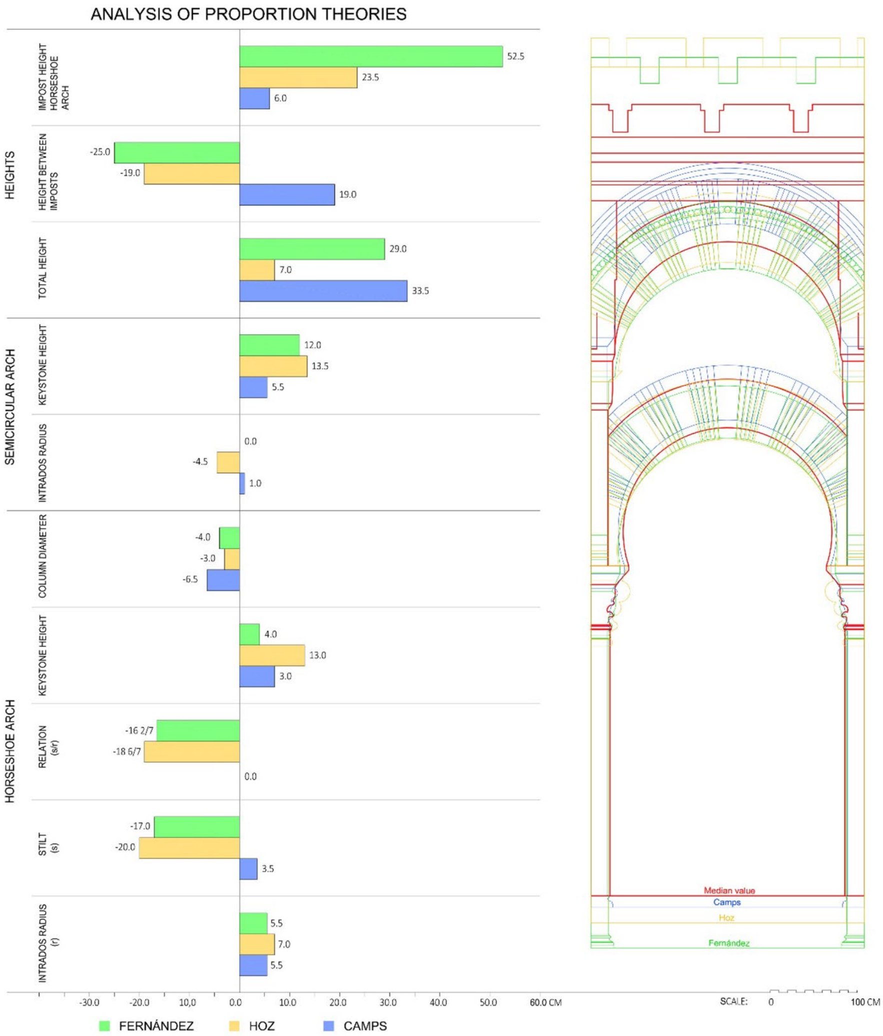


Fig. 8 Analysis of the degree of precision of the three theories on proportions (Camps; Hoz; Fernández)

Therefore, the lack of convergence between the three theories is clear. The quantification of the deviations evidences that the larger difference with respect to the Mean Value correspond to Fernández, where the closest is that by Camps. As the differences are excessive, none of these theories can be applied to the arcades of the Al-Hakam II extension (Fig. 8).

Analysis and Quantification of Deformations

This research considers deformations as dimensional variations suffered by an element, which may be due to construction technicalities, subsequent restorations, other transformations, or simply time. To achieve their dimensional analysis, each row of arches was labelled as "E" or "W" (east or west view) and each column identified with a number, preceded by "e" or "w" depending on their location view: east (e) or west (w). The analyzed variables match the ones defined in the theory by Camps, and the floor level was taken into account in order to analyze its slope.

The Mean Value of the geometric model, obtained as an arithmetic mean of each variable, results in an archetypical row of arches which can be graphically depicted. There are two values for the radii of the upper arches – E78 and W78 – which were not considered in the calculation due to their excessive difference to the rest (Table 2; Fig. 9).

The analysis of the floor level in the east and west arcades reveals values above and below the mean value as there is a deformation or difference of 8.0 cm between both ends of the arcade.

The average distance between the column axes is 315 cm and their diameter 44 cm, presenting a difference of –4.0 and + 3.0 cm in the first one, and –5.0 and + 4.0 cm in the second, under and over the value, respectively. It is important to highlight here that it has been mistakenly claimed that all columns in the Al-Hakam II extension "have around 64 cm of diameter, and a distance between axes of 2.70 m. approximately, without bases, they were specially carved".[9]

The relation between the stilt height and the average radius is 1/3, as shown in seven of the thirteen arches. However, the stilt ranges between + 9.0 cm (over the value) in W45 and –6.0 cm (under the value) in E56 with respect to the mean value of 39 cm.

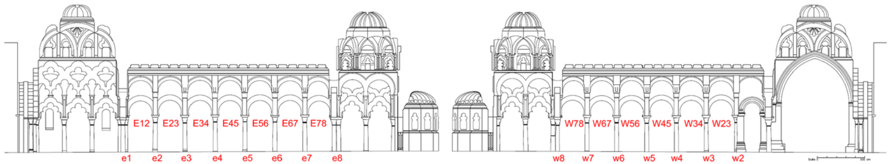
The semicircular arch shows major deformations in its radius affecting E78 and W78, respectively 19.0 and 12.5 cm under the value in relation to 130 cm. This significant reduction reflects the need to solve the intersection with the perpendicular arches perpendicularly adjacent to the mihrab.

The floor slope affects the height of the horseshoe arch spring line. Its relation with the intercolumn width ranges between 1.202 and 1.255 (with mean value of 1.231), with five arches exceeding this relation. This height, subtracting the floor's difference level, allows the highest point of the cymatia to be obtained, which only shows differences of 4.4 cm above the value in column e5 and 4.8 cm below the value in w6.

The variable of greater construction and composition importance, independent to the floor level and ceiling height, is the height between the imposts in the

[9] When referring to columns, capitals, and cymatia in the extension of Al-Hakam II, Nieto-Cumplido observes: … *Todas las columnas de esta ampliación, de unos 64 cm. de diámetro, a una distancia entre ejes de 2.70 m. aproximadamente, también sin basas, se labraron expresamente para ella* (Nieto-Cumplido 2001: 210, our Eng. trans.).

Table 2 Dimensional analysis of the mihrab nave arcades



Mihrab Nave	Inter column or column	Floor level	Distance between axes	Horseshoe arch					Semicircular arch		Heights		
				Intrados radius (r)	Stilt (s)	Relation (s/r)	Keystone height	Column diameter	Intrados radius	Keystone height	Total height	Height between imposts	Impost height horseshoe arch
3D Geometric model from point cloud	E12 or e1	11	313	121	33	1/4	58	46	131	48	757	250	393
	E23 or e2	10	311	119	40	1/3	57	46	134	43	761	244	391
	E34 or e3	9	316	120	37	1/3	57	41	131	44	757	241	390
	E45 or e4	7	315	120	33	2/7	60	44	127	44	755	242	392
	E56 or e5	6	318	121	35	2/7	60	45	129	48	752	245	388
	E67 or e6	5	313	119	41	1/3	60	48	131	51	748	241	383
	E78 or e7	4	314	114	35	1/3	59	47	111	55	743	241	386
	e8	3						44					385
	W23 or w2	10	316	122	43	1/3	53	39	131	47	750	242	391
	W34 or w3	9	316	121	42	1/3	52	44	130	47	750	242	389
	W45 or w4	8	313	121	48	2/5	52	44	128	43	753	244	385
	W56 or w5	6	317	122	40	1/3	52	44	127	45	755	245	382
	W67 or w6	5	313	121	36	2/7	54	45	131	47	754	242	384
	W78 or w7	4	315	121	40	1/3	51	42	117	46	754	241	385
	w8	3						42					386
	Mean value	7	315	120	39	1/3	56	44	130	47	753	243	**387**

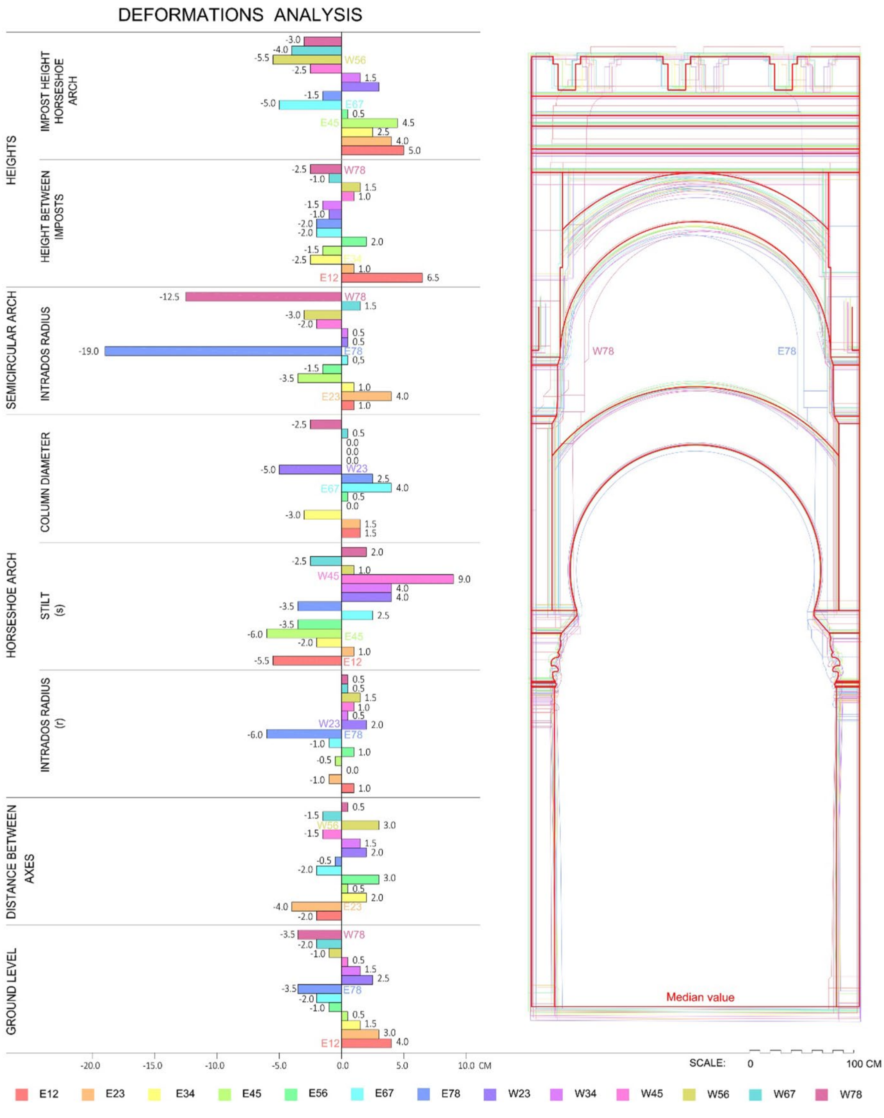


Fig. 9 Analysis and quantification of deformations

two levels of arches (between 241 and 250 cm). It presents deformations of up to 6.5 cm in E12, which indicates a relation with the intercolumn width between 1.252 and 1.314; and as such six arches exceed 1.300, resulting in 1.294 as the mean value.

Conclusions

To analyze the dimensions of the arcades in the central nave central of Al-Hakam II expansion at the Mosque-Cathedral of Cordoba, this research accomplished its own digital geometric model from the point cloud obtained with a 3D laser scanner. Subsequently, a meticulous graphic transcription of the three major theories on arcades proportions was conducted to compare them through their superimposition over the rigorous model of the survey in its current state.

This first approach has proved that the graphic results of these three theories show different patterns between them, and they also differ from reality. The highest point of both the floor and the ceiling display major differences in Camps and Fernández. It must be noted here that both elements have suffered major transformations; in the 18th century plaster vaults and skylights were added, later dismantled in restorations carried out in the 20th century, when floorings were also modified.

In other respects, the average stilt of the horseshoe arches, 1/3 according to Camps, does not correspond with the one obtained in the theories of Hoz and Fernández. The same occurs in the relation between the distance of the intercolumn and the height of the imposts line in the two levels of arches, whose mean value is 1.294, thus being almost square in Camps (6/5 = 1.200), and greater in Hoz and Fernández (1.405 and 1.443 respectively). All this indicates that the theories on proportions regarding the area commissioned by Abd al-Rahman I do not apply to the Al-Hakam II's Mosque extension.

Furthermore, the digital model allowed the major and current deformations of the analyzed arcades to be quantified. It can be affirmed that the average diameter and distance between columns is 44 and 315 cm, respectively, and not 66 and 270 as indicated in the aforementioned publications. Furthermore, the significant differences in the altimetric level of the cymatia, 4.5 cm above the value and 5.0 cm below it, are quite revealing. The major deformation corresponds to the intercolumns in the south end, where the radius of the upper semicircular arch is reduced 19.0 and 12.5 cm in the east and west arcade, respectively, in their intersection with the perpendicular row of arches. Another key fact, not considered so far in the scientific literature on the Mosque-Cathedral, is the slope of 0.4% between both ends of the arcade, with a total difference in height of 8.0 cm in the sections analyzed, decreasing southwards.

Finally, from the analysis of the theories of proportions and the rigorous survey carried out, there is evidence to suggest the existence of a relation of 1.3 between the width of the intercolumn and the height between the impost lines of the two levels of arches. This relation is the one displaying the greatest compositional and constructive importance found so far (referred to as "main proportion" in to Fig. 10) since it does not depend on the transformations or restorations of the floor elevation and ceiling height. Its dimensional analysis reveals a mean value of this relation of 1.294, though 6 of the 13 surpass the relation of 1.300. It is currently unknown why the arch designers would have used this relation related to the classical or Pythagorean proportions discussed by Fernández (Fig. 11a) and the theories of

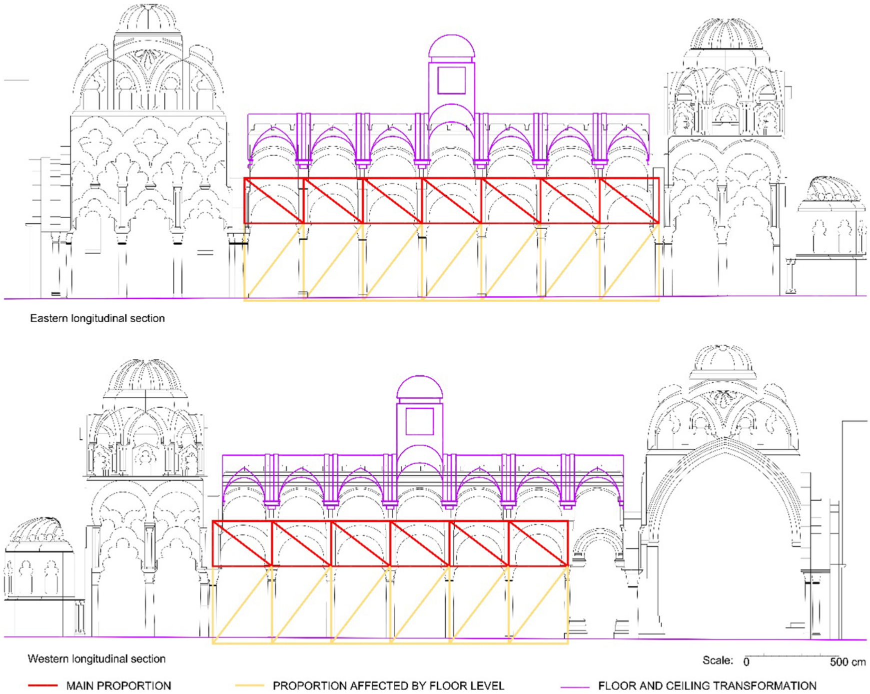


Fig. 10 Relations in the arcades of the central nave, Al-Hakam II extension

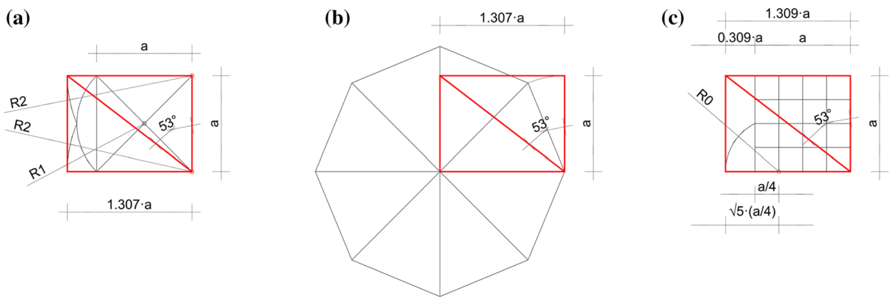


Fig. 11 Outlines showing proportions close to 1.3. **a** Fernández (1.307), **b** Hoz (1.307), **c** another simple method (1.309)

proportions based on the number 1.307 which Hoz called *proporción cordobesa*. In the first case, Fernández applies this relation to obtain the capital height from a square and the tracing of two arcs (R1 and R2); and in the second (Fig. 11b), Hoz derives it from the ratio between the circumradius and side length of a regular octagon. However, there is another simple method (Fig. 11c) based on the tracing of an arc (R0) with the center to ¼ of the side, from which an approximate relation of 1.309 is obtained.

Another relation affected by the grade of slope of the floor, and the hypothesis that part of the columns is buried, is the height of the spring line of the horseshoe arch. Its ratio with the intercolumn width has a mean value of 1.231, though five arches slightly surpass this relation and could reach the 1.300 proportion when considering the architectonic transformations already mentioned.

Finally, after superimposing and comparing the theories of Camps, Hoz, and Fernández with the digital model created for this study and quantifying the deformations which were not previously considered in the related scientific literature, it is apparent that not all the arcades are identical. No metric pattern is repeated in their different areas and thus no general theory on architectural proportions can be deduced for the arcades of this singular monument.

This research foresees carrying out new surveys in the Abd al-Rahman I zone, and in the extensions built by Abd al-Rahman II and Almanzor to further analyse the theories of proportions mentioned, as well as studying the metrology and measurement units at the construction level to enable a better understanding of the geometry and proportions of the Mosque-Cathedral of Córdoba.

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