Flowmeter random error estimation by an analytical variance estimation method: a simple test bed

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Abstract

This paper uses an experimental method to estimate the variance of flowmeter errors (random errors or Type A errors), focusing on problems of detecting defects existing in sensors even with very small magnitudes. An inexpensive and simple but accurate test bed is shown, based on the detailed experimental estimation method submitted. Some of the difficulties and shortcomings of this estimation are highlighted and a simulation and a real estimation is given.

Keywords: Flowmeter test bed; Sensor fault diagnosis; Variance estimation

1. Introduction

Many fault diagnosis methods have been developed in the last few years. Recently, statistical methods have developed rapidly and attracts many researchers. One of these interesting methods, the generalized likelihood ratio (GLR), introduced for the first time by Willsky and Jones (1974) and then developed by Chafoux, Aituche, and Wang (1994) and Menéndez, Biscarri, and Gómez (1998), enables the detection of defects existing on meters or sensors even with very small magnitudes. However, in practice, the results of direct detection by this method are not very satisfying when the variance of errors (random errors or Type A errors) is not perfectly known. In order to improve the test results, a method for evaluating the variance of errors could be applied previously.

The aim of this paper is to explore the use of a new experimental method to estimate the variance of errors in linear systems through the use of analytical redundancy. This study is useful to perform an optimal flow estimation and also allows us to monitor and improve an on-line diagnosis of meters, keeping in mind that the most detection methods, such as the GLR methods, assume as a preliminary hypothesis that the variance of errors (random errors) is known. The theoretical results are illustrated by means of two examples: a simulated test, in Section 6, and a real test, in Section 7.

2. Measurement uncertainty

This paper agrees with the work by Abernethy published in the US Air Force handbook on measurement uncertainty (Abernethy et al., 1973). That work was careful to divide error sources into two types: bias (or systematic errors) and precision (or random errors). Uncertainty variations are due to bias or/and precision variations. This paper covers random errors on line because they focus on a continuous estimation of random uncertainties.

In this way, the first major error type considered is the random error. Sources of random error will add a component to the result which is unknown but, with repeated measurements, changes in a random fashion. The error component added to the second measurement is uncorrelated to the one which was added to the first measurement. This occurs in the same way with each successive measurement. None of the components are correlated.

Random error components are drawn from an error distribution which is a Gaussian distribution or normal

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distribution (ANSI/ASME PTC 19.1, 1985):

$$F(X) = \frac{1}{\alpha\sqrt{2\pi}} e^{-(X-\mu)^2/2/\sigma^2}. \tag{1}$$

The term σ describes the scatter in the X values of the infinite population about its average, μ. However, an experimenter never has all the data in the infinite population, but rather only a sample of n data points with which to calculate the standard deviation. The scatter in the data is characterized by the sample standard deviation, S_X. But S_X^2 is not always the variance of error in X measurement. When the true value of X, noted X_{true}, is not a constant during the test,

$$S_X^2 = S^2_{X_{meas}-X_{true}} + S^2_{X_{true}}, \tag{2}$$

where X_{meas} is the measured value of X and $(X_{meas} - X_{true})$ is the measurement error. The sample standard deviation of measurement errors in X, $S_{X_{error}}$, is only due to the meter and it is defined by

$$S_{X_{error}} = S_{X_{meas}-X_{true}}. \tag{3}$$

The main objective of all uncertainty analysis is to work out $S_{X_{error}}$ to establish random error limits (random uncertainty).

The fact is that systematic errors do not cause scatter in the test results (they are not observable in the test data). Coming up with an estimate of the magnitude for a systematic uncertainty source is often difficult. ANSI/ASME PTC (1985) provides five methods for obtaining systematic uncertainties: the use of a calibration standard, the use of independent methods to measure the same thing,… . From the point of view of the statistical data analysis, constant systematic errors during the test are always assumed (EAL-R2, 1997). Thus, these errors do not have any influence on variance error estimation.

3. Flow optimal estimation by means of redundancy

A data reconciliation applied on a statistical process defined by linear equations is the study principle for the mass or energy flow measurements in a network. For example, the technique of the reconciliation of data named (FLow Optimal eStimation) (FLOS) (Menéndez et al., 1998), based on the GLR method, enables one to obtain a flow estimation in function of the slanted measure. It requires some suppositions: unbiased errors, not interrelated, with a known variance matrix half-defined positive and with a determinist balance equation set:

$$Q_{estimated} = [H(H^{\mathrm{T}}RH)^{-1}H^{\mathrm{T}}R^{-1}]Q_{meas}, \tag{4}$$

where $Q_{estimated}$ is the vector of estimated flow in the measurement network, Q_{meas} is the vector of measured flow in the measurement network, H is the matrix of the system incidence (resume node flow equation in a distribution network) and R is the variance matrix.

In the example of Fig. 1, where || symbolizes a flowmeter, $Q1$ is the flow across flowmeter 1 and $Q2$ is the flow across flowmeter 2, the FLOS algorithm allows us to get the best estimate

$$Q_{estimated} = \frac{S^2_{Q2_{error}} Q1_{meas} + S^2_{Q1_{error}} Q2_{meas}}{S^2_{Q2_{error}} + S^2_{Q1_{error}}}. \tag{5}$$

However, before it is necessary to have knowledge of the sample standard deviation of errors in meters, $S_{Q1_{error}}$ and $S_{Q2_{error}}$.

4. Random uncertainty estimation

The authors present a new method to obtain the sample standard deviation of errors ($S_{Q_{error}}$). Taking up again the network shown in Fig. 1, only the additional cost of adding a new meter (Fig. 2) is required.

Experimentally, we can measure $Q1$, $Q2$ and $Q3$ with errors which are independent. Standard deviation differences between two redundant measurements are:

$$\begin{aligned}
\sigma_{\Delta1,2} &= \sigma_{(Q1_{meas}-Q2_{meas})} = [(\sigma_{Q1_{error}})^2 + (\sigma_{Q2_{error}})^2]^{1/2},\\
\sigma_{\Delta2,3} &= \sigma_{(Q2_{meas}-Q3_{meas})} = [(\sigma_{Q2_{error}})^2 + (\sigma_{Q3_{error}})^2]^{1/2},\\
\sigma_{\Delta1,3} &= \sigma_{(Q1_{meas}-Q3_{meas})} = [(\sigma_{Q1error})^2 + (\sigma_{Q3error})^2]^{1/2}.
\end{aligned} \tag{6}$$

Even if the process varies, as long as the instruments are observing the same process parameter at the same time, their difference can be used to infer the standard deviation of the three instruments. Thus, it can be deduced that:

$$\begin{aligned}
\sigma_{Q1error} &= [0.5(\sigma^2_{\Delta1,2} + \sigma^2_{\Delta1,3} - \sigma^2_{\Delta2,3})]^{1/2},\\
\sigma_{Q2error} &= [0.5(\sigma^2_{\Delta1,2} + \sigma^2_{\Delta2,3} - \sigma^2_{\Delta1,3})]^{1/2},\\
\sigma_{Q3error} &= [0.5(\sigma^2_{\Delta1,3} + \sigma^2_{\Delta2,3} - \sigma^2_{\Delta1,2})]^{12}.
\end{aligned} \tag{7}$$

Using a sample of measured flows, leads to an estimation of σ_{Q1}, σ_{Q2} and σ_{Q3}:

$$\begin{aligned}
S_{Q1error} &= [0.5(S^2_{\Delta1,2} + S^2_{\Delta1,3} - S^2_{\Delta2,3})]^{12},\\
S_{Q2error} &= [0.5(S^2_{\Delta1,2} + S^2_{\Delta2,3} - S^2_{\Delta1,3})]^{1/2},\\
S_{Q3error} &= [0.5(S^2_{\Delta1,3} + S^2_{\Delta2,3} - S^2_{\Delta1,2})]^{1/2},
\end{aligned} \tag{8}$$

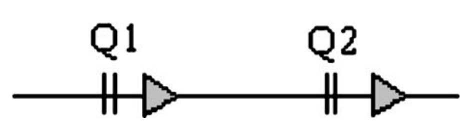

Fig. 1. Two meters in the same pipe.

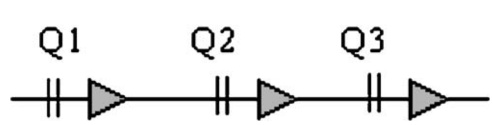

Fig. 2. Three meters in the same pipe.

Table 1
Confidence intervals for the variance

Population variance	Sample variance	Confidence interval for σ^2
$\sigma^2_{Q1_{error}}$	$S^2_{Q1_{error}}$	$\left[\frac{1}{2}(a_{12}+a_{13}-b_{23}),\ \frac{1}{2}(b_{12}+b_{13}-a_{23})\right]$
$\sigma^2_{Q2_{error}}$	$S\sigma^2_{Q2_{error}}$	$\left[\frac{1}{2}(a_{12}+a_{23}-b_{13}),\ \frac{1}{2}(b_{12}+b_{23}-a_{13})\right]$
$\sigma^2_{Q3_{error}}$	$S\sigma^2_{Q3_{error}}$	$\left[\frac{1}{2}(a_{13}+a_{23}-b_{12}),\ \frac{1}{2}(b_{13}+b_{23}-a_{12})\right]$

where

$$S^2_{\Delta p,q} = \frac{1}{N-1}\sum_{k=1}^{N}\left[Qp_k - Qq_k - (\overline{Qp} - \overline{Qq})\right]^2$$
$$\text{for } pq = \{12, 13, 23\},$$

Qj_k is the kth measurement of Q_j $(j = 1,2,3)$, $\overline{Qj}$ is the average of Q_j $(j = 1,2,3)$ and N is the number of data being averaged.

5. Confidence intervals for the estimated standard deviation

A statistical supplementary analysis allows us to infer the confidence interval of $\sigma_{Q1_{error}}$, $\sigma_{Q2_{error}}$ and $\sigma_{Q3_{error}}$ for a fixed level of confidence.

It may be assumed that $\Delta p,q = Q_p - Q_q$ follows a Gaussian distribution of error, with standard deviation $\sigma_{\Delta p,q} = \sigma_{(Q_p - Q_q)} = (\sigma^2_{Q_p} + \sigma^2_{Q_q})^{1/2}$ and mean μ,

$$\Delta p,q \sim N(\mu, \sigma_{\Delta p,q}). \tag{9}$$

Thus, it may be deduced that:

$[(N-1)(S_{\Delta p,q})^2]/(\sigma_{\Delta p,q})^2$ follows a chi-square distribution with $v = N-1$ degrees of freedom,

$$\frac{(N-1)(S_{\Delta p,q})^2}{(\sigma_{\Delta p,q})^2} \sim \chi^2_{N-1}, \tag{10}$$

where N is the number of data points with which to calculate $S_{\Delta p,q}$, and χ^2_{N-1} denotes a chi-square variable with $N-1$ degrees of freedom.

Therefore,

$$P\left[\chi^2_{1-\alpha/2,N-1} \leqslant \frac{(N-1)S^2_{\Delta p,q}}{\sigma^2_{\Delta p,q}} \leqslant \chi^2_{\alpha/2,N-1}\right] = 1-\alpha, \tag{11}$$

where $1-\alpha$ is the confidence coefficient and $P[X]$ symbolizes the probability of X, $\chi^2_{\alpha/2,N-1}$ is the upper $100\alpha/2$ per cent point, so $P[\chi^2_{N-1} \geqslant \chi^2_{\alpha/2,N-1}] = \alpha/2$, and $\chi^2_{1-\alpha/2,N-1}$ is the lower $100\alpha/2$ per cent point, so $P[\chi^2_{N-1} \geqslant \chi^2_{1-\alpha/2,N-1}] = 1-\alpha/2$.

It is clear from this expression that

$$P[a_{pq} \leqslant \sigma^2_{\Delta p,q} \leqslant b_{pq}] = 1-\alpha, \tag{12}$$

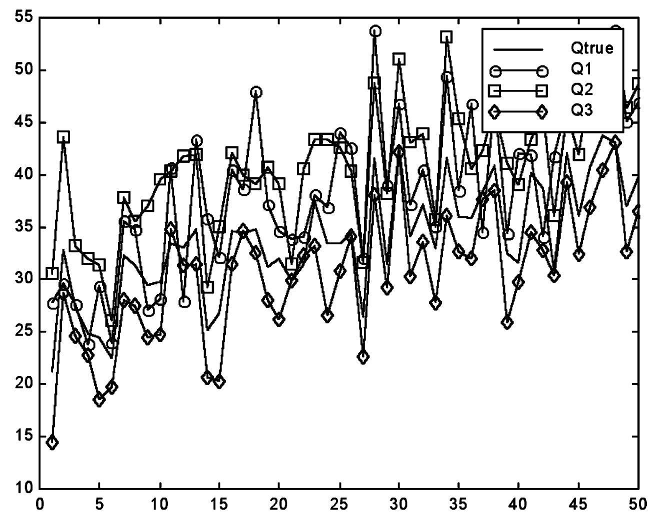

Fig. 3. Simulated aleatory sample.

where

$$a_{pq} = \frac{(N-1)S^2_{\Delta p,q}}{\chi^2_{\alpha/2,N-1}},$$
$$b_{pq} = \frac{(N-1)S^2_{\Delta p,q}}{\chi^2_{1-\alpha/2,N-1}}. \tag{13}$$

The cumulative distribution of χ^2_{N-1} is tabled in Pearson and Hartley (1954). Confidence intervals for $\sigma_{Q1_{error}}$, $\sigma_{Q2_{error}}$ and $\sigma_{Q3_{error}}$ may be obtained from Eqs. (7) and (13). Table 1 gives the obtained results.

The probability that the confidence interval covers the unknown parameter σ_{Qj} $(j = 1,2,3)$ is $100(1-\beta)$ per cent, with $\beta = 1-(1-\alpha)^3$.

6. Simulation

The simple network shown in Fig. 2 is considered in the following. Here the new estimator (Eq. (7)) was applied to a set of simulated flow measurements: $Q1$, $Q2$ and $Q3$ (Fig. 3).

Also presented in Fig. 3 is the simulated true value of flow across the line, Q_{true}. Note that it is always unknown. This variable emphasizes that the variability in Q_j $(j = 1,2,3)$ is due to variability in Q_{true} plus the

Table 2
Simulated flow and added errors

	Added systematic error (m^3)	Added random error (m^3)
Q_{true}	zero	zero
Q_1	+5	$\sigma_{Q1_{error}} = 5$
Q_2	+7	$\sigma_{Q2_{error}} = 3$
Q_3	−3	$\sigma_{Q3_{error}} = 2$

Table 3
Results of the estimation process

	Estimation of the systematic error (m^3)	Estimation of the standard deviation of errors (m^3)*	Estimation of the standard deviation of measurements (m^3)**
Q_{true}	zero	zero	zero
Q_1	—	4.71 [3.74, 5.69]	7.58
Q_2	—	3.26 [1.93, 4.51]	7.56
Q_3	—	1.94 [0.00, 3.63]	6.27

* In brackets, the confidence interval for a level of confidence fixed to 90%.

** It is defined by

$$S_{Qj_{meas}} = \sqrt{\frac{\sum_{k=1}^{N}(Q_{j_k} - \overline{Q_j})^2}{N-1}} \quad j = 1, 2, 3.$$

random error in meter j. If the measured variable cannot be held constant, the random error of flowmeters are not linearly dependent on the standard deviation of Q_j measurements.

Table 2 labels the simulated flow measurements. Using this sample of data (Fig. 3), Table 3 shows the estimation of standard deviation and confidence intervals for a fixed confidence coefficient $(1 - \beta) = 0.90$.

7. Industrial application: calibration test bed

An inexpensive, practical, simple and accurate flowmeter calibration test bed is shown in Fig. 4. The test bed is made up of a tank, a pump, almost three flowmeters and a control valve. Liquid flowmeter installations should be in an orientation which ensures that the flowmeters remain full of liquid when a measurement is desired. Also, the flow profile should be predictable and not distorted. This is accomplished by positioning a straight pipe upstream and downstream of the flowmeters.

7.1. Test-bed considerations

The test bed presented has several advantages over the self-validation scheme: a master flowmeter system

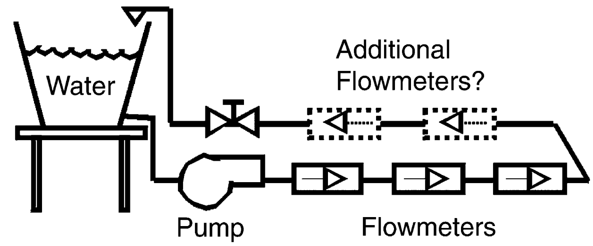

Fig. 4. Flowmeter and water meter test bed.

or a calibrated volumetric tank are not required, and the flow during the test is not exactly known. It is just necessary that sample standard deviations of errors ($S_{Q_{error}}$) are constant over a range of measured flows.

Test bed yield the totalized flow. Flowmeters are commonly used to totalize flows, most often for charging batches, for internal custody transfer and for billing purposes. In custody transfer applications, flow totalization provides the only basis for the cost of the total fluid transfer. Pulse record is easily automated (by a data-logger or a computer) and the installation evaluates, without distinction, flowmeters and water meters.

7.2. Standard $\frac{1}{2}''$ water meter installation

7.2.1. Water meter installation detail

To evaluate three domestic standard water meters (C1, C2 and C3), a simple test bed is made (Fig. 5).

Water meters are designed and prepared for remote reading and correspond to ISO 4064 standards, of class B. This meter was tested and approved by the Spanish Department of Weights and Measures. Limits on the accuracy are set by the standards established for the water industry by the ISO (Europe) and the American Water Works Association (USA). The accuracy of the meter is guaranteed by its manufacturer when it is purchased by the water utility. Fig. 6 shows error tolerance versus flow curve in the meter provided by our manufacturer.

7.2.2. Experimental sample test

The experimental test has been made between Q_t and Q_{max} (2% zone). Figs. 7–9 show data samples.

The first assumption considered is that random errors in C1, C2 and C3 are independent. That is why the difference between two redundant measurements must follow a Gaussian distribution of error. It is appropriate to test this normality. A probability plot or a histogram can help us to check it (Figs. 10–12).

A quantitative test for normality is the Kolmogorov–Smirnov Statistical Test (Bowker & Lieberman, 1972). This and other tests provide fine quantitative answers to the question of normality. However, it is often more

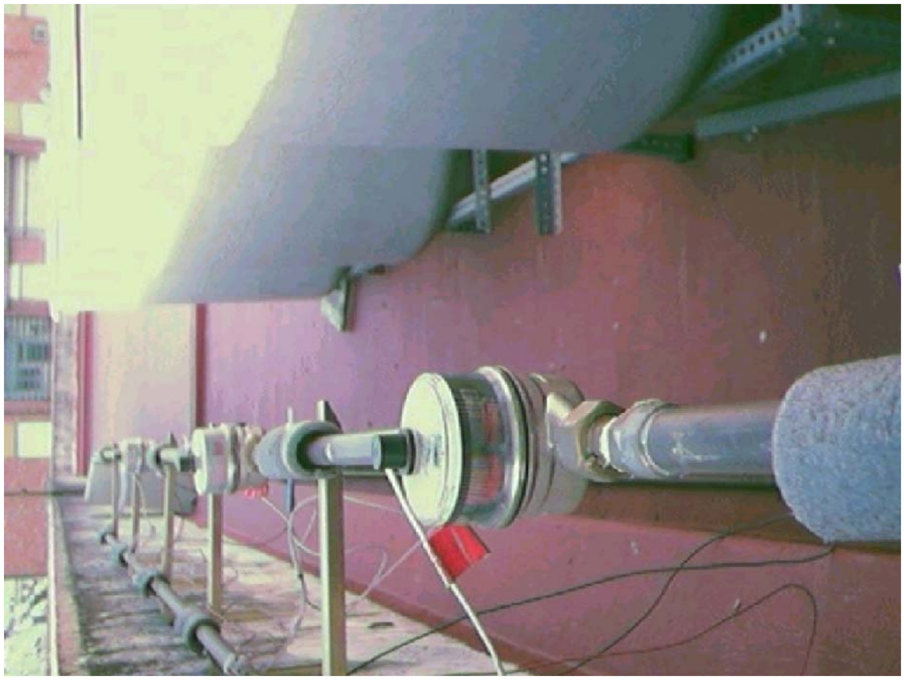

Fig. 5. Experimental test bed.

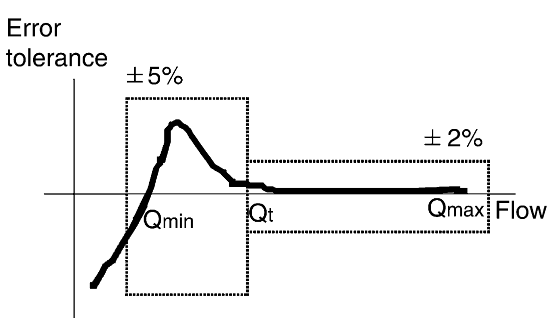

Fig. 6. Error tolerance versus flow.

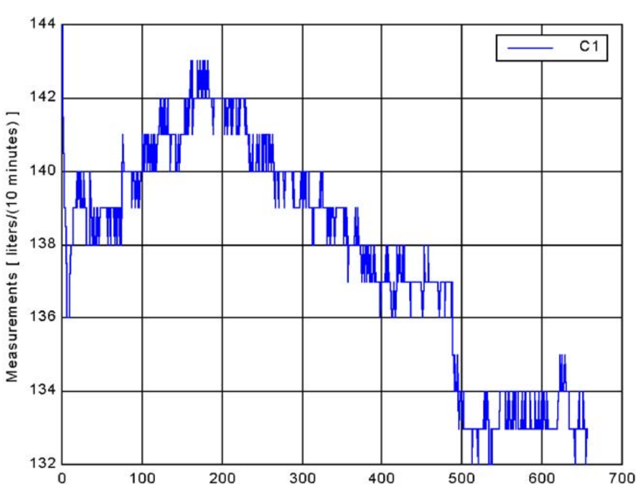

Fig. 7. C1 measurements.

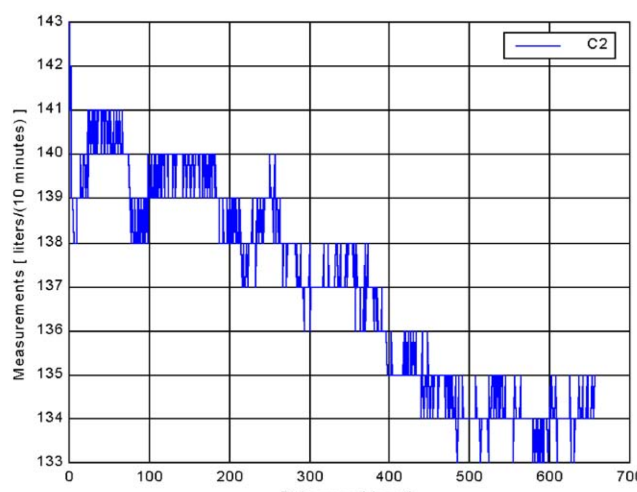

Fig. 8. C2 measurements.

practical and instructive to use a more qualitative technique such as probability plotting.

Now consider the data in Figs. 10–12. At first it appears to be normally distributed. Average values different from zero demonstrate low systematic errors (bias). These errors are constant during the test and have no influence on variance error estimation. Absolute systematic error is not in the test data. If a significant systematic uncertainty is detected, an additional systematic analysis will be necessary to ensure low global uncertainty.

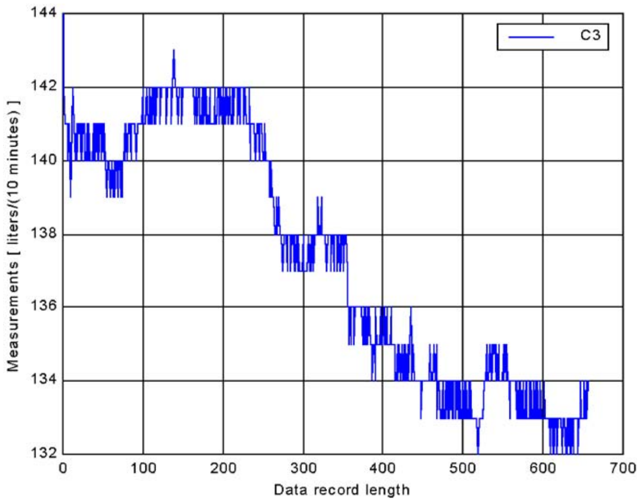

Fig. 9. C3 measurements.

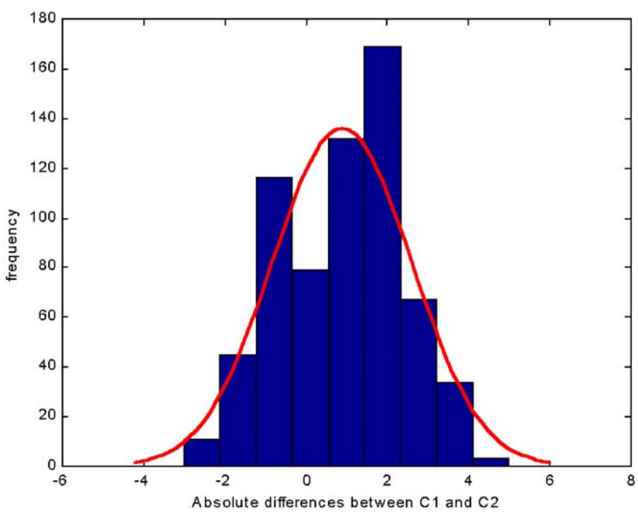

Fig. 10. Histogram of C1, C2 data.

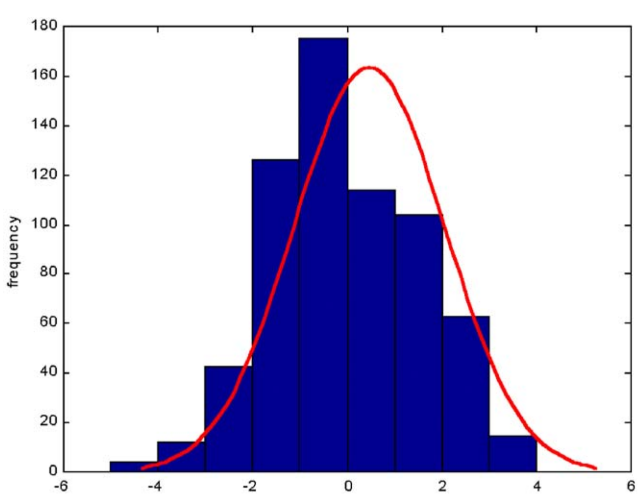

Fig. 11. Histogram of C1–C3 data.

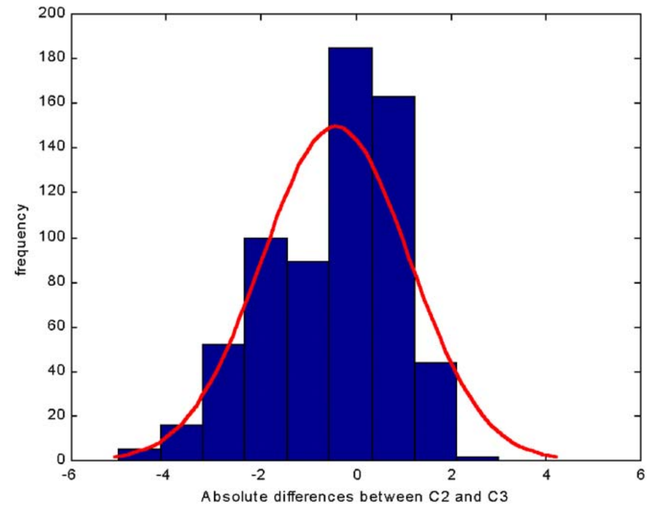

Fig. 12. Histogram of C2, C3 data.

Table 4
Sample standard deviation of errors and confidence intervals

	$S_{X_{error}}$ (l)	Confidence intervals (l)
C1	1.24	[1.04, 1.42]
C2	1.18	[0.97, 1.36]
C3	1.01	[0.76, 1.22]

Table 5
Maximum random error tolerance

	Meter tolerance (l)	Maximum random error tolerance (%)
C1	2.47	1.79
C2	2.35	1.72
C3	2.03	1.47

7.2.3. *Results*

Thus, given this sample of data, Table 4 shows estimated standard deviation of errors ($S_{C1_{error}}$, $S_{C2_{error}}$ and $S_{C3_{error}}$) and confidence intervals.

If Eqs. (14) and (15) are considered,

$$\text{Meter tolerance} = t_{95,650} S_{X_{error}} \ (l) \tag{14}$$

and

$$\text{Maximum random error tolerance (\%)} = 100 \frac{\text{Meter tolerance}}{\bar{V}_{Ci}} \% \tag{15}$$

are the model of errors, where $\bar{V}_{Ci}$ is the sample average of Ci ($i = 1, 2, 3$). Table 5 presents the results obtained.

All water meters tested comply with the manufacturer's specification sheet.

8. Continuous monitoring and fault diagnosis

In order to discriminate normal and abnormal behaviour, statistical confidence limits for each of the measured variables must be calculated. Violations of these limits are considered to be indicative of abnormal behaviour (Gertler, 1998; Xun Wang, Uwe Kruger, & Barry Lennox, 2003). The confidence limits relate generally to the statistical properties of the process variables and are defined typically in terms of the percentage of data points which fall outside certain thresholds in a given time interval.

Gallagher, Wise, Butler, White, and Barna (1997) emphasized that most industrial processes are time-variable and thus require an adaptative rather than a fixed model. For the monitoring of such processes, it is required that the model may be updated to accommodate for time-varying behaviour while still being able to detect abnormal behaviour according to confidence limits which may also have to vary with time.

Applications of this paper for industrial process monitoring are based on a meter error estimation which has been produced from the analysis of measured data on the process and allows us to establish the limit of abnormal behaviour. This application provides a continuous and easily monitored error estimation based on measured data. Time- and flow-varying confidence limits were set up.

Thus, from the point of view of stationary process behaviour, this paper provides confidence limits varying with flow (Fig. 6) and from a non-stationary point of view, confidence limits which also vary with time, because of ageing and small shifts.

9. Conclusions

The new method presented in this paper allows us to deduce the variance of errors (random uncertainty or Type A) in flowmeters $Q1$, $Q2$ and $Q3$ (Fig. 2) from a sample of data with random and systematic errors. This result can be extended to the other flowmeters in a branched or multibranched line.

Random uncertainties make it possible to perform a flow optimal estimation. Also, the study can be used as a useful tool for:

- Testing the meters.
- Preventive maintenance.
- Bearing wear and/or damage detection.

On the other hand, the added flowmeter ($Q3$ in Fig. 2) should be accessible and may require some routine service to minimize systematic error.

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