Three-Dimensional Modelling of the Tide-Induced Dispersion of Radionuclides in the Sea

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ABSTRACT

A three-dimensional marine dispersion model for radionuclides has been developed. The model solves the three-dimensional hydrodynamic equations and, simultaneously, the three-dimensional advective–diffusive dispersion equation. Thus, the tide-induced dispersion of radionuclides is obtained. The equations are solved using a finite difference explicit scheme, using a time step of a few seconds, with appropriate boundary conditions. As an example, the model has been applied to study the dispersion of ^{137}Cs in the Irish Sea, which is released from the nuclear fuel reprocessing plant at Sellafield. Tidal amplitudes and current profiles obtained with the model have been compared with observations in the Irish Sea: both sets of data are in good agreement. Observed and computed ^{137}Cs distributions in waters have also been compared. The model gives a good representation of the dispersion of this radionuclide in the Irish Sea.

INTRODUCTION

Over the past few years, there has been an increasing use of three-dimensional models, rather than two-dimensional hydrodynamic models, to solve oceanographic problems (Davies *et al.*, 1997; Proctor and James, 1996; Davies and Lawrence, 1994). The reason is related to the limitations of two-dimensional models, which for instance, cannot give information about the vertical profile of currents. In recent years, the use of three-dimensional models has also increased due to the dramatic rise in computing power.

Three-dimensional models can also be applied to study the dispersion of radionuclides in aquatic environments. Indeed, Prandle *et al.* (1993) have shown that vertical structure can be of primary concern when typical

simulated periods are in the order of one month or less. Moreover, the vertical variability is important for such typical times, even in shallow waters (depth ~ 50 m), if tidal mixing keeps the vertical diffusion coefficient smaller than $10^{-3}\,\mathrm{m^2\,s^{-1}}$ (Prandle *et al.*, 1993) (a value $\sim 10^{-3}\,\mathrm{m^2 s^{-1}}$ can be considered representative of a strong tidal action). In this situation, a three-dimensional model should be used to simulate accurately the dispersion of radionuclides. Of course, in places with weak tides and in lakes, the value of the vertical diffusion coefficient can be greatly reduced (Prandle *et al.*, 1993) and the use of three-dimensional models will be necessary to simulate the dispersion of tracers. The model of Nies *et al.* (1997), for instance, studies the three-dimensional transport of radionuclides in the Artic Ocean but, unfortunately, the model results could not be compared with observations.

Thus, the objective of this paper is to present and test a relatively simple, but effective, three-dimensional dispersion model for radionuclides. The model solves the three-dimensional hydrodynamic equations, using a time step of a few seconds, and solves simultaneously the three-dimensional advection–diffusion dispersion equation. The tide-induced transport of radionuclides is then obtained. As an application, the model has been used to study the dispersion of radionuclides in the Irish Sea. These radionuclides are released from the nuclear fuel reprocessing plant at Sellafield. The Irish Sea has been chosen since there is enough oceanographic information (tidal elevations and currents, current profiles) to compare with the results of the model. Also, there is a well-known source of radionuclides and there are measurements of activities over the sea. These measurements can be compared with model predictions. It must be noted that by application of the model to the dispersion of Sellafield-released radionuclides we are validating our numerical model, and, at the same time, also developing an useful predictive tool that can be applied, for instance, in the case of an accidental release from the reprocessing plant. In the case of an accident, the model can give information about the radionuclide concentrations in the area at desired times immediately after the accident (hours, days and even months).

In the next section, the model equations are presented. Then the numerical method is briefly described and finally, in the last section, the model application to the Irish Sea is presented.

EQUATIONS

The three-dimensional equation of continuity for an homogeneous sea and for an incompressible flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

where u, v and w are the components of the water velocity along the x, y and z coordinates. x- and y-axis are in the horizontal plane and z is measured upwards from the undisturbed water level.

This equation can be integrated from the sea bottom ($z = -h$) to the surface ($z = \zeta$) to obtain the equation for water elevation. h is the undisturbed depth of water and ζ is water elevation above the mean level due to tidal oscillations:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\int_{-h}^{\zeta} u \, dz + \frac{\partial}{\partial y}\int_{-h}^{\zeta} v \, dz = 0 \tag{2}$$

The equations of motion are

$$\frac{\partial u}{\partial t} + g\frac{\partial \zeta}{\partial x} - \Omega v = \frac{\partial}{\partial z}\left(N\frac{\partial u}{\partial z}\right) \tag{3}$$

$$\frac{\partial v}{\partial t} + g\frac{\partial \zeta}{\partial y} + \Omega u = \frac{\partial}{\partial z}\left(N\frac{\partial v}{\partial z}\right) \tag{4}$$

where g is the acceleration due to gravity, Ω is the Coriolis parameter, $\Omega = 2w \sin \phi$ (w being earth rotational angular velocity and ϕ the latitude), and N is the coefficient of vertical eddy viscosity. The last term in the equations is a diffusion term (N would be the diffusion coefficient) and represents the dissipation of energy due to turbulence. The non-linear advective terms have been removed from the equations since Charnock and Crease (1957) have shown, by dimensional analysis, that they are important only when the water elevation above the mean level is comparable with the mean depth. In this situation, a three-dimensional model would not be necessary. The non-linear terms, however, generate a residual current that may affect the radionuclide transport when studying long-term dispersion (time scale in the order of years). However, the effect of such weak current can be neglected if time scale of interest is in the other of weeks, as is the case. Moreover, Prandle (1984) excludes the advective terms in his long-term dispersion model since their effect is to add additional structure to residual distributions, but this structure is often exaggerated due to poor topographic resolution (Prandle, 1984).

To solve eqns (1)–(4) for ζ, u, v and w, surface and sea-bed boundary conditions must be specified. The surface boundary condition is

$$\rho\left(N\frac{\partial u}{\partial z}\right)_{z=\zeta} = F_s \tag{5}$$

$$\rho\left(N\frac{\partial v}{\partial z}\right)_{z=\zeta} = G_s \tag{6}$$

where F_s and G_s denote the components of wind stress acting on the water surface in the x and y directions and ρ is the water density. This stress can be written as in Pugh (1987) and Periáñez *et al.* (1994). Similarly, at the sea bed

$$\rho\left(N\frac{\partial u}{\partial z}\right)_{z=-h} = F_b \tag{7}$$

$$\rho\left(N\frac{\partial v}{\partial z}\right)_{z=-h} = G_b \tag{8}$$

where F_b and G_b are the components of the bottom stress. Assuming a linear law of bottom friction:

$$F_b = \rho k u_b \tag{9}$$

$$G_b = \rho k v_b \tag{10}$$

where k is a friction coefficient and u_b and v_b are the components of the water velocity at a given height above the sea bed, which is usually 1 m (Davies and Stephens, 1983). A quadratic law for bottom friction could also be assumed. An alternative bottom boundary conditions is a no slip condition, which consists of imposing zero water velocity at the sea bed. Details can be seen, for instance, in Davies and Stephens (1983). Although the use of a quadratic bottom friction is now more extended than a linear formulation, the linear law has been used since a faster convergence of the equations was obtained than with a quadratic model. Indeed, a linear friction is more appropriate in linear models (Davies, 1985).

The three-dimensional advection–diffusion dispersion equation for dissolved radionuclides is

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(K_h\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_h\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_v\frac{\partial C}{\partial z}\right)-\lambda C \tag{11}$$

where C is the radionuclide concentration, λ is the radioactive decay constant and K_h and K_v are the horizontal and vertical diffusion coefficients, respectively. The external source of radionuclides, where it exists, should be included to this equation.

The horizontal diffusion is considered to be the same in both the x and y directions. The vertical diffusion coefficient is usually written as a function of the eddy diffusivity (Kowalik and Murty, 1993)

$$K_v = \sigma N \tag{12}$$

where the non-dimensional number σ ranges from 0·1 to 0·5.

NUMERICAL SOLUTION

All the equations are solved using finite differences. A finite difference grid is employed in both the horizontal and vertical directions (Fig. 1). The grid has uniform spacing in the horizontal, Δx and Δy, and in the vertical, Δz. Each grid cell is identified by three numbers (i, j, l), where the pair i, j denotes the position in the horizontal plane and l denotes the vertical layer number. Horizontal water velocities are specified on the centre of the lateral faces of the grid box, vertical water velocity is specified on the centre of the upper face and the concentration of radionuclides is specified at the centre of the grid box. Water elevation, ζ, is only specified on the upper face of the top grid box. Eddy viscosity and the vertical diffusion coefficient are specified at the same places as the vertical water velocity and the horizontal diffusion coefficients at the same place as the horizontal water velocities.

The model is started from rest and eqns (1)–(4) are integrated forward through time, computing the values of ζ, u, v and w (in this sequence) from their values at the previous time step (an explicit scheme has been adopted). The explicit numerical scheme for two-dimensional advection–diffusion presented in Periáñez *et al.* (1994) can be extended in a straightforward manner to three-dimensional dispersion.

The explicit scheme implies that some conditions must be satisfied to obtain a stable numerical solution of the equations. In the case of the hydrodynamic equations we have the CFL criterion (Kowalik and Murty, 1993; Prandle, 1974):

$$\Delta t < \frac{\Delta x_i}{\sqrt{2gh_{\max}}} \tag{13}$$

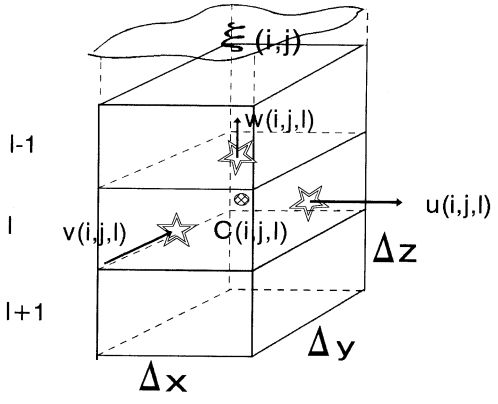

Fig. 1. Three dimensional representation of the finite differences grid and the position of grid points.

where Δt is the time step, $h_{\max}$ is the maximum water depth and Δx_i represents Δx and Δy. We also have an extra condition due to the energy diffusion term (Kowalik and Murty, 1993):

$$\Delta t < \frac{\Delta z^2}{2N}. \tag{14}$$

The advection–diffusion dispersion equation also imposes two stability conditions (Kowalik and Murty, 1993):

$$\Delta t < \frac{\Delta x_i}{u_{i,\max}} \tag{15}$$

where $u_{i,\max}$ represents the maximun values of u and v and

$$\Delta t < \frac{\Delta x_i^2}{6K_i} \tag{16}$$

where the i index represents the three directions in space.

On the other hand, it is known that the upstream difference scheme used for the advective terms in the dispersion equation introduces numerical diffusion. It has been shown (Prandle, 1984; Periáñez *et al.*, 1994) that the magnitude of numerical diffusion is equivalent to increasing the diffusion coefficients K_i by K_i', where

$$K_i' = \tfrac{1}{2}(u_i \Delta x_i - u_i^2 \Delta t). \tag{17}$$

Due to the small values of the vertical water velocities (see below) numerical diffusion in the vertical direction can be neglected, that is, $K_{\mathrm{v}}' \ll K_{\mathrm{v}}$. In the horizontal directions numerical diffusion has been corrected by subtracting the instantaneous value of K_{h}' to K_{h} (K_{h}' is different along the x and y directions). This technique has been successfully used before (Abril, 1990).

Some boundary conditions are required along both closed and open borders of the computational domain. For closed borders, a no flux condition is imposed,

$$u_i = 0 \tag{18}$$

where u_i is the current component which is normal to the boundary. Similarly, there is no flux of radionuclides through a closed boundary,

$$\frac{\partial C}{\partial x_i} = 0 \tag{19}$$

along the boundary, where x_i is the normal direction to the boundary.

Along open boundaries, water elevations are specified from observations (tidal charts) and the normal component of the surface water velocity is obtained from a radiation condition (Kowalik and Murty, 1993)

$$u_i = \zeta \sqrt{\frac{g}{h}} \tag{20}$$

where u_i is the normal component of the current. This condition allows the tide wave to propagate outward from the domain without any reflection. For the dispersion equation, the boundary condition described in Periáñez *et al.* (1994) was applied:

$$C_i = \alpha C_{i-1} \tag{21}$$

where C_i is the concentration in the open boundary and C_{i-1} represents the concentration just inside the computational domain. The nondimensional number α is obtained from calibration.

APPLICATION OF THE MODEL

As said in the Introduction, the model has been applied to study the dispersion of radionuclides in the eastern Irish Sea. The three-dimensional model has an horizontal resolution of $\Delta x = \Delta y = 5000$ m and a vertical resolution of $\Delta z = 5$ m. The time step is $\Delta t = 100$ s. With these resolutions all the stability conditions are satisfied. The computational domain is shown in Fig. 2, where the points where tide elevations and currents have been measured can also be seen. The water depths, which have been introduced from bathymetric maps, range from 55 m in the west to a shallower area along the British coast.

The FORTRAN code, written by the author in its totality, was implemented on a HP Exemplar X-Class computer.

Water elevations are specified along the open boundary from observations (Howarth, 1990). Only the main component, M_2, has been considered. The friction coefficient [eqns (9) and (10)] has been obtained from a calibration exercise: $k = 0{\cdot}0045$ and the profile shown in Fig. 3 has been used for the eddy viscosity. This profile is similar to that used by Davies and Stephens (1983). Wind effects have not been considered, thus, $F_s = G_s = 0$.

The mean value 0·3 has been used for σ [eqn (12)] and the horizontal diffusion coefficient was taken as $K_h = 500\ \mathrm{m^2\ s^{-1}}$. For the open boundary condition of the dispersion equation [eqn (20)], good results are obtained taking $\alpha = 0{\cdot}9$.

The dispersion of ^{137}Cs released at Sellafield has been simulated. The real input rate from the source has been used (Jefferies and Steele, 1989). As

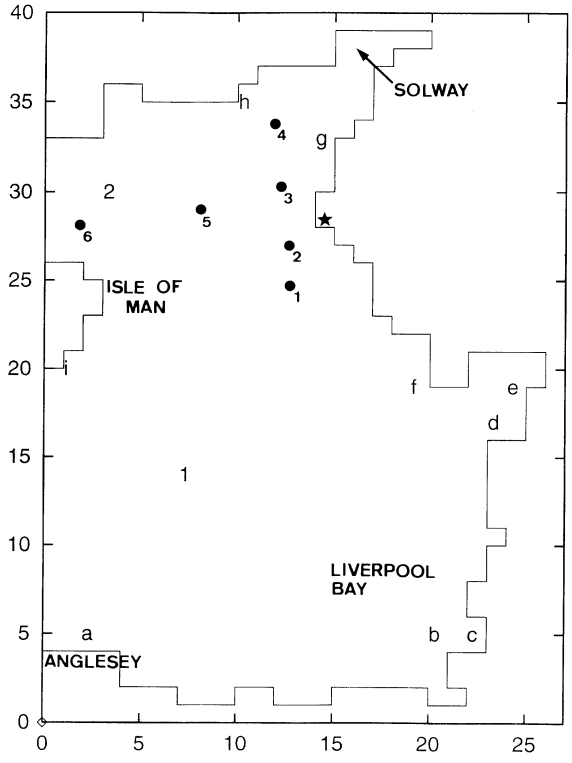

Fig. 2. Computational domain. Letters are the ports where tidal amplitudes and phases have been measured and numbers are the points where current profiles have been measured. The star is Sellafield nuclear fuel reprocessing plant. Numbered circles denote points where ^{137}Cs concentrations were obtained from surface and bottom waters. Each unit in the x- and y-axis is 5000 m.

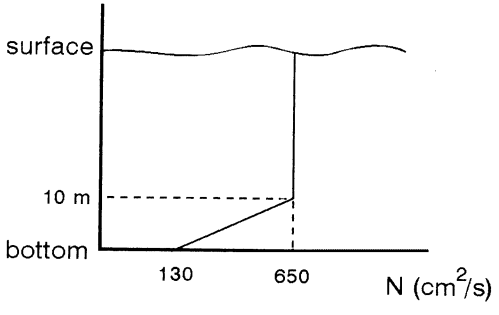

Fig. 3. Eddy viscosity profile used in the model.

a first approach to the three-dimensional dispersion problem, ^{137}Cs has considered to be perfectly conservative, which is usual in some models (Prandle, 1984; Abril and García-León, 1992).

Water circulation

The M_2 corange chart is presented in Fig. 4 (lines join points with the same tidal amplitude). It is not significantly different from that based upon observations (Howarth, 1990), with tidal amplitudes increasing rapidly as we move towards the east. Indeed, a comparison of computed and observed tidal amplitudes and phases is presented in Table 1. Although in the case of the port of Douglas a difference of 40 cm is obtained (which means an error of 17%), there is good agreement for the rest of the ports, errors being below 5% in all cases (except Douglas). In the case of tidal phases, errors are below 8% in all cases.

The time evolution of water elevations and surface and bottom currents at point 1 (see map in Fig. 2) is presented in Fig. 5. It can be seen that bottom currents are smaller than surface currents, which is due to friction

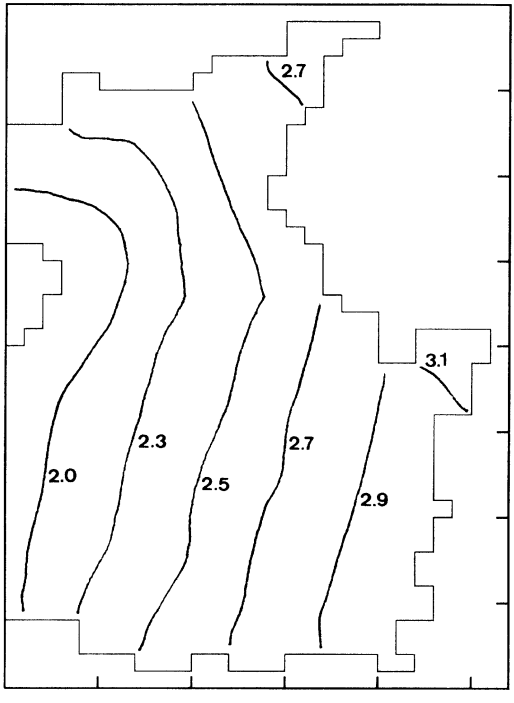

Fig. 4. Computed corange chart (tide amplitudes in m).

TABLE 1

Observed and Computed Tidal Amplitudes (m) and the Difference Between Computed and Observed Amplitudes and Tidal Phases (Δg, degrees) at the Ports Shown in Fig. 2. $30^\circ = 1{\cdot}04$ h

Code	*Port*	*Observed amplitude*	*Computed amplitude*	*Difference comp-obs*	*Phase* Δg
a	Wylfa Head	2·06	1·98	− 0·08	− 21
b	Hilbre	2·92	3·02	0·10	− 4
c	Birkenhead	3·11	3·06	− 0·05	− 8
d	Fleetwood	3·05	3·08	0·03	6
e	Heysham	3·15	3·14	− 0·01	7
f	Barrow	3·08	2·93	− 0·15	0
g	Workington	2·73	2·66	− 0·07	6
h	Hestan	2·76	2·68	− 0·08	7
i	Douglas	2·30	1·90	− 0·40	− 17

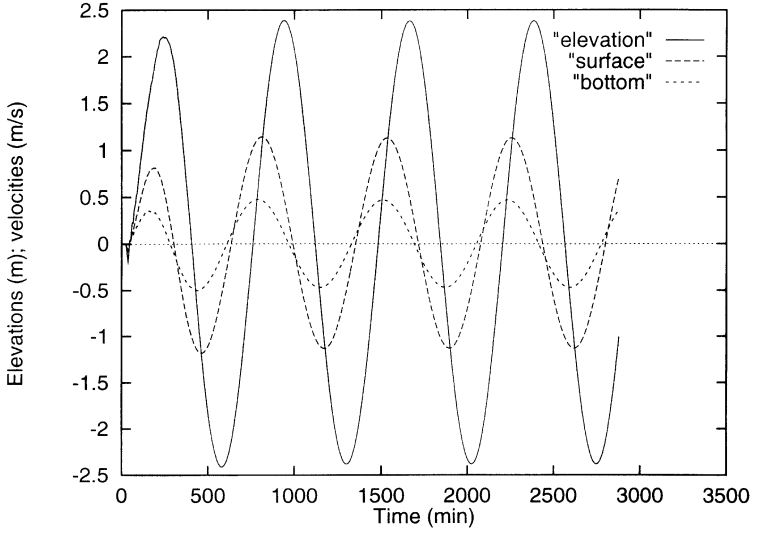

Fig. 5. Time evolution of the water elevation and the u component of the surface and sea-bed water current at point 1.

with the sea bed. On the other hand, it can be seen that zero currents are obtained aproximately during low and high water, as is expected. The vertical water velocity, w, at point 1 at the middle of the water column is shown in Fig. 6. As can be seen, maximum vertical velocity is of the order of

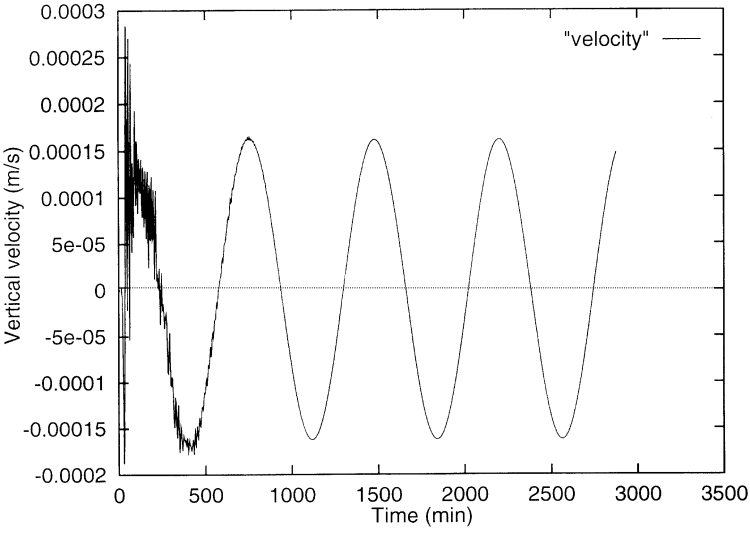

Fig. 6. Time evolution of the vertical water velocity, w, in the middle of the water column at point 1.

10^{-4} m s^{-1}, which is the order of magnitude that can be found in literature for this velocity (Kowalick and Murty, 1993).

Surface current maps when water level is increasing and decreasing at north Anglesey are given in Fig. 7. These maps are again similar to those obtained from observations (Howarth, 1990) or previous tidal models (Jones and Davies, 1996), showing strong currents (over 1·5 m s^{-1}) between Anglesey and the Isle of Man and weaker currents (of the order of 0·5 m s^{-1}) in the Liverpool Bay region.

It is evident that u and v points (see Fig. 1) do not coincide with the sea surface or the sea bed. However, sea bed and surface currents can be linearly extrapolated from the interior points (Davies, 1985).

Computed and observed current profiles at points 1 and 2 (see Fig. 2) are presented in Fig. 8. The shape of the profile is reproduced by the model at both points and for both the u and v components of the water velocity. However, it seems that the model overestimates slightly the magnitude of the current at point 1. This can be a consequence of the simple model that has been used for the eddy viscosity. Probably, the use of a TCM (Turbulent Closure Model) could improve the results (Kowalick and Murty, 1993). Nevertheless, our main objective is to study the dispersion of radionuclides and the results of the hydrodynamic part of the model seem good enough to allow an adequate description of the dispersion processes.

^{137}Cs dispersion

The major ^{137}Cs source to the Irish Sea has been the discharges from Sellafield. These discharges, for the period 1964–1985, can be seen in Jefferies and Steele (1989). Other sources like nuclear weapon test fallout contributed less than 1% of the total input (Jefferies and Steele, 1989).

The dispersion model was tested by balancing at each time step the number of particles in the computational domain. Thus, for each time step the particle number inside the grid should be equal to the particles which entered the grid from the external source. After a simulation over four tidal cycles, the difference in the number of particles is always smaller than 0·3%.

As a first numerical experiment, an arbitrary instantaneous discharge was released at the sea surface at compartment (7, 29). The time evolution of concentrations in the surface box, the grid box at the middle of the water column and in the deepest box can be seen in Fig. 9. Surface concentration

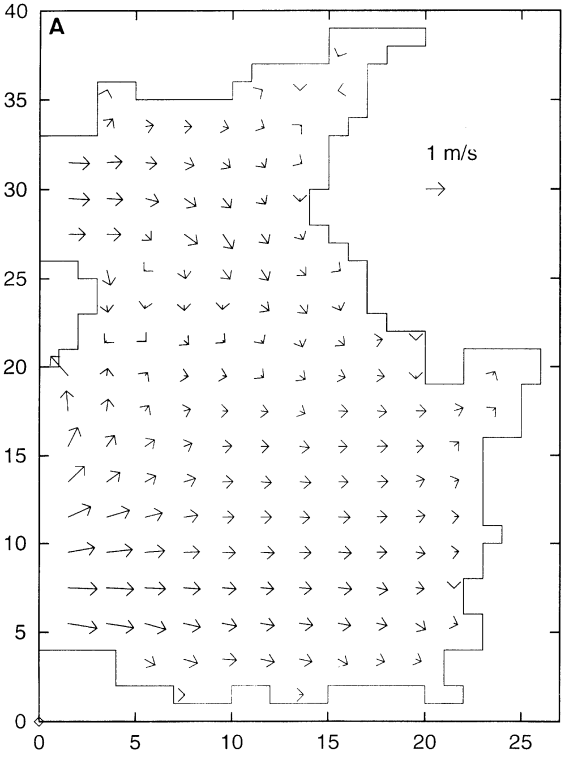

Fig. 7. Surface current maps when water level is increasing (a) and decreasing (b) at north Anglesey.

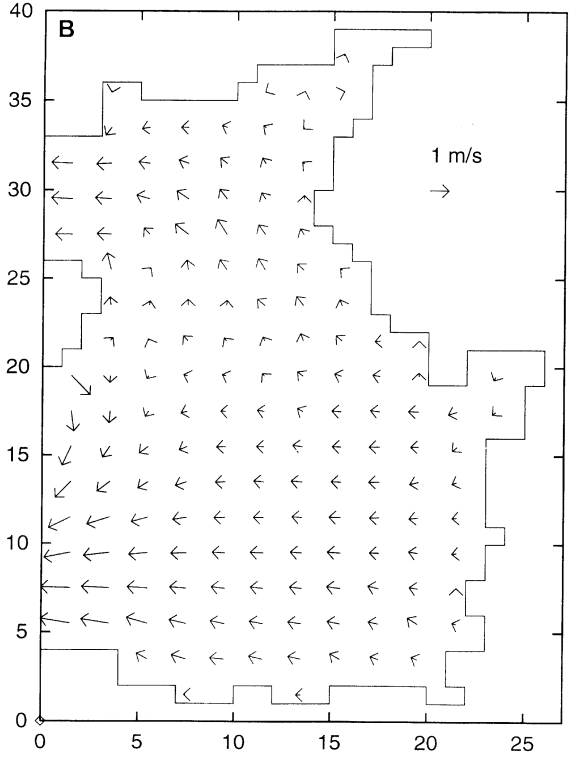

Fig. 7. Continued.

decreases as a result of horizontal and vertical advection and diffusion. Due to vertical dispersion, concentration increases in the middle of the water column and later in the deepest water layer. It can be seen that after some 400 min (6·7 h) vertical mixing has occurred. It is interesting to notice that some oscillations appear in the surface concentrations; they are due to tidal oscillations and have already been observed in a similar experiment carried out with a two-dimensional model (Periáñez *et al.*, 1996*a*). However, these oscillations are not noticeable in bottom water (which must be an effect of the weak water velocities close to the sea bed). Thus, as can be seen in Fig. 9, some vertical structure appears and disappears alternately. A vertical structure parameter, VS, has been defined as

$$VS = \frac{|C_{\text{sur}} - C_{\text{bot}}|}{C_{\text{sur}}} \tag{22}$$

which is the difference between the concentration of radionuclides in the surface and bottom waters relative to the concentration in the surface. In the case of perfectly mixed waters $VS = 0$. The time evolution of VS at

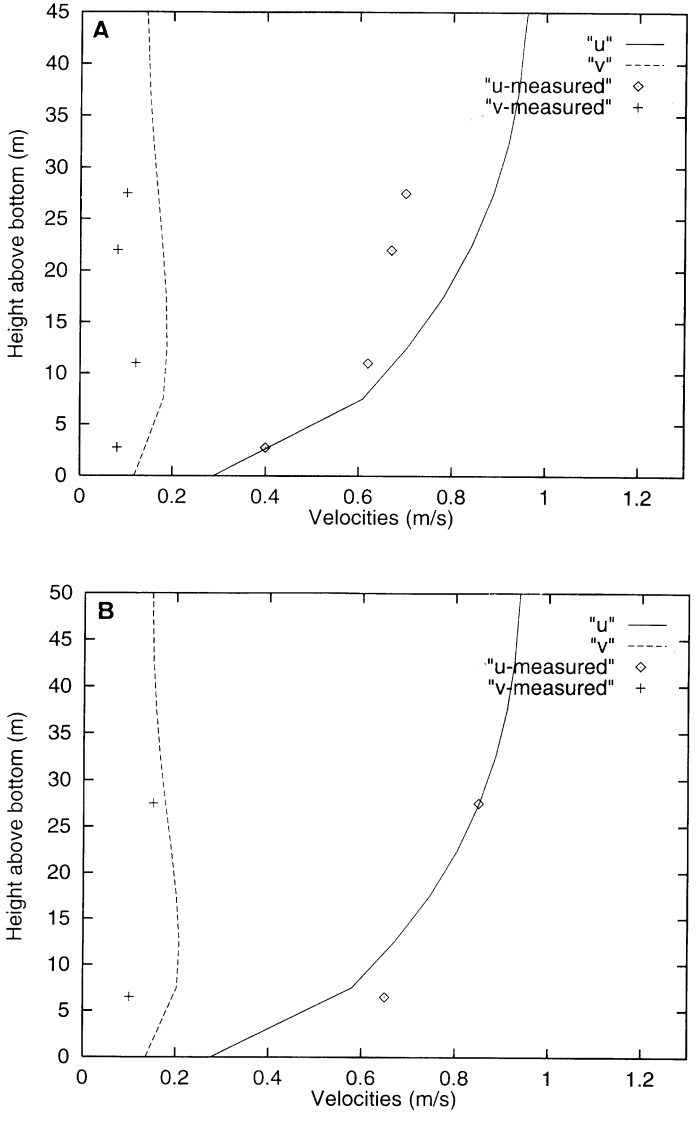

Fig. 8. Current profiles at points 1(a) and 2(b).

compartment (7, 29) can be seen in Fig. 10a. VS oscillates at the same frequency as tidal oscillations; vertical mixing occurs during low water while vertical structure is formed soon after high water. The correlation between water elevation and VS can be seen clearly in Fig. 10b. Initially, $VS = 1$, which is the maximum value (a relative difference of 100%), since an instantaneous discharge has been carried out at the surface. Then VS

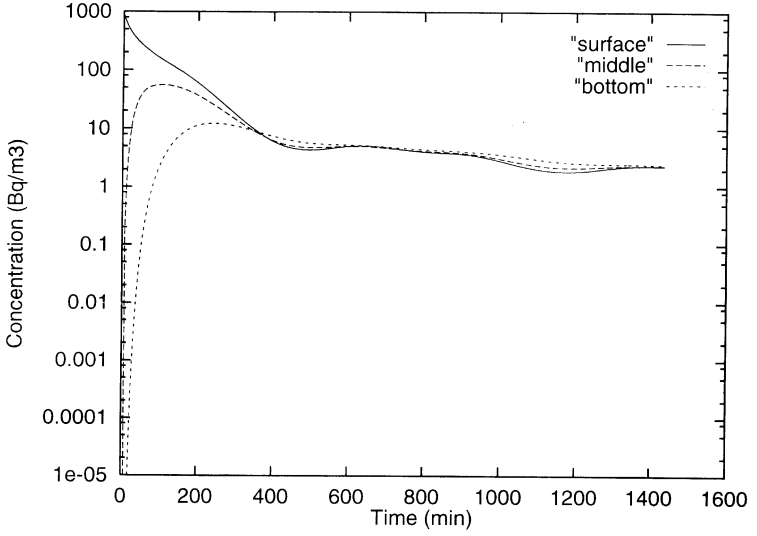

Fig. 9. Time evolution of radionuclide concentration in the upper grid box, a grid box at the middle of the water column and in the deepest box.

decreases as a result of vertical mixing and begins to oscillate. Minimum *VS* values are obtained for minimum water elevations (low water) and maximum *VS* are obtained for maximum elevations (high water). The amplitude of the oscillations decreases in time: from 50% in the first oscillation to 20% in the last. Indeed, after a simulation over 85 tidal cycles the amplitude of *VS* was 4·34%. This attenuation is probably due to the smoothing of horizontal concentration gradients, which leads to a progressive homogenization of contaminants over the sea.

A transverse section of the sea from the west open boundary to the British coast (line $y = 29$ in Fig. 2) is presented in Fig. 11. The distribution of ^{137}Cs at four times after the instantaneous discharge is shown. The effect of vertical dispersion can be seen, as well as the effect of horizontal advection and diffusion.

Observed ^{137}Cs distributions have also been obtained with the model. The real input from Sellafield, which is 325 TBq year^{-1} (Jefferies and Steele, 1989) for 1985, was introduced at the sea surface at point (14, 29). This mean input rate is equivalent to $1{\cdot}04 \times 10^9$ Bq per time step. However, the input has been taking place since the sixties. Thus, instead of starting the model from zero concentration, we have assumed an uniform background of 950 Bq m^{-3} over the sea. This background represents the effect of previous discharges. In previous modelling work (Periáñez *et al.*, 1994), it was shown that model results do not depend upon the way the background is created. Thus, the same results are obtained by assuming uniform concentration

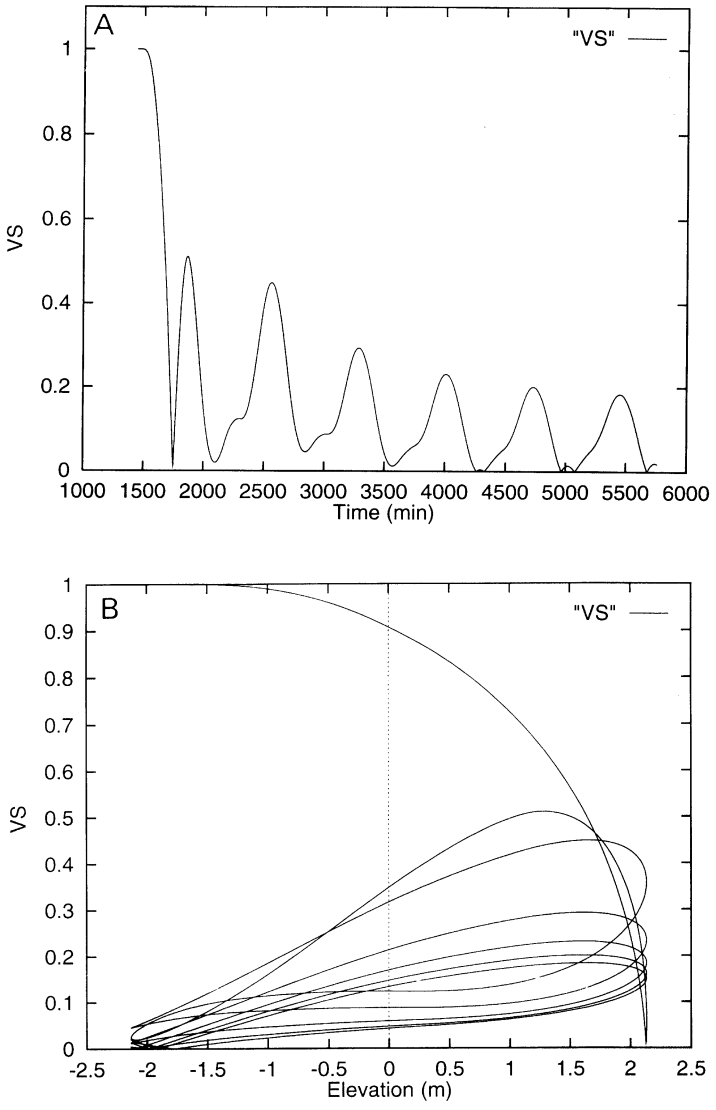

Fig. 10. (a): Time evolution of the vertical structure parameter in compartment (7, 29). (b): Vertical structure parameter vs. water elevation (m).

over the study area or by performing a large discharge and allowing some time to elapse so that this discharge is dispersed through the sea. To save CPU time, the uniform background option was chosen. Thus, discharges from Sellafield are carried out over this uniform background and model results are obtained after a simulation period of 25 days. These results are then compared with observations.

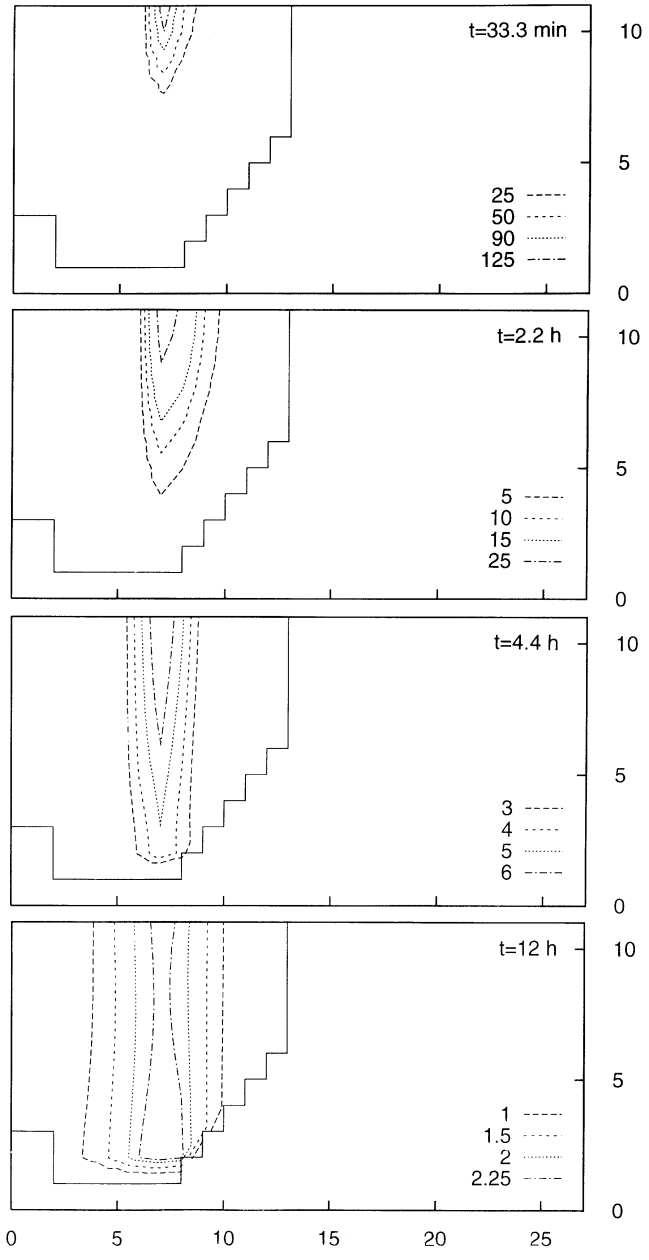

Fig. 11. Transverse sections of the sea at four different times. Concentrations are given in Bq m^{-3}. Each unit in the x and y axis is 5000 m and 5 m, respectively.

A ^{137}Cs distribution map in surface water is presented in Fig. 12. This map is not significatively different from that obtained from observations. It seems that the model overestimates slightly the concentrations in the area north from Sellafield (the Solway) and that the model underestimates the concentrations in the region to the south of Sellafield. However, it must be said that the observed ^{137}Cs map has been drawn by interpolation from several sampling points (see Fig. 12) and there are no sampling points in these regions (the Solway and Morecambe Bay). Thus, we cannot be sure if the model is really overestimating and underestimating the concentrations in these regions. In the rest of the sea, there is a good agreement between the observed and computed concentrations.

Observed and computed concentrations in waters of the British coastline, north and south from Sellafield, can be seen in Fig. 13. There is good agreement between observed and computed concentrations. The model gives a good indication of the tide-induced distribution of ^{137}Cs along the coast.

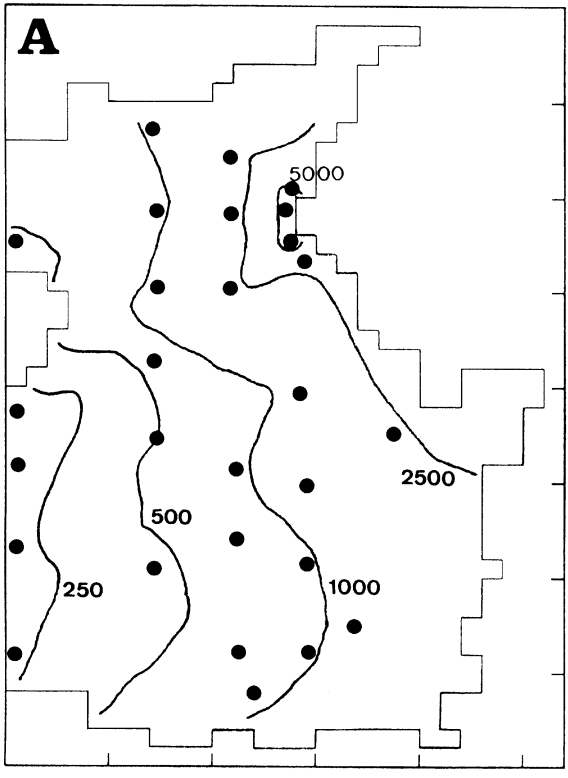

Fig. 12. Observed (a) and computed (b) ^{137}Cs distribution maps in surface waters. Concentrations are given in Bq m^{-3}. Sampling points are also shown in (a).

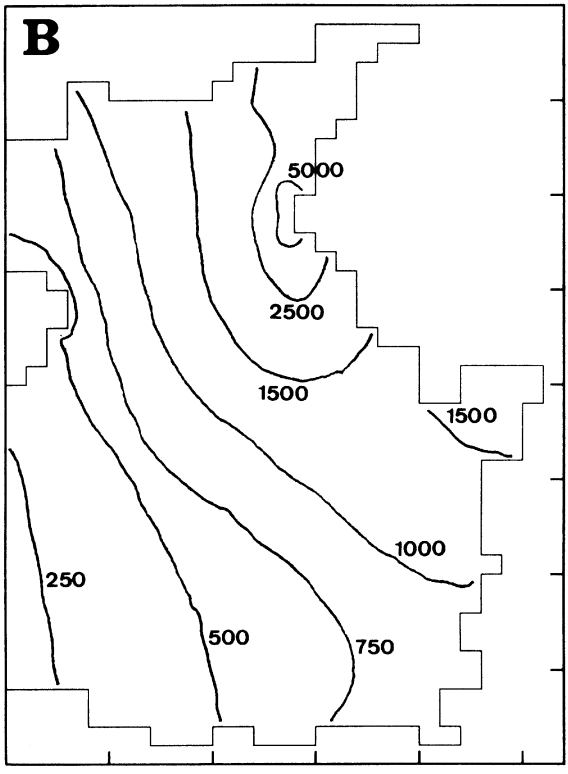

Fig. 12. Continued.

Surface and bottom water samples were collected from the Irish sea in January 1992 (Cook *et al.*, 1997). The sampling points are shown as numbered circles in Fig. 2. The model has been applied to reproduce the measured ^{137}Cs concentrations. The input from Sellafield in 1992, 15 TBq year^{-1}, was obtained from Hunt *et al.* (1997) and the model was started from an uniform background of 150 Bq m^{-3} over the sea. Model results were obtained after a simulation over 20 tidal cycles. Measured and computed ^{137}Cs concentrations in surface and bottom water samples are presented in Fig. 14. The model gives the general distribution of ^{137}Cs in both sets of samples. Indeed, a slight increase of specific activity was detected in the mouth of the Solway (point 4), which is in agreement with the model results. It must be said that the instant of time (tidal state) in which model results are obtained had to be selected by trial and error. This is due to the fact that, as we have seen previously, there are alternating periods of well mixed and structured waters in the vertical. Thus, depending on the tidal state, in a given tidal cycle, when results are obtained, a different

vertical distribution is obtained. Since the tidal state when samples were collected is not known, model results had to be obtained from the model in the moment when they are in the best possible agreement with observations.

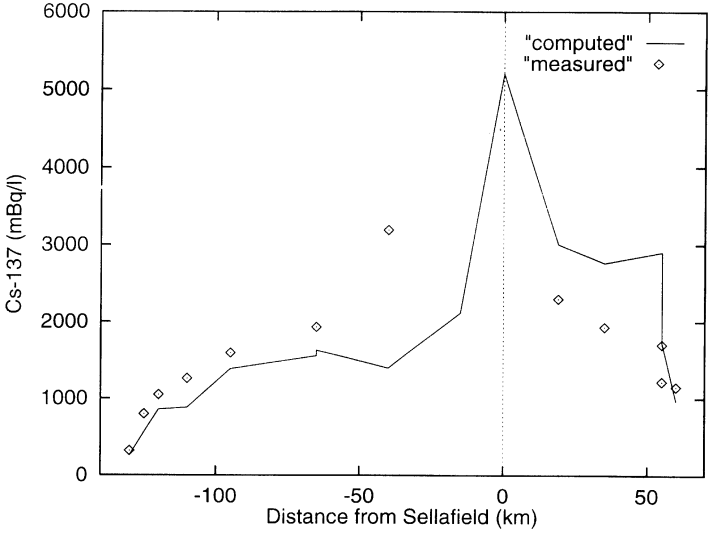

Fig. 13. ^{137}Cs distribution along the coastline north (positive distances) and south (negative distances) from Sellafield.

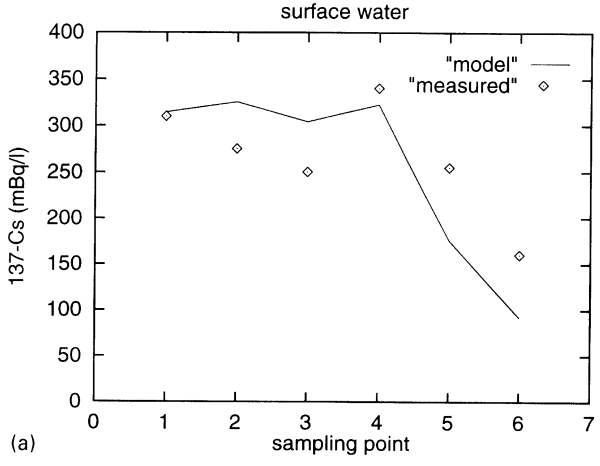

Fig. 14. Measured and computed ^{137}Cs concentrations in surface and bottom waters collected from points shown in Fig. 2.

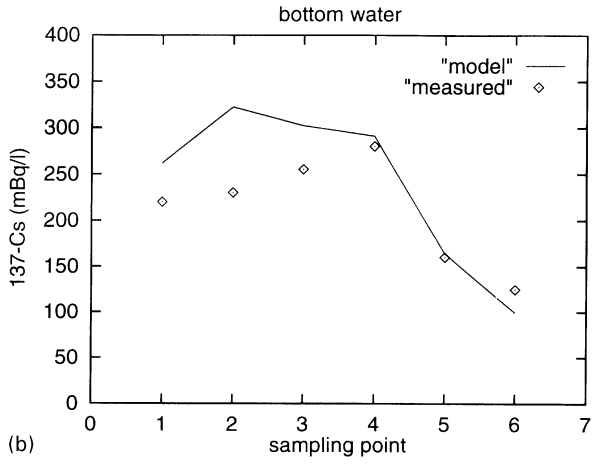

Fig. 14. Continued.

As said in the Introduction, once the model has been validated it could be used as a predictive tool that can give information on the ^{137}Cs concentrations in waters after an hypothetical accidental release.

CONCLUSIONS

A three-dimensional dispersion model for radionuclides has been developed. The model solves the three-dimensional hydrodynamic equations and, simultaneously, the three-dimensional dispersion equation. Thus, the tide induced dispersion of radionuclides is obtained with the model. The model has been applied to the Irish Sea: the hydrodynamic part of the model has been validated by comparing computed water elevations and currents with observations and the dispersion part has been validated by studying ^{137}Cs dispersion, which is released from a nuclear fuel reprocessing plant at Sellafield. Results in good agreement with observations have been obtained for both the hydrodynamic and the dispersion parts of the model.

An interesting alternating effect of well mixed and vertically structured periods with the same frequency as tidal oscillations has also been found.

The results of the hydrodynamic part of the model are realistic and representative of the studied site. Thus, the dispersion model can be implemented over the hydrodynamic model. However, if necessary, the hydrodynamic model could be improved:

- Improving the description of the eddy viscosity by using a turbulent closure model.

- Incorporating salinity and temperature gradients in the equations, so that the model can also be applied to stratified seas.

The model could also be extended to non-conservative radionuclides, following the description in Periáñez *et al.* (1996*b*).

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