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Contact conditions and stresses induced during fretting fatigue

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Abstract

This paper analyses the contact conditions and stresses induced during fretting fatigue tests of uniaxial specimens using fretting bridges with spherical-tip contact pads. This analysis considers the displacement of the stick zone induced by the variable bulk stress applied to the specimen. It describes an approach to, analytically, calculate the eccentricity of the stick zone and the stress tensor under the contact zone including this stick zone displacement. Two different situations have been considered in the specimen, plane stress and plane strain. The error in the calculation of the stresses using this approach has been estimated and has been found to be very small. A comparison to the results obtained experimentally is presented. Finally, the analytically calculated stresses are compared to those obtained using the classical Mindlin-Cattaneo approach which does not consider the bulk stress. It has been found that stresses calculated using the classical approach may be underestimated even by 10%, which is a very critical quantity for a fatigue analysis.

1 Introduction

Fretting is a particular case of fatigue in which the principal loads are due to the contact between two solids that tend to slide. In addition to local stress fields in the vicinity of contact, one or both of the solids may be subjected to bulk stresses caused by cyclic loads. This initiates many cracks in the process zone. These cracks may give rise to the deterioration of the surface by spallation or they may grow until final fracture occurs across one of the elements under contact.

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Locations where fretting fatigue is observed in service include riveted and bolted joints, shrink-fitted couplings, metal ropes and cables, etc. (Waterhouse[1]).

In order to study this phenomenon, the first step is understanding how the loads are transmitted from one body to the other and the shape of the contact zone. The distribution of these loads and the contact zone shape can be very different and depend on the material, the normal and tangential loads applied, the geometry of the bodies, etc.

The first satisfactory analysis of the stresses at the contact of two elastic solids is due to Hertz, 1882. Various researches have undertaken analytical and computational studies of the contact problem, but much about the phenomenon needs to be studied.

The contact problem becomes more complicated when a tangential load (not high enough to produce global sliding of the two bodies)is applied. Cattaneo[2] and Mindlin[3,4] were the first to obtain, independently, a solution for this problem in the case of the contact between a sphere and a plane surface. Hills and Nowell[5] solved the problem of cylindrical contact when, in addition to the other loads, a bulk stress is applied to one of the bodies as it happens in many fretting fatigue situations.

In this paper, a solution for the contact stresses and the shape of the contact zone in spherical contact with bulk stress is obtained.

2 Spherical contact without bulk stress

The radius, a, of the contact zone between a flat surface and a sphere, R, for a normal load, N, and the distribution of normal pressure are, from Hertz theory

$$a = \left(\frac{3NR}{4E^*} \right)^{1/3} \tag{1}$$

$$p(r) = \frac{3N}{2\pi a^2} \sqrt{1 - \frac{r^2}{a^2}} = p_0 \sqrt{1 - \frac{r^2}{a^2}} \tag{2}$$

where p_0 is the maximum normal stress in the contact zone.

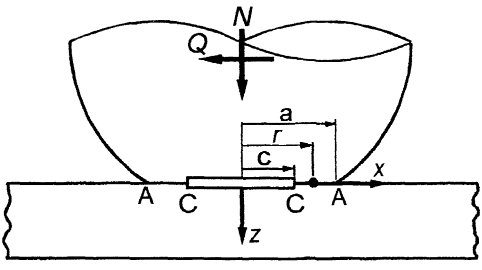

Figure 1: A schematic of the contact zone in plane $y = 0$.

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If a tangential load is applied, $Q<\mu N$, the contact zone separates in two zones, an outer concentric annulus where sliding takes place called slip zone and an inner circle, of radius c, called stick zone, Figure 1.

If the mechanical properties of the two bodies are the same, the normal and tangential problems are not coupled and they can be solved separately. The distribution of normal stresses is the one given by Hertz, eqn (2), and the tangential stresses at the surface are assumed to be parallel to Q. To find this distribution, Mindlin superposes the effect of two distributions

$$q_x'(r) = \frac{3\mu N}{2\pi a^2}\sqrt{1-\frac{r^2}{a^2}} \qquad 0 \le r \le a \tag{3}$$

which corresponds to the case of globlal sliding, plus

$$q_x''(r) = -\frac{c}{a}\frac{3\mu N}{2\pi a^2}\sqrt{1-\frac{r^2}{c^2}} \qquad 0 \le r \le c \tag{4}$$

The sum of the two distributions yields zero slip in a circle of radius c, the stick zone. The size of this zone, c, can be obtained by forcing the integral of the tangential stresses over the contact zone to be equal to Q.

$$\frac{c}{a} = \left(1-\frac{Q}{\mu N}\right)^{1/3} \tag{5}$$

Once the normal and tangential stresses on the surface are known, the stresses and displacements at any point may be calculated using the solution of Hamilton and Goodman[6]. The explicit expressions were later obtained by Hamilton[7] and by Sackfield and Hills[8], which are easier to handle.

3 Cylindrical contact with bulk stress

If, after applying the normal load N, an axial load is applied to one of the bodies, a tangential load Q at the interface will be produced simultaneously (figure 2). Its value will depend on the magnitude of the axial load and the stiffness of both elements in contact (Dominguez[9]).

Under these conditions, the axial stress produces a displacement, e, of the stick zone. To analyze this effect in cylindrical contact, Hills and Nowell[5] treated the problem as a perturbation of the Mindlin problem. They added the deformation due to the axial load, P, to one of the solids.

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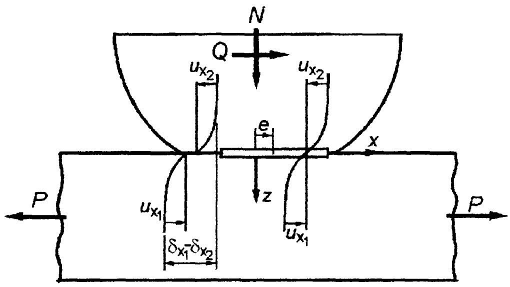

Figure 2: A schematic of the displacements at the contact zone.

The distribution of tangential stresses in the slip zone is

$$q(x) = \mu p_0 \sqrt{1 - \frac{x^2}{a^2}} \tag{6}$$

and in the stick zone

$$q(x) = \mu p_0 \sqrt{1 - \frac{x^2}{a^2}} - \frac{b}{a} \mu p_0 \sqrt{1 - \frac{(x-e)^2}{b^2}} \qquad |x - e| \leq b \tag{7}$$

The value they found for the eccentricity of the stick zone was

$$e = \frac{\sigma_{ax} a}{4 \mu p_0} \tag{8}$$

where σ_{ax} is the stress due to the force P.

4 Spherical contact with bulk stress

This problem can be solved using a perturbation of the Mindlin problem. The tangential stresses are the sum of two distributions, as Mindlin did, but one of them is displaced a distance e from the center of the contact zone. The two distributions are

$$q'_x = \frac{3 \mu N}{2 \pi a^3} \left(a^2 - r^2\right)^{\frac{1}{2}} = \mu p(r) \qquad r \leq a \tag{9}$$

and

$$q''_x = -\frac{c}{a} \frac{3 \mu N}{2 \pi a^2} \sqrt{1 - \frac{r_e^2}{c^2}} \qquad r_e \leq c \tag{10}$$

where

$$r_e = \sqrt{(x-e)^2 + y^2} \tag{11}$$

The x and z axis are the same as those that appear in figure 2 and the y axis is perpendicular to both of them. Hereafter, subscript 1 will refer to the flat surface and subscript 2 to the sphere. The traction distribution given by eqns (9) and (10) yields the following displacements at the surface, in the stick zone (Johnson[10])

$$u_{x2} = -\frac{\pi\mu p_0}{32Ga}\left[4(2-\nu)\left(a^2-c^2\right)-(4-3\nu)\left(2ex-e^2\right)\right] \tag{12}$$

$$u_{x1} = \frac{\pi\mu p_0}{32Ga}\left[4(2-\nu)\left(a^2-c^2\right)-(4-3\nu)\left(2ex-e^2\right)\right]+\frac{\sigma_a}{2G(1+\nu)}x \tag{13}$$

The last term of eqn (13) comes from the deformation produced by the axial stress σ_a under plane stress assumption.

By definition, the slip must be zero in the stick zone. This condition implies that, in the x axis,

$$s_x = u_{x1} - u_{x2} - \delta_x = 0 \qquad \forall x, y \tag{14}$$

where δ_x is a constant that represents the displacement in x direction between two distant points.

Introducing eqns (12) and (13) into eqn (14) gives the eccentricity for the stick zone in plane stress

$$e = \frac{4\sigma_a a}{\pi\mu p_0}\frac{1}{(1+\nu)(4-3\nu)} \tag{15}$$

Only one of the conditions of no slip has been used to obtain e. The other one is

$$s_y = u_{y1} - u_{y2} = 0 \qquad \forall x, y \tag{16}$$

Finding the displacements in the stick zone in y direction (Johnson[10]) and substituting into eqn (16) yields

$$u_{y1} - u_{y2} = -\frac{\sigma_a}{2G}\frac{\nu}{1+\nu}3\frac{1-\nu}{4-3\nu}y \neq 0 \tag{17}$$

This means that the surfaces tend to slide in the y direction. To impede this sliding, a new component of stress will appear, q_y, that was supposed to be zero.

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This new component will affect the direction and distribution of the stresses over the contact zone.

The slip that appears is symmetric with respect to the x axis and zero in $y = 0$. Therefore, the two conditions of no slip are true on this axis and the effect over the size of the stick zone in this axis will be negligible. Very close to this axis there would be slip, eqn (17), but the stress that appears, q_y, is so small that q_x plus q_y is not high enough to produce sliding. Close to the limit of the stick zone q_x plus q_y reaches the value of the limit of friction and so those points enter the slip zone. The size of the stick zone is smaller and its shape is no longer a circle.

In order to find whether this new component, q_y, is significant or not, a comparison between the maximum slip in y, s_y, and the slip that would be in x when the radius of the stick zone tends to zero and there is no axial stress, s'_x, is made. The reason for using this comparison is that there must be a direct relation between relative slip and the stresses that appear to cancel this slip in the limit of the stick zone. If the material used is Aluminum (ν=0.33) and the value of the eccentricity is small (e/c = 0.1), this ratio is

$$\frac{s_{y(x=0,y=c)}}{s'_{x(x=c,y=0)}} = \frac{-\dfrac{\sigma_a}{2G}\dfrac{\nu}{1+\nu}3\dfrac{1-\nu}{4-3\nu}c}{-2\dfrac{\pi\mu p_0}{32Ga}(4-3\nu)c^2} = 6\nu\frac{1-\nu}{4-3\nu}\frac{e}{c} = 0.44\frac{e}{c} = 0.044 \tag{18}$$

Therefore, an estimation of the ratio between the stress components at the edge of the stick zone is

$$\frac{q_y}{q_x} \approx \frac{0.044\dfrac{c}{a}\mu p_0}{\mu p_0} \approx 0.04 \tag{19}$$

This means that the error in the solution due to the fact that the stresses q_y have been assumed to be zero is very small.

The solution shown corresponds to the case of plane stress. If plane strain is assumed, the equations vary lightly. In this case, the displacements due to the axial stress are different

$$u^a_{x1} = \frac{\sigma_a}{E}(1-\nu^2)x = \frac{\sigma_a}{2G}(1-\nu)x \tag{20}$$

As before, an expression for the displacement of the stick zone can be obtained, which for plane strain is

$$e = \frac{4\sigma_a a}{\pi\mu p_0}\frac{1-\nu}{4-3\nu} \tag{21}$$

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The difference of the solution in the two cases is small, for Aluminum the relation between them is

$$\frac{e_{\varepsilon_y^a=0}}{e_{\sigma_y^a=0}} = 1-\nu^2 = 0.89 \tag{22}$$

In plane strain, the slip s_y is smaller than in plane stress. The expression equivalent to eqn (18) is

$$\frac{s_{y(x=0,y=c)}}{s'_{x(x=c,y=0)}} = \frac{\frac{\sigma_a}{2G}\nu\frac{1-\nu}{4-3\nu}c}{-2\frac{\pi\mu p_0}{32Ga}(4-3\nu)c^2} = \frac{2\nu}{4-3\nu}\frac{e}{c} = 0.0219 \tag{23}$$

And the relation between the tangential stresses in the border of the stick zone is approximately

$$\frac{q_y}{q_x} \approx \frac{0.0219\frac{c}{a}\mu p_0}{\mu p_0} \approx 0.02 \tag{24}$$

In both cases q_y is one order of magnitude lower than q_x and errors made assuming $q_y = 0$ are small.

Figure 3 shows a photograph of the scar made on an Aluminum specimen in a fretting test where the contact and the stick zone can be appreciated clearly, Wittkowsky[11].

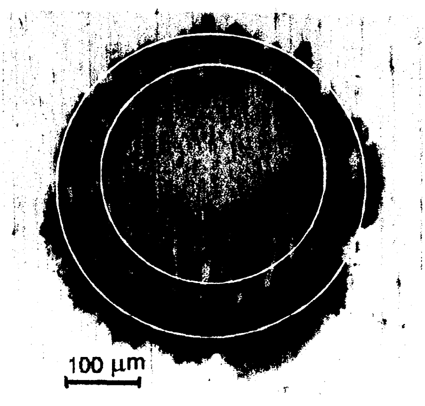

Figure 3: Fretting scar produced in a test.

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5 Comparison with the classical approach

The stresses calculated under the contact zone are different depending on whether or not the eccentricity of the stick zone is taken into account. The classical approach (Mindlin-Cattaneo) does not calculate it and the stresses may be underestimated.

In order to analyze this effect, a specific case with typical values is going to be studied. The problem is the contact between a flat surface and a sphere of radius R = 25.4 mm, the material is Aluminum and the contact loads are N = 12 N, $Q = \pm 9$ N and $\sigma_a = \pm 83$ MPa.

Figure 4 shows the range of variation of the stress σ_x for $y = 0$, $z = 0$. The x axis has been normalized with respect to the size of the contact zone and the stresses with respect to the maximum normal pressure p_0. It can be seen that, using the classical approach, the maximum range of σ_x is underestimated by a 6%. This error is even larger if the stress analyzed is the Von Mises equivalent stress, 8% as shown in figure 5.

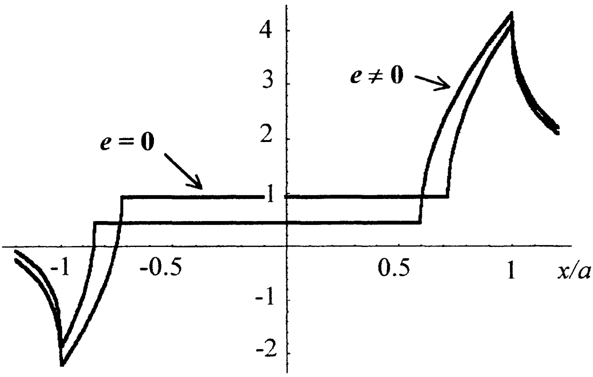

Figure 4: $\Delta\sigma_x$ in the axis $y = 0$, $z = 0$.

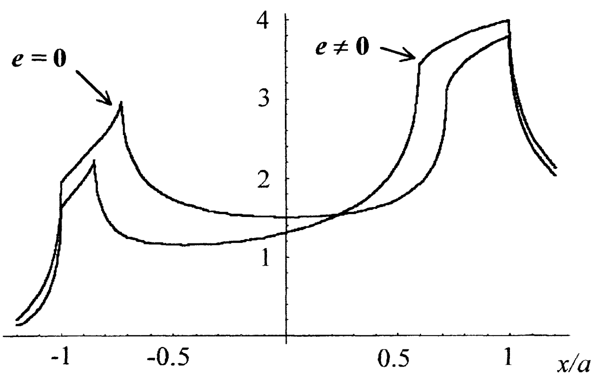

Figure 5: Range of Von Mises equivalent parameter in $y = 0$, $z = 0$.

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A similar analysis may be carried out at a different depth, $z = 0.1a$. The value of the stresses are lower in this case, although the error in the maximum stress with the classical approach is larger. Figure 6 shows that the error in $\Delta\sigma_x$ is 9%, for the Von Mises stress this error goes up to 12%, figure 7.

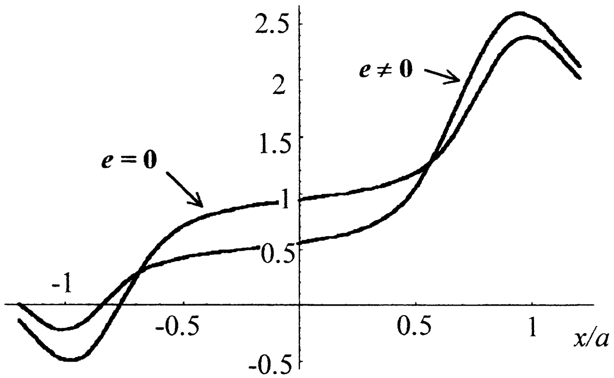

Figure 6: $\Delta\sigma_x$ in the axis $y = 0$, $z = 0.1a$.

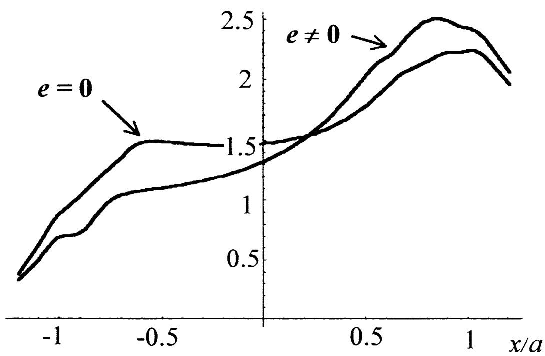

Figure 7: Range of Von Mises equivalent parameter in $y = 0$, $z = 0.1a$.

6 Conclusions

An expression for the eccentricity of the stick zone in a fretting fatigue test with spherical contact and a bulk stress applied has been found. Two different situations have been distinguished in the flat surface, plane stress and plane strain.

These expressions are approximations to the real situation because the tangential stresses in the direction perpendicular to the applied load, Q, have been assumed to be zero, and it has been found that this is not exact. Nevertheless, these stresses, q_y, are very low compared to q_x, and the solution found is not far from reality.

It has been proved that not taking into account the displacement of the stick zone has the consequence of underestimating the stresses under the contact zone. This error can be significant, up to 10%.

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