

Implementation of a Hybrid Classical-Quantum Annealing Algorithm For Logistic Network Design

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The logistic network design is an abstract optimization problem that, under the assumption of minimal cost, seeks the optimal configuration of the supply chain's infrastructures and facilities based on customer demand. Key economic decisions are taken about the location, number, and size of manufacturing facilities and warehouses based on the optimal solution. Therefore, improvements in the methods to address this question, which is known to be in the NP-hard complexity class, would have relevant financial consequences. Here, we implement in the D-Wave quantum annealer a hybrid classical-quantum annealing algorithm. The cost function with constraints is translated to a spin Hamiltonian, whose ground state encodes the searched result. As a benchmark, we measure the accuracy of results for a set of paradigmatic problems against the optimal published solutions (the error is on average below 1%), and the performance is compared against the classical algorithm, showing a remarkable reduction in the number of iterations. This work shows that state-of-the-art quantum annealers may codify and solve relevant supply-chain problems even still far from useful quantum supremacy.

I. INTRODUCTION

Calculating the global minimum (maximum) of a multi-variable function is in general arduous, especially when the number of variables and constraint conditions grows and the objective function is highly non-linear. The problem is in the NP-hard complexity class since it is complicated even to verify whether a given solution is the optimal one. As a branch of the optimization problem, the logistic network design problem (NDP) covers a massive set of decision-making problems for management issue [1], e.g., where to place facilities and how to assign customers minimizing the total cost, or how to redistribute driving paths of vehicles to reduce traffic jams. There is an endless list of classical algorithms for optimization problems, e.g., branch and bound, context partition and dynamical programming, metaheuristic algorithm based on a single solution such as hill-climbing, simulated annealing [2], and tabu search [3, 4], or intelligent optimization by genetic algorithms [5], ant colony optimization [6, 7], and artificial neural networks [8]. This algorithm has been applied to study NDP and provided some preliminary results [9–12]. However, these algorithms' mathematical principles for finding global minima are not systematically established and, in most cases, it requires experience adjusting parameters. This raises the demand for developing an interpretable algorithm for

solving logistic NDPs efficiently.

Quantum annealing is an optimization technique especially suitable for satisfiability problems, which makes use of quantum tunneling of potential barriers to enhance the performance of the classical algorithm [13, 14]. The ground state codifying the solution of the problem is attained expectedly employing a shorter annealing time than the classical algorithm and afterward decoded to achieve the optimal solution with respect to the objective function [15]. Current D-Wave cloud quantum annealer comprises 2048 qubits distributed in a hardware architecture according to the Chimera graph. The constraints imposed by the architecture generally allow for codifying only relatively small problems, which can be enhanced when combined with classical algorithms. Additionally, this quantum device is affected by thermal fluctuations, decoherence, and I/O errors, which reduces the signal-to-noise ratio and consequently prolongs the computation time due to the extra sampling required for canceling the noise (otherwise, the accuracy of the solution would be affected). Nonetheless, quantum annealers have proven their capability to codify hard problems in different fields, such as condensed matter physics [16–20], engineering [21], cryptography [22, 23], biology [24] and finance [25–27], among others. This shows that current quantum annealing technology, although incoherent and still far from reaching useful quantum advantage, is ready to implement relevant small-scale optimization problems.

In this Article, we experimentally address a set of paradigmatic logistic NDPs employing a hybrid classical-quantum annealing algorithm, showing a remarkable accuracy (less than 1% error) despite the device incoherence and better performance in the number of iterations with

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respect to the classical one (about one-tenth, when classical annealing algorithm can reach global minimum with adequate hyperparameters). Inspired in Ref. [30], we propose an alternative approach, which we call *combined quantum annealing algorithm*, that makes use of two layers with feedback-control interaction between them. The approach is tested in 12 paradigmatic logistic NDPs employing both the simulator and the D-Wave cloud quantum annealer, achieving the aforementioned remarkable results when compared with the classical ones. This supports the extended idea that a hybrid quantum-classical algorithm will allow us to enlarge the class of solvable problems with quantum computers (quantum annealers), accelerating the development of quantum information processing tasks in quantum technologies.

This manuscript is organized as follows: in Sec. II, we formulate the logistic NDP as a constrained 0 – 1 programming problem, which will afterwards allow us to map it into D-Wave’s Chimera architecture. In Sec. III, we introduce the fundamental elements of quantum annealing and of the combined quantum annealing algorithm. Afterwards, in Sec. IV, we experimentally tests the optimization of logistic NDPs by comparing the results against the best known classical ones, as well as those given by a classical algorithm for finding global minimum. Finally, in Sec. V, the results are analyzed and possible alternative software and hardware approaches for further enhancement are discussed. The conclusions are finally listed in Sec. VI.

II. LOGISTIC MODEL

Although a logistic NDP could be described in an abstract framework, we choose the customer-facility picture because it is illustrative. Let us assume that there are at most m sites for potential facilities and n customers to be allocated. The indices $J = \{1, 2, \dots, m\}$ and $I = \{1, 2, \dots, n\}$ denote the set of potential location sites and the set of customers, respectively. It costs f_j to build a facility with production capacity v_j placed at site $j \in J$. When a customer $i \in I$ with product demand d_i is served by facility j , the transportation process brings c_{ij} to the cost. One has to find a way to allocate customers with a minimum total cost that ensures all customers are served, and none of the facilities is overflowed. To formulate the model, one has to minimize the following cost function

$$\text{cost}(x_j, y_{ij}) = \sum_j f_j x_j + \sum_i \sum_j c_{ij} y_{ij}, \quad (1)$$

where x_j and y_{ij} are binary variables that represent the allocation configuration. A facility is built at site j when $x_j = 1$, and obviously, customers can only be served from sites with facilities. Accordingly, $y_{ij} = 1$ means customer i is assigned to the facility in site j , and other facilities are no longer available for this customer. These constraints

can be expressed as

$$\sum_j y_{ij} = 1, \quad \forall i \in I, \quad (2)$$

$$\sum_i d_i y_{ij} < v_j, \quad \forall j \in J, \quad (3)$$

$$y_{ij} \leq x_j, \quad \forall i \in I, \quad \forall j \in J. \quad (4)$$

Minimizing the cost function under the constraints above is proven to be an NP-hard problem, i.e., obtaining its global minimum value is computationally challenging and highly time-consuming. Additionally, it is also hard to verify if a given network is the solution that minimizes the cost function. These features lead to the demand for specific algorithms that accelerate the searching process and enhance the solution’s quality.

III. COMBINED QUANTUM ANNEALING ALGORITHM

The solution of satisfiability problems can be codified in the ground state of a problem spin Hamiltonian H_{pro} [28]. Quantum annealing is a quantum algorithm based on the adiabatic theorem [29], which ensures that if we start in the ground state of a known Hamiltonian H_0 , by slowly modifying a parameter t transforming H_0 into H_{pro} , the system remains in its ground state, providing us with the solution for the satisfiability problem. For example, in a spin-1/2 annealer, the total Hamiltonian is split into a tunneling Hamiltonian and a problem Hamiltonian, which codifies the solution for the problem,

$$H = \left(1 - \frac{t}{t_f} \right) \sum_i \hat{\sigma}_i^x + \frac{t}{t_f} \left(\sum_i h_i \hat{\sigma}_i^z + \sum_{i>j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z \right).$$

The system is supposed to be in the ground state of the problem Hamiltonian when $t = t_f$. If we introduce the qubit operator \hat{x} with eigenvalues 0 and 1, such that $\hat{x}|0\rangle = |0\rangle$ and $\hat{x}|1\rangle = |1\rangle$, respectively, *quadratic unconstrained binary optimization* (QUBO) problem can be mapped to a spin-1/2 quantum annealing problem by $\hat{x} = (1 + \hat{\sigma}^z)/2$.

Looking at the cost function given by Eq. (1), we understand that it cannot be directly optimized by quantum annealing since there are constraints. The cost function and all constraints are linear, so they are all relatively easy to be converted to a QUBO formulation. For example, Eq. (4) can be satisfied by introducing penalty terms in the form $\alpha(x_j - y_{ij} - z_{ij})^2$, where z_{ij} are auxiliary binary variables. One can verify that the penalty is unavoidable only when $x_j = 0$ and $y_{ij} = 1$. With an effective QUBO formulation satisfying Eq. (2) and Eq. (3), we propose a hybrid quantum-classical algorithm inspired in

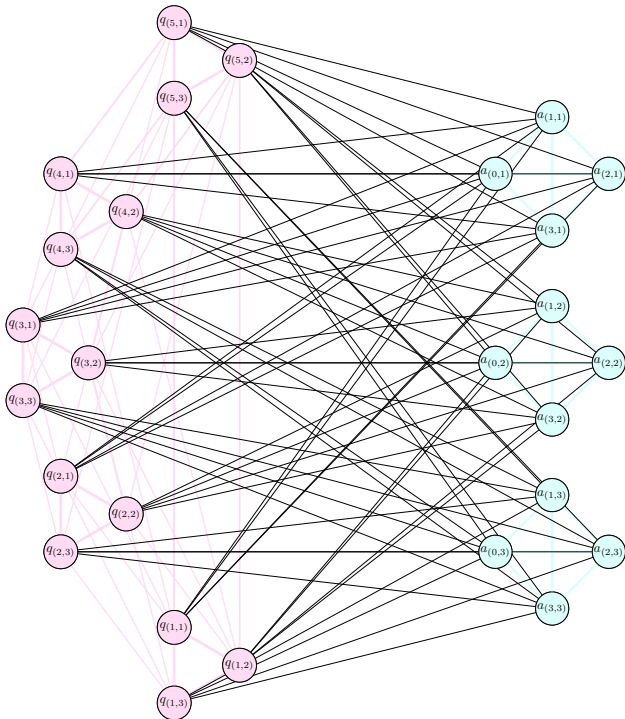


FIG. 1. A scheme of how binary variables and constraint conditions are encoded by logical and ancilla qubits. Here, we represent a simple NDP problem comprising 3 facilities and 5 customers, as an illustrative example, while different colors and thicknesses classify the couplers according to their properties.

Ref. [30] for obtaining the global (or quasi-global) minimum. We apply a combined quantum annealing algorithm comprising two layers, namely, the outer and the inner layers. In the former, a simulated annealing algorithm performs the optimization for facilities' locations while, in the latter, the quantum annealing process runs.

In the outer layer, a list with elements x_j , sorted by index j in ascending order, denotes the neighboring configuration of facilities, while a neighboring function operates on the list to generate a new configuration in three ways: (i) pick a x_j with value 1, and set it to 0, which means a facility is randomly closed; (ii) pick a x_j with value 0, and set it to 1, which means a facility is randomly built on a site; (iii) swap the values of x_j and x_k , if their values are different, which means a facility moves to another site. Dice will be rolled to decide which operation is carried out by the neighboring function on the list, while all the operations should be allowed, e.g., when all facilities are open, operation (ii) is no longer available. In the inner layer, we perform quantum annealing to minimize $\sum_i \sum_j c_{ij} y_{ij}$ under constraint given by Eq. (2), Eq. (3), and Eq. (4). As we mentioned before, the latest quantum annealer is designed to solve QUBO problems, i.e. to minimize the objective function $\text{obj}(\mathbf{x}) = \mathbf{x}^T \mathbf{Q} \mathbf{x}$, that Q is the QUBO matrix and \mathbf{x} is the binary vector. We map binary variables y_{ij} to qubit $q_{(i,j)}$, and allowed j are

the sites on which facilities are built. In order to introduce the constraint given by Eq. (2), we employ weighted penalty functions, with a reasonable λ_i , yielding an effective unconstrained Hamiltonian for customer i , e.g. $\lambda_i (\sum_j q_{(i,j)} - 1)^2$, which guarantees that customer i is associated only to one facility. Accordingly, Eq. (3) may also be written as $\mu_j (\sum_i d_i q_{(i,j)} + \langle \mathbf{2}, \mathbf{a}_j \rangle - v_j)^2$ for facility j , with slack variables encoded by ancilla qubits. Here, $\langle \mathbf{2}, \mathbf{a}_j \rangle$ denotes the binary expansion $\sum_{l=0}^k 2^l a_{(l,j)}$, which implies that the number of ancilla qubits required to introduce the constraint is $k_j = \lceil \log_2 v_j \rceil$. One can verify that any violation of Eq. (2) and (3) introduce extra squared cost, being scaled by λ_i and μ_i . For example, if customer i is not assigned to any facility, i.e., $\sum_j q_{(i,j)} = 0$, the violation of Eq. (2) punishes a cost of λ_i , which should be larger than the profit from saving the transportation cost c_{ij} . The problem Hamiltonian is hence given by

$$H_P = \sum_i \sum_j c_{ij} q_{(i,j)} + \sum_i \lambda_i \left(\sum_j q_{(i,j)} - 1 \right)^2 + \sum_j \mu_j \left(\sum_i d_i q_{(i,j)} + \langle \mathbf{2}, \mathbf{a}_j \rangle - v_j \right)^2, \quad (5)$$

which could be mapped to a solvable spin-1/2 Hamiltonian for the annealer. The ground state of the problem Hamiltonian is supposed to be the configuration that minimize the classical objective function with reasonable penalty coupling strengths λ_i and μ_j .

This combined quantum annealing algorithm works as follows: (i) Set the optimal cost to infinity and generate an initial configuration as the optimal configuration of facilities. Schedule the annealing process, i.e., the cooling rate, initial temperature, final temperature, etc. (ii) Flip the values of variables x_j in list by neighboring function and obtain a new configuration \tilde{x}_j . (iii) Perform the quantum annealing process according to the neighboring configuration, decode the state of qubits to \tilde{y}_{ij} , and calculate value of the cost function $\text{cost}(\tilde{x}_j, \tilde{y}_{ij})$. (iv) Apply Metropolis algorithm such that, if $\text{cost}(x_j, y_{ij}) > \text{cost}(\tilde{x}_j, \tilde{y}_{ij})$, we accept the new configuration and value as the optimal ones and go for next step. Otherwise, randomize $\rho \in (0, 1)$, if $\rho < \exp(-(\text{cost}(\tilde{x}_j, \tilde{y}_{ij}) - \text{cost}(x_j, y_{ij}))/T)$, we also accept them and continue. We go for the next step without operations if the new configuration and value are denied. (v) Increase the iteration index by one and check if it meets the upper limit. Once it is larger than the maximum iteration number, we adjust the annealing temperature according to the schedule, reset the iteration index, and go on for the next step. Otherwise, return to step (ii). (vi) Output the optimal solution if the current temperature is not higher than the target temperature. Otherwise, return to step (ii).

Thus, if all parameters in the outer and inner layers were correct and the quantum annealer was noiseless, the places for building facilities, the allocation of customers, and the optimized total cost would be obtained by this

combined annealing algorithm. In any case, error correction can be applied to provide a quasi-optimal solution, which is a relevant advantage since we are employing a noisy and incoherent quantum annealer.

IV. EXPERIMENTS

We choose the same twelve problems tested by Ref. [30], which are open-source NDP test problems from OR-Library [31]. The optimal solution is given by the author using *Lindo* software. We encoded $m \times n$ logical qubits by $q_{(i,j)}$ and $\sum k_j$ ancilla qubits by $a_{(l,j)}$ for NDP problem with m potential sites for building facilities with capacities v_j and n customers to be served. The connectivity of these qubits is high, with a number of couplers approximately given by

$$\frac{(m-1)mn}{2} + \frac{(n-1)nm}{2} + \sum_j \frac{(k_j-1)kn}{2} + \sum_j nk_jm, \quad (6)$$

where terms denote the numbers of couplers between logical qubits given by Eq. (2), Eq. (3), couplers between ancilla qubits and couplers that link an ancilla qubit to a logical qubit (see Fig. 1), respectively. The minimal number of qubits for solving this problem is $n \times m + k \times m$, if the structure of the quantum device is exactly the same as we showed in Fig. 1. Notice that this number could substantially grow if qubits are connected differently from the problem's connectivity graph since additional ancillary qubits are required to implement minor embedding. The smallest problem for testing requires 1056 qubits with 40320 couplers, which cannot be directly implemented in the 2048-qubit D-Wave quantum annealer due to its limited connectivity (6016 couplers). The open-source software *qbsolv* developed by D-Wave [32] allows for splitting a large QUBO problem into smaller embeddable sub-problems, which can subsequently be solved in either a local simulator with tabu algorithm or the real quantum annealer under authorized license. The QUBO problem is submitted to a server for queueing, and the result is retrieved after the annealing is performed. Although the computing time in a real annealer is negligible, one trial of solving a large QUBO matrix could be very time-consuming (about 15 minutes for a 900×900 QUBO matrix) due to the queueing time. We also emphasize that x_j vanishes since we employ a hybrid algorithm instead of a standard approach. In the inner layer where quantum annealing performs, we solve a subproblem as follows: how customers should be assigned according to a given facility configuration? Some variables y_{ij} are constantly zero if there is no facility built at site x_j . In our practice, these variables are already removed in each quantum annealing process, making embedding or solving by *qbsolv* easier and more efficient. Thus, in each iteration that generates a new facility configuration, the number of logical qubits required is $\tilde{m} \times n$ with a maximum of $m \times n$, where \tilde{m} denotes the number of facilities

Problem	Size	Lindo	SA	QA
cap71	16 × 50	932615.7500	1460909.750	933172.1000
cap72	16 × 50	977799.4000	1395389.538	977988.1000
cap73	16 × 50	1010641.450	1585875.550	1010641.450
cap74	16 × 50	1034976.975	1390963.787	1034976.975
cap101	25 × 50	796648.4400	1182235.563	797656.2875
cap102	25 × 50	854704.2000	1282306.175	854952.5125
cap103	25 × 50	893782.1125	1395701.200	894872.1125
cap104	25 × 50	928941.7500	1458550.450	928941.7500
cap131	50 × 50	793439.5620	1167543.950	796066.6500
cap132	50 × 50	851495.3250	1132436.300	852291.9375
cap133	50 × 50	893076.7120	1126423.238	893521.4125
cap134	50 × 50	928941.7500	1321380.713	928941.7500

TABLE I. The combined quantum annealing algorithm is tested by 12 NDP problems from OR-Library while the results are listed under QA. The optimal solutions are compared with the global optimal solutions from *Lindo* given by the author of Ref. [30] and classical simulated annealing algorithm with the same annealing schedules and iteration numbers denoted by SA. In our experiments, the combined quantum annealing algorithm outperforms simulated annealing as a well-known metaheuristic algorithm massively under the same iteration numbers.

being built.

The initial temperature of simulated annealing is set to be 10000, which is scaled to a reasonable value considering the deviation between the new and old value of the cost function. For simplicity, we choose $\alpha = 0.5$ as cooling rate and the schedule as scale cooling. The maximum iteration number is m , and the target temperature is 1. To obtain the optimal solution according to a given configuration of facilities, the penalty strength should be sufficiently strong, i.e., it should ensure that the problem Hamiltonian's ground state satisfies all constraint conditions. Hence, if the quantum annealer were noiseless, the parameters would follow $\lambda_j \gg \mu_i \gg c_{ij}$, which ensures that every customer is assigned to only one facility, none of the facility is overflowed, and the configuration corresponds to the minimum total cost. However, in practice, the quantum device is affected by thermal fluctuation, inaccuracy of magnetic field tunneling, and energy excitation by nonadiabatic effects. Thus the result in general satisfies the constraints, but the solution is not optimal. Consequently, the penalty strength is set slightly larger than c_{ij} for the possibility of obtaining an optimal solution. Although sometimes constraints are also not fulfilled due to the noise, this could be corrected by repeating the quantum annealing process until every constraint is satisfied. After studying the dataset, the penalty strength λ_i are set to be $\min_j(c_{ij})$, while μ_i are almost negligible considering the demand and capacities.

The results are presented in Table. I, while the hyper-parameters for simulated annealing algorithm are the same as those in outer layer of our quantum-classical hybrid algorithm. For a fair comparison, we take m it-

erations in each cooling step (the simulated annealing algorithm is discussed in Appendix. A). Consequently, both simulated annealing and combined quantum annealing algorithms take $m \times \lceil \log_{0.5} 1e - 4 \rceil = 14m$ runs for Metropolis acceptance criterion. Results presented here are not deliberately selected via several trials, which means one can obtain different results fluctuating with a small range. We also show how these two algorithms work for different logistic NDPs by depicting the evolution of optimal costs by iteration steps in Fig. 2 (a-c). We notice that results given by the classical algorithm are still far from optimal with the same iterations. To obtain a global minimum (or near), one should evaluate about $10m$ new configurations in each cooling step instead of m , with no guarantee of success because the classical algorithm highly depends on others' choice hyperparameters.

V. DISCUSSION

Let us analyze the experimental results for improving our understanding of the protocol before any further discussion about how to enhance its performance. As we mentioned above, solving a 900×900 QUBO matrix by *qbsolv* takes about 15 minutes, summing up to approximately 56 hours for Problem cap71 (even more for larger problems). This would be solved in case of having local access to the quantum device and, of course, if the QPU would have more qubits and better/customized connectivity for directly running quantum annealing instead of employing *qbsolv*. We highlight that, even though our quantum device is noisy and incoherent, one can still find a result with a remarkable deviation smaller than 0.5% when compared against the global optimal result obtained from the exhaustive search. We run the quantum annealing process with a large sampling number, selecting the configuration with minimum energy among all samples, even if this configuration might not appear that frequently due to energy fluctuations during the non-adiabatic process. Once the optimal configuration for facilities is obtained, the optimal solution can be checked/refined by repeatedly running the quantum annealing process to check if a customer allocation with a lower cost can be attained. The reason is that the search space by exhaustive searching for a QUBO matrix encoded in $n \times m$ logical qubits and $k \times m$ ancilla qubits is $2^{(n+k)m}$, but the search subspace is dramatically reduced once the quantum annealer excludes most of the states with higher energy. In this way, the quantum annealing algorithm works for searching a global minimum even in noisy intermediate-scale quantum devices. The price to pay is a more extensive sampling in the quantum annealing process.

Now, we evaluate several improvements to make the algorithm more efficient. From the perspective of algorithm, the outer layer applies a classical simulated annealing algorithm which theoretically obtains global

minimum with adequate annealing parameters. As we mentioned before, the inappropriate annealing schedule will stuck the algorithm to a local minimum and we should find the initial temperature for simulated annealing. The total number of iterations to guarantee a global minimum is enormous which is proved by Ref. [33], while the transition probability from state i to state j is denoted by $P_{ij} = G_{ij}A_{ij}$. The acceptance probability follows the Metropolis acceptance criterion that $A_{ij} = \exp(-(E_j - E_i)/T)$ when $E_j > E_i$, otherwise $A_{ij} = 1$. We assume that the generation probability is symmetric $G_{ij} = G_{ji}$ and the Markov chain of a given temperature is acyclic and irreducible, then the system follows a Maxwell-Boltzmann distribution that

$$\pi_i = \frac{|N(i)| \exp(-E_i/T)}{\sum_j |N(j)| \exp(-E_j/T)}, \quad (7)$$

where $N(i)$ and $N(j)$ denote the sets of neighbours of state i and j , respectively, and generation probability P_{ij} is distributed uniformly among neighbours of state i (i.e., $G_{ij} = 1/|N(i)|$ if $j \in N(i)$). Accordingly, the acceptance probability $\chi(T)$ and an iteration algorithm to obtain the proper annealing schedule are provided and proved in Ref. [34]. In this way, we could analyze the dataset and find optimal parameters for the outer layer algorithm before solving the NDP problem with combined quantum annealing algorithm. In the inner layer, quantum annealing for a large QUBO matrix cannot be implemented directly which requires *qbsolv* for generating subproblems. However, the partition algorithm in the main loop of *qbsolv* sometimes leads to local minimum that requires better method for splitting the matrix. Alternative algorithms could be introduced, e.g. embedding larger subproblems which exploit the resources provided by the quantum annealer [35] or emphasizing the importance of mitigating the embedding cost [36]. Meanwhile, the quantum annealing algorithm in the inner layer is affected by the penalty strength, while these parameters are very tricky to be decided. One should scale them according to the dataset and the noise of the hardware, for obtaining an acceptable solution that satisfies all constraint conditions. We notice that the optimized penalty strength for a QUBO problem could be given with the combination of machine learning algorithm, e.g., gradient descent as the most trivial idea, by quantum annealing with variant parameter vectors λ and μ and stop at an optimal solution.

As briefly analyzed in Sec. III, the minimization of cost function (1) under constraint conditions (2), (3), and (4) can be encoded in QUBO formulation. Specifically, Eq. (4) can be implemented in a similar way as penalty terms by using auxiliary qubits $b_{(i,j)}$, for ensuring that customers are not assigned to sites without facility. Thus, one can solve the NDP by standard approach, performing quantum annealing of the problem Hamiltonian,

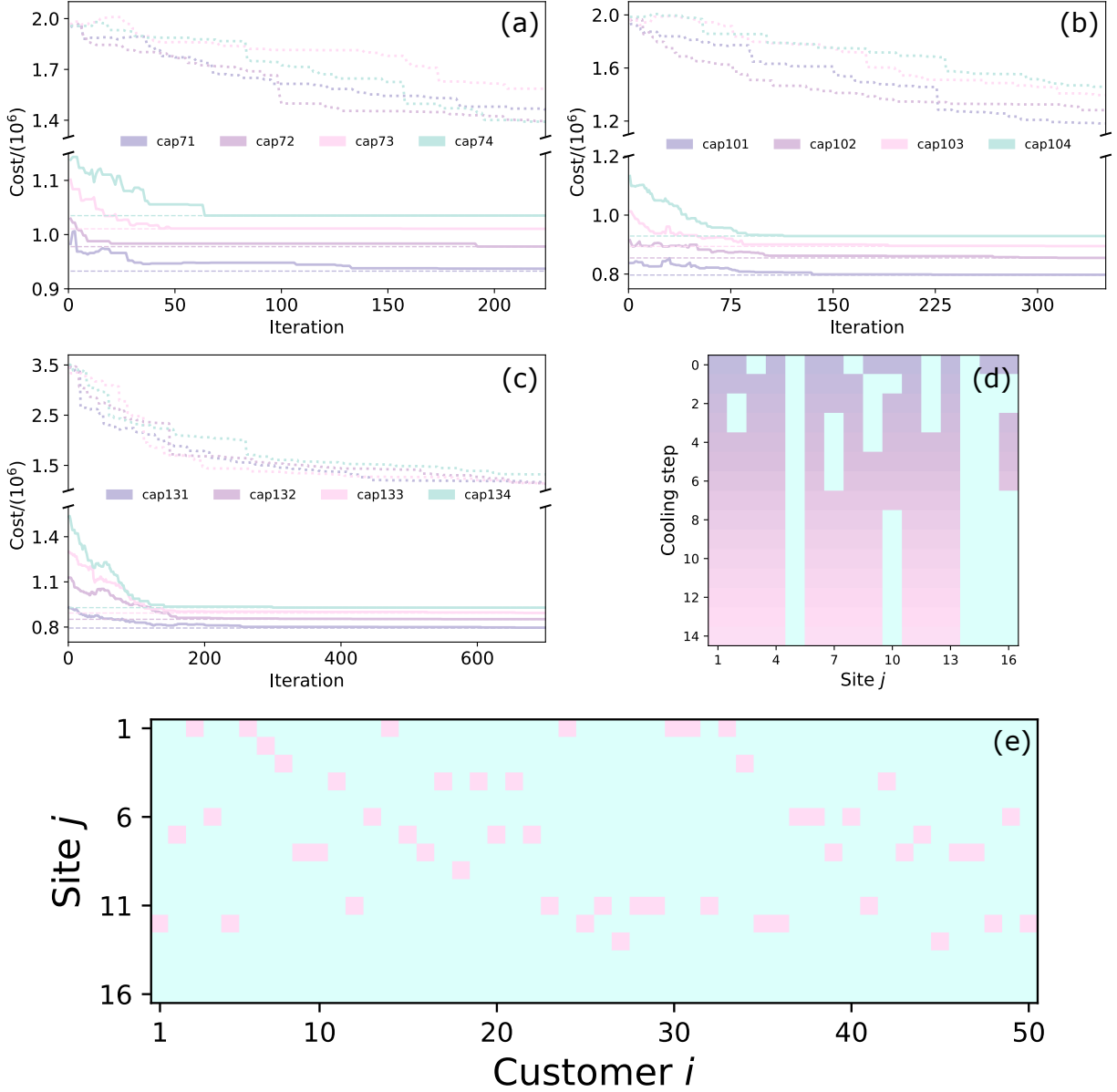


FIG. 2. (a-c) Optimal costs versus iteration numbers, with different problem sizes. Different problems with the same size are in the same color. The evolutions of the combined annealing algorithm and simulated annealing algorithm are plotted by lines and dotted lines, with ground energies by dashed lines for comparison. (d,e) Facility configuration at each cooling step and the allocation of customers of Problem cap71 shown in Table I, with a minimum cost of 933172.1000. A block in warm/cold color denotes $x_j, y_{ij} = 1$ or 0.

which reads as

$$\begin{aligned}
 H_P = & \sum_j f_j \tilde{q}_{(j)} + \sum_i \sum_j c_{ij} q_{(i,j)} + \sum_i \lambda_i \left(\sum_j q_{(i,j)} - 1 \right)^2 \\
 & + \sum_j \mu_j \left(\sum_i d_i q_{(i,j)} + \langle \mathbf{2}, \mathbf{a}_j \rangle - v_j \right)^2 \\
 & + \sum_j \alpha_j \left(\tilde{q}_{(j)} - \sum_j q_{(i,j)} - \sum_j b_{(i,j)} \right)^2. \quad (8)
 \end{aligned}$$

Here $\tilde{q}_{(j)}$ denote the configurations of facilities at site x_j .

The penalty scaled by α_j is only activated when $\tilde{q}_{(j)} = 0$ and $\sum_j q_{(i,j)} \geq 1$, i.e., customers are assigned to sites where no facility is built. The standard approach requires additional auxiliary qubits of $m \times n$ and enormous couplers for implementing the $\tilde{q}_{(j)} q_{(i,j)}$, $\tilde{q}_{(j)} b_{(i,j)}$, and $q_{(i,j)} b_{(i,j)}$ interaction, which is too complicated for minor embedding, which aims at find an effective Hamiltonian with the same low-energy subspace of Eq. (8). Meanwhile, diadiabatic effect can cause energy excitation during the annealing process, resulting in failure of prepar-

ing the ground state of the effective Hamiltonian. In this way, one may obtain quasi-optimal solutions or even solutions that violate the constraint conditions, if the energy gaps between low-energy states are not sufficiently large. This requires a systematic study of optimizing the quantum annealing of the problem Hamiltonian (8) and its effective Hamiltonian for minor embedding, e.g., the customized annealing schedules and embedding algorithms, which goes beyond the scope of this work. Although we have compared the outcome of the hybrid algorithm with the global optimal result, a comparison of the performance between our hybrid classical-quantum algorithm and the standard classical approach (8) would be relevant for a future research. We will undoubtedly consider this issue as a key milestone for a future followup project.

On the hardware side, the priority is to own a D-Wave quantum annealer instead of using a cloud quantum annealer, considering that the inner annealing time does not contribute a lot to the whole computation time. An advance could be controlling the quantum annealing process in the inner layer. Generally, the Hamiltonian of a quantum annealer $H = A(t)H_T + B(t)H_P$ is constrained by boundary conditions that $A(0) = 1$, $A(t_f) = 0$, $B(0) = 0$ and $B(t_f) = 1$, to start with initial tunneling Hamiltonian and result in the ground state (or low-energy state) of the final problem Hamiltonian. A customized quantum annealing process might shorten the annealing time while reducing the energy excitation to generate a better solution within less time. This annealing protocol could be given by control theory or other optimal methods, e.g., shortcut to adiabaticity in spin system [37–40], that controls the preparation and evolution of qubits in the quantum annealer. Quality of the solutions could also be improved by quantum annealer with more qubits, larger connectivity, and less noise, which will be released by D-Wave in mid-2020 named Pegasus [41]. Alternative

hardware will be coherent quantum annealer, which is still far from the practical application but could be built with current technologies while providing preliminary results [42–44].

VI. CONCLUSION

We proposed a combined quantum annealing algorithm inspired in Ref. [30] to solve logistic network design problems, but which can also be applied to a large variety of optimization problems. The algorithm is tested with 12 NDP problems, and the results are in very good agreement with the already-known best solutions given by *Lindo*. This research is another convincing evidence for the feasibility of applying quantum annealing for optimization problems, even when the quantum devices are limited by the number of qubits, the connectivity, and the noise.

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Appendix A: Simulated Annealing Algorithm for NDPs

Here we introduce how to perform simulated annealing algorithm for solving NDPs. We keep the same cooling schedule and iteration number at each cooling step for a

fair comparison. The initial state is an allocation of customers that randomly distributed to different sites and its according configuration of facilities. If a site is visited by at least once, a facility should be built on this site. The neighboring function will be, select an arbitrary customer among all n customers, and randomly assign the

customer to a site. With the neighboring function and definition of state, one can evaluate the cost difference, updating the solution by Metropolis acceptance criterion as we do in outer layer of combined quantum annealing algorithm.