The P versus NP problem: Unconventional insights from Membrane Computing

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The P versus NP problem (I)

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- Finding solutions versus checking the correctness of solutions.
- Proofs versus verifying their correctness.

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P = NP?

- Finding solutions versus checking the correctness of solutions.
- Proofs versus verifying their correctness.

- This is essentially the central problem of Computational Complexity theory

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Attacking the P versus NP problem

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- $\mathbf{P} \neq \mathbf{NP}$.
  - Find an **NP**-complete poblem such that it does not belong to the class **P**.
- $\mathbf{P} = \mathbf{NP}$.
  - Find an **NP**-complete poblem such that it belongs to the class **P**.

Goal:

- Unconventional approaches/tools to attack the **P versus NP problem** are given by using **Membrane Computing**.

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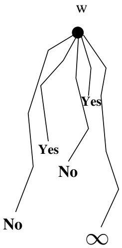
  - The devices of this paradigm (*P systems* or membrane systems), provide distributed parallel and nondeterministic computing models.

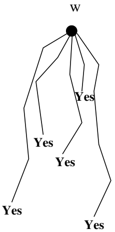
- A computational complexity theory in Membrane Computing is proposed.
  - Polynomial complexity classes associated with (cell–like and tissue–like) P systems are presented.
    - A notion of acceptance must be defined in the new framework (different than the classical one for nondeterministic Turing machines)

The notion of acceptance

Nondeterministic Turing machines



Membrane systems



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- In the real-life, many abstract problems are combinatorial optimization problems not decision problems.
- Every decision problem has associated a language in a natural way.
- The solvability of decision problems is defined through the **recognition** of the languages associated with them.

Recognizer Membrane Systems

- Cell-like P systems: $\Pi = (\Gamma, \Sigma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
- Tissue-like P systems: $\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$

RGNC

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  - The working alphabet contains two distinguished elements *yes* and *no*.
  - All computations halt.
  - For any computation of the system, either object *yes* or object *no* (but not both) must have been sent to the output region of the system, and only at the last step of the computation.

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- Accepting/rejecting computations for recognizer P systems

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- Computation.
  - *Halting computation* (accepting or rejecting)

Polynomial time solvability

- A decision problem X is *solvable in polynomial time* by a family of recognizer membrane systems $\mathbf{\Pi} = \{\Pi(n) : n \in \mathbf{N}\}$, iff:
  - ★ The family $\mathbf{\Pi}$ is polynomially uniform by Turing machines, that is, there exists a DTM working in polynomial time which constructs the system $\Pi(n)$ from $n \in \mathbf{N}$.

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  (a) for each instance $u \in I_X$, $s(u)$ is a natural number and $cod(u)$ is an input multiset of the system $\Pi(s(u))$;

  (b) for each $n \in \mathbf{N}$, $s^{-1}(n)$ is a finite set;

  (c) the family $\mathbf{\Pi}$ is polynomially bounded with regard to (X, cod, s), that is, there exists a polynomial function p, such that for each $u \in I_X$ every computation of $\Pi(s(u))$ with input $cod(u)$ is halting and it performs at most $p(|u|)$ steps;

  (d) the family $\mathbf{\Pi}$ is sound with regard to (X, cod, s), that is, for each $u \in I_X$, if there exists an accepting computation of $\Pi(s(u))$ with input $cod(u)$, then $\theta_X(u) = 1$;

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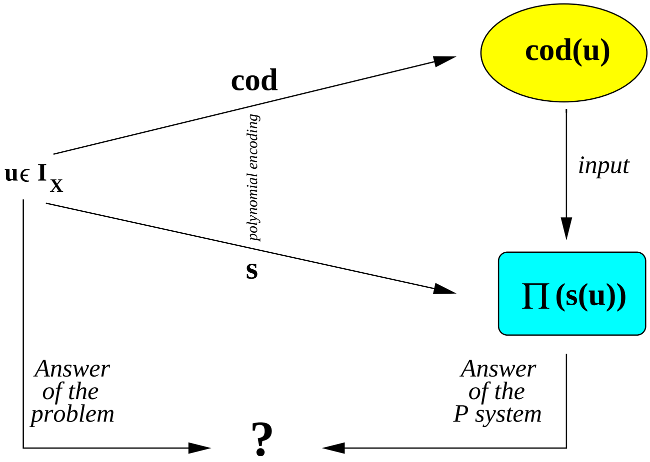
- We denote it by $X \in \mathbf{PMC}_{\mathcal{R}}$

- $\mathbf{PMC}_{\mathcal{R}}$ is closed under complement and polynomial–time reductions.

Solvability of a decision problem



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Frontiers of the efficiency:

- M_1 efficient.
- M_2 non efficient.
- $M_2 \subseteq M_1$: each solution S of a problem X in M_2 is also a solution in M_1.

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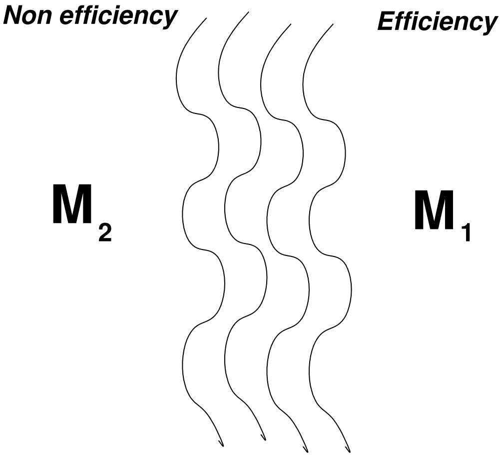
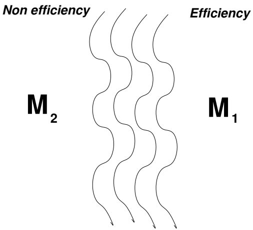
Passing from M_2 to M_1 amounts to passing from non efficiency to efficiency.

Managing frontiers of the efficiency



Attacking the P versus NP problem



P = NP

- Finding an **NP**-complete problem efficiently solvable in M_2.
  - Traslating a polynomial time solution of an **NP**-complete problem in M_1, to a a polynomial time solution in M_2.

P ≠ NP

- Finding an **NP**-complete problem that is not polynomial time solvable in M_2.

Basic cell-like membrane systems

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- *Basic transition* P systems:
  - $[\ u\ ]_h \rightarrow [\ v\ ]_h$ (evolution rules).
  - $[\ u\ ]_h \rightarrow v\ [\ \ ]_h$ and $u\ [\ \ ]_h \rightarrow [\ v\ ]_h$ (communication rules).
  - $[\ u\ ]_h \rightarrow v$ (dissolution rules).
- $\mathcal{T}$: class of recognizer basic transition P systems.

Efficiency of cell-like membrane systems

- **Proposition 1** (`Sevilla theorem`, 2004)
  Every DTM working in polynomial time can be simulated in polynomial time by a family of recognizer basic transition P systems.

- **Proposition 2** (`Milano theorem`, 2000)
  If a decision problem is solvable in polynomial time by a family of recognizer basic transition P systems with input membrane, then there exists a DTM solving it in polynomial time.

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- **Theorem:** $\mathbf{P} = \mathbf{PMC}_{\mathcal{T}}$ (Sevilla team, 2004).

  - **Corollary:** $\mathbf{P} \neq \mathbf{NP}$ if and only if every, or at least one, **NP**–complete problem is not in $\mathbf{PMC}_{\mathcal{T}}$.

P systems with active membranes

- Electrical charges associated with membranes.
- Type of rules:

  (a) $[\, a \to u \,]_h^{\alpha}$ (*object evolution* rules).

  (b) $a\, [\ ]_h^{\alpha_1} \to [\, b \,]_h^{\alpha_2}$ (*send–in communication* rules).

  (c) $[\, a \,]_h^{\alpha_1} \to [\ ]_h^{\alpha_2}\ b$ (*send–out communication* rules).

  (d) $[\, a \,]_h^{\alpha} \to b$ (*dissolution* rules).

  (e) $[\, a \,]_h^{\alpha_1} \to [\, b \,]_h^{\alpha_2}\ [\, c \,]_h^{\alpha_3}$ (*division* rules for *elementary membranes*).

  (f) $[\, [\,]_{h_1}^{\alpha_1}\ [\,]_{h_2}^{\alpha_2} \,]_h^{\alpha} \to [\, [\,]_{h_1}^{\alpha_3} \,]_h^{\beta}\ [\, [\,]_{h_2}^{\alpha_4} \,]_h^{\gamma}$ (*division* rules for *non–elementary membranes*).

- Non cooperation.
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- The sets $\mathcal{NAM}$, $\mathcal{AM}(-ne)$ and $\mathcal{AM}(+ne)$.

Efficiency of P systems with active membranes

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- Bounds for the complexity class $\mathbf{PMC}_{\mathcal{AM}(+ne)}$:
  - $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}(+ne)} \subseteq \mathbf{EXP}$ (A.E. Porreca, G. Mauri and C. Zandron, 2006).

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- **Conclusion:** $\mathcal{AM}$ is too powerful from the complexity point of view.

Polarizationless P systems with active membranes

- $\Pi = (\Gamma, \Sigma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$:

  (a) $[\, a \to u \,]_h$ (*object evolution rules*).

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  (e) $[\, a \,]_h \to [\, b \,]_h\ [\, c \,]_h$ (*division rules for elementary membranes*).

  (f) $[\, [\,]_{h_1}\ [\,]_{h_2}\,]_h \to [\, [\,]_{h_1}]_h\ [\, [\,]_{h_2}]_h$ (*division rules for non–elementary membranes*).

- The sets $\mathcal{NAM}^0$, $\mathcal{AM}^0(\alpha, \beta)$,where $\alpha \in \{-d, +d\}$ and $\beta \in \{-ne, +ne\}$.

A Păun's conjecture

At the beginning of 2005, Gh. Păun (problem **F** from [1]) wrote:

> *My favorite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible – and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.*

The so–called Păun's conjecture can be formally formulated:

$$\boxed{\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0(+d,-ne)}}$$

[1] Gh. Păun: Further twenty six open problems in membrane computing. *Third Brainstorming Week on Membrane Computing* (M.A. Gutiérrez et al. eds.), Fénix Editora, Sevilla, 2005, pp. 249–262.

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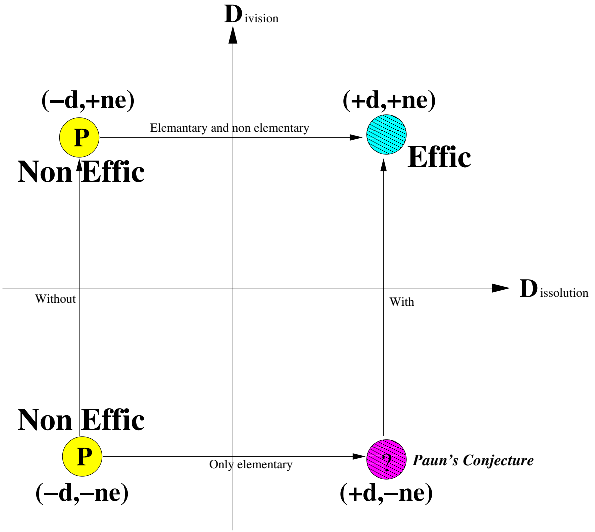
A borderline of the efficiency

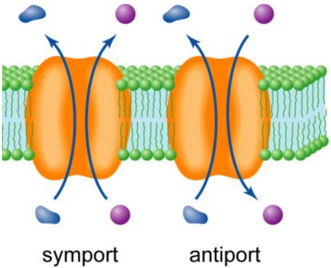
- dissolution rules in $\mathcal{AM}^0(+ne)$.

On efficiency of polarizationless P systems with active membranes



Tissue-like membrane systems (I)

- $\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
- Basic tissue P systems:
  - $(i, u/v, j)$, for $i, j \in \{0, 1, \ldots, q\}$, $i \neq j$, and $u, v \in \Gamma^*$ (*symport-antiport rules*).
  - *Length* of the rule $(i, u/v, j)$: $|u| + |v|$.



The set $\mathcal{TC}$.

Tissue-like membrane systems (II)

- Tissue P systems with cell division:
  - *Symport-antiport rules*.
  - $[\ a\ ]_i \rightarrow [\ b\ ]_i [\ c\ ]_i$, where $i \in \{1, 2, \ldots, q\}$ and $a, b, c \in \Gamma$ (*division rules*).
- Tissue P systems with cell separation:
  - *Symport-antiport rules*.
  - $[\ a\ ]_i \rightarrow [\ \Gamma_1\ ]_i\ [\ \Gamma_2\ ]_i$, where $i \in \{1, 2, \ldots, q\}$, $a \in \Gamma$, $i \neq i_{out}$ and $\{\Gamma_1, \Gamma_2\}$ is a fixed partition of Γ (*separation rules*).

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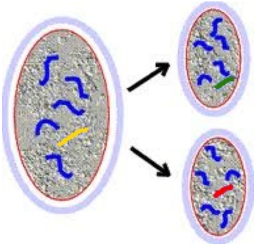
- The sets $\mathcal{TDC}$, $\mathcal{TSC}$, and $\mathcal{TDC}(k)$, $\mathcal{TSC}(k)$, for each $k \geq 1$.

- *Cell Division*.



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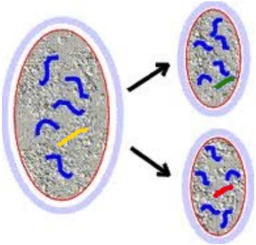
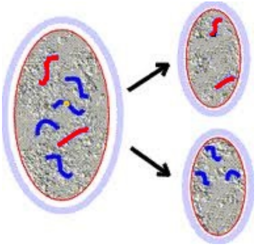


- *Cell Separation*.



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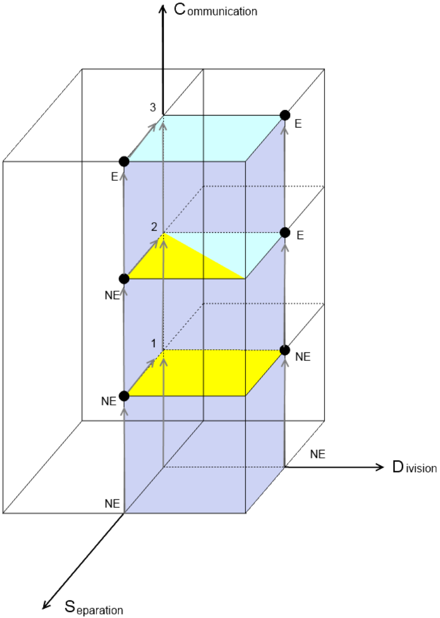

Tissue P systems without environment

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- Tissue-like P systems: $\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
  - The objects of $\mathcal{E}$ initially appear located in the environment in an arbitrary number of copies.
- Tissue-like P systems without environment: $\mathcal{E} = \emptyset$.
- The classes $\widehat{\mathcal{TC}}$, $\widehat{\mathcal{TDC}}$, $\widehat{\mathcal{TSC}}$, and $\widehat{\mathcal{TC}(k)}$, $\widehat{\mathcal{TDC}(k)}$, $\widehat{\mathcal{TSC}(k)}$, for each $k \geq 1$.

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Each of them provides a new way to attack the P versus NP problem.

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