CMMoST 2019

5th INTERNATIONAL CONFERENCE ON

Mechanical Models in Structural Engineering

Alicante, SPAIN
23 - 25 October 2019
Escuela Politécnica Superior
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Edita: Editorial Club Universitario
C/ Decano, n.º 4 – 03690 San Vicente (Alicante)
www.ecu.fm
original@ecu.fm

ISBN: 978–84–17924–58–4
ISBN papel: 978–84–17924–22–5

Printed in Spain

Organizan:

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ADHESIVES·SEALANTS·CHEMICAL PRODUCTS FOR BUILDING

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CONTENTS

A TOPOLOGICAL ENTROPY-BASED APPROACH FOR DAMAGE DETECTION OF CIVIL ENGINEERING STRUCTURES

Jiménez-Alonso, Javier Fernando[1]; López-Martínez, Javier[2], Blanco-Claraco, José Luis[3], González-Díaz, Rocío[4] and Sáez, Andrés[5]

ABSTRACT

In this study, a new method based on topological entropy, the so-called persistent entropy, is presented for addressing the damage detection problem in civil engineering structures. The efficiency of the algorithm is certificated by the stability theorem for persistent entropy. The method, which has been previously used for the classification of DC electrical motors, has been adapted herein to solve the supervised classification damage detection problem. For this purpose, a laboratory footbridge was designed, built and subsequently damaged. Two states of the structure were considered: (i) undamaged and (ii) damaged. Its response under ambient conditions was recorded both numerically and experimentally. Later, its persistent entropy was computed under the different situations. Finally, it was checked that there is a clear relationship between the stiffness of the structure and its persistent entropy. Therefore, the proposed method can be considered as an available tool for the damage detection of civil engineering structures.

Keywords: damage detection, structural health monitoring, topological data analysis, persistent homology, persistent entropy, supervised classification

1. INTRODUCTION

Civil engineering structures play a crucial role in modern societies, to the extent that maintenance plans must be established in order to know the state of conservation of these infrastructures. Among the different method, structural health monitoring (SHM) represents an important tool in management activities [1]. Health monitoring permits to identify early and progressive structural damage. In a broad sense, damage identification can be broken down hierarchically into three categories [1]: (i) damage detection, (ii) damage diagnosis (location, type and severity) and damage prognosis. On the other hand, there are possibly two main approaches for structural health monitoring [1]: (i) the physics-based and (ii) the data-based. The physics-based approach makes use of the inverse problem techniques to update numerical models and attempts to identify damage by comparing the measured data from the structure with estimated data from the updated models. The data-based approach is based on the machine learning field where machine learning algorithms are used to learn the behaviour of the structure from the experience (recorded data). Machine learning algorithms are usually classified as: (i) supervised and (ii) unsupervised learning. Supervised learning

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refers to the case where data from the undamaged and damaged structure are available to train the algorithms. Unsupervised learning refers to the case where training data are only available from the undamaged conditions. The data-based approach presents as main advantage a lower computational cost and a greater simplicity. This approach, under supervised learning, is usually based on the comparison between two signals [2]. There are several methods for measuring the similarity among signals [3]. To the best of the author's knowledge, these methods may be organized mainly in five groups: (i) distance-based methods; (ii) time-frequency analysis, (iii) wavelet analysis, (iv) global-description distance method and (v) local-feature distance method.

This manuscript focuses on using computational topology [3] as a tool to compute the similarity between signals and thus to assess its possible use as a data-based supervised learning method for damage detection. For this purpose, a topological entropy-based method [3], previously used to classify DC electrical motors under a supervised learning approach, has been implemented herein to identify severe damage in civil engineering structures. As benchmark, a laboratory steel structure has been designed, built and damaged. The performance of the topological entropy-based approach has been assessed numerically and experimentally. The response of the structure (accelerations) in several characteristic points has been recorded for this purpose. The structure has only been excited by ambient vibration. As reference the variation of the modal properties of the structure in terms of the level of damage has been provided. As result of this study, the topological entropy-based approach allows identify the change of the response of the structure associated with the occurrence of a severe damage highlighting its available use as tool for damage detection. Nevertheless, further studies are needed in order to analyse the sensibility of the persistent entropy to the change of the environmental and operational conditions and to extend the use of this technique to other steps inside the structural health monitoring categories, as for instance, the damage localization.

The manuscript is organized as follows; in section 2 some basics about computational topology are introduced. Subsequently, in section 3, the methodology used to damage detection based on the topological comparison of signals is described in detail. Later, in section 4, a numerical and experimental case study is presented in order to check and validate the proposed methodology. Finally, in section 5, some remarkable conclusions are included to finish the manuscript.

2. BASICS OF COMPUTATIONAL TOPOLOGY

A topological space is a powerful tool to describe the connectivity of a space. A topological space can be defined informally as a set of points equipped with the notion of neighbouring (formally endowed with a topology) [4]. A topology on a set of points $\mathbf{X}$ is a family of subsets $T \subseteq 2^{\mathbf{X}}$ which satisfies : (i) if $S_1, S_2 \in T$, then $S_1 \cap S_2 \in T$; (ii) if $S_j (j \in J) \subseteq T$ then $\bigcup_{j \in J} S_j \in T$; and (iii) $\emptyset, \mathbf{X} \in T$. Thus, the pair $(\mathbf{X}, T)$ of a set $\mathbf{X}$ and a topology T is a topological space E. A topological space may be decomposed into simple pieces of lower dimension (abstract simplicial complex). Each abstract simplicial complex K is represented by: (i) a set $V$of 0-simplices (called vertices); (ii) for each $k \geq 1$ a set of $k-$ simplices $\sigma = \{v_0, v_1, \dots, v_k\}$ where $v_i \in V$; (iii) each $k-$ simplex has $k+1$ faces obtained by removing one of the vertices; and (iv) if σ belongs to K, then all the faces of σ must belong to K [3]. Finally, a simplicial complex $K$is the geometric representation of an abstract simplicial complex K. For the rest of the manuscript, both the abstract simplicial complex and the simplicial complex will be called simplicial complex and denoted as K for the sake of the clarity of the exposition [4].

Homology is an algebraic tool which allows describing a topological space E. The $k-$Betti number represents the rank of the $k-$dimensional homology group. In this manner, β_k, counts the number of

k −dimensional holes characterizing E (β_0 is the number or connected components, β_1 is the number of holes in 2D and β_2 is the number of voids in 3D). Persistent homology is a method for computing k −dimensional holes at different spatial resolutions. In order to compute persistent homology, a distance function, which can be obtained constructing a filtration on the simplicial complex, is necessary. In this manner, a filtered simplicial complex is a collection of sub-complexes $\{K(t): t \in \mathbb{R}\}$ of K such that $K(t) \subset K(s)$ for $t < s$ and there is $t_{max} \in \mathbb{R}$ such that $K_{t_{max}} = K$. Persistent homology describes how the homology of K changes along a filtration [4]. The set of intervals representing birth and death times of homology classes is called the *persistence barcode* associated to the corresponding filtration. Instead of bars, a set of points in the plan, called *persistence diagram*, such that a point $(xy) \in \mathbb{R}^2$ (with $x < y$) corresponds to a bar (x, y) in the persistence barcode. Finally, the measurement of the order of the construction of the filtered simplicial complex is performed using the so-called persistent entropy. The persistent entropy may be defined as follows [3]. Given a filtered simplicial complex $\{K: t \in \mathbb{R}\}$ and the corresponding persistent diagram $D = \{a_i = (x_i, y_i): i \in I\}$ with $x_i < y_i$ for all $i \in I$, the persistent entropy H of the filtered simplicial complex may be computed as follows:

$$H = -\sum_{i \in I} p_i \log(p_i) \tag{1}$$

where $p_i = \frac{l_i}{L}$, $l_i = y_i - x_i$ and $L = \sum_{i \in I} l_i$. The maximum persistent entropy corresponds to the case in which all the intervals in the barcode are of equal length, $H = \log n$, being n the number of elements of I. Aversely, the value of the persistent entropy decreases as more intervals of different length are presented.

3. TOPOLOGY COMPARISON OF SIGNALS

The aim of this study is to address the problem of damage detection of civil engineering structures based on the comparison between the shapes of the signals recorded under two different states (undamaged and damaged). For this purpose, the topology of the shape of the signals has been analysed. The method is based on algebraic topology [4]. The method consists of the following steps: (i) the signals are transformed into filtered 1-dimensional simplicial complexes; (ii) the persistent homology of these simplicial complexes is analysed; (iii) subsequently the persistent entropy is computed from the homological groups; and (iv) the relative difference between the persistent entropy of the two considered states is determined as one-dimensional feature that characterize the damage. Figure 1 summarizes the process needed to compute the persistent diagram of a signal.

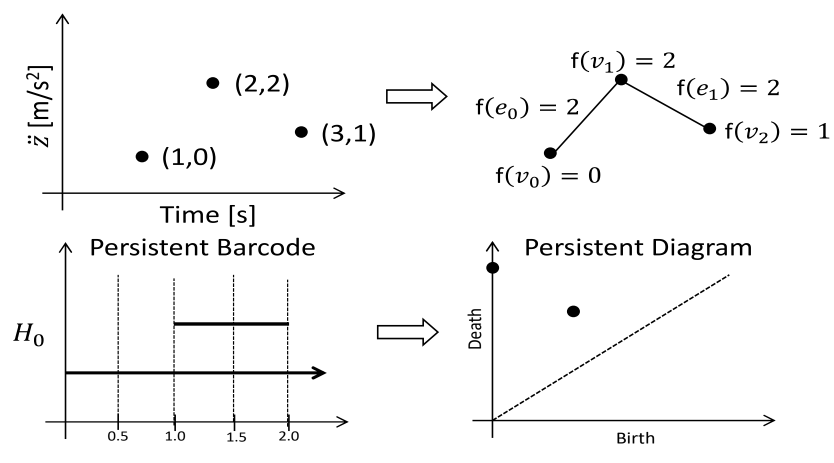

Figure 1. *Graphical representation of the topological signal analysis method [3].*

4. CASE STUDY: DAMAGE DETECTION OF A LABORATORY STEEL STRUCTURE

As benchmark, a steel laboratory was designed, built and damaged to assess the performance of the topology entropy-based method. The structure consists in two parallel Warren trusses braced laterally by strut elements (Figure 2). The structure has a single span of 6 m of length. The depth of the structure is about 0.30 m. The two lateral Warren trusses are separated 0.60 m. All the elements of the structure are square tube profiles with dimensions 30x30x2 mm (Figure 3).

Figure 2. *Laboratory footbridge at the Universidad de Almeria.*

A finite element model of the structure was performed in the software Ansys [5]. The numerical models consist of 46 nodes linked by 107 beam elements (BEAM188). The structure is simply supported in its four extreme lower nodes. The mechanical properties considered for the steel material are: (i) density, $\rho_s = 7850$ kg/m^3; (ii) Young's modulus, $E_s = 210000$;MPa and (iii) Poisson's ratio, $\upsilon_s = 0.3$.

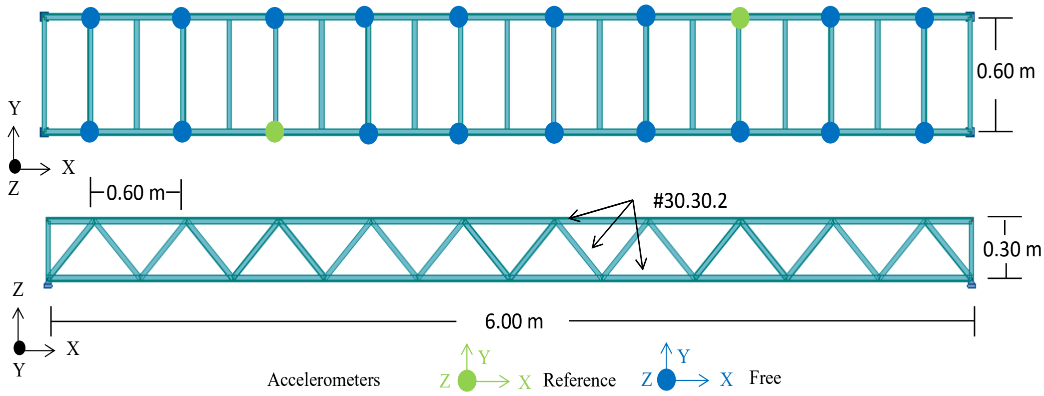

Figure 3. *Numerical and experimental ambient vibration test*

In order to assess the performance of the topological entropy-based method, the structure was damaged numerically and physically. The persistent entropy of the response of the structure under the two states (undamaged and damaged) was compared. The study was organized in two steps: (i) numerically and (ii) experimentally. Additionally, the modal properties of the structure under the two states were identified both numerically and experimentally.

The stiffness of the upper chord at mid-span was reduced by a severe damage (both numerically and experimentally). An ambient vibration test was performed numerically and experimentally to assess the dynamic behavior of the structure under the two considered states. The dynamic response (accelerations) of the nodes of the upper chords of the structure was recorded. Two points were considered as references. Each ambient vibration test was organized in 18 set-ups with a sampling frequency of 100 Hz and a duration of each step-up of 300 s. Three tri-axial force balanced accelerometers were used for the experimental ambient vibration tests. Figure 3 illustrates the gridline of the ambient vibration tests.

The signal recorded during the ambient vibration tests were processed by an operational modal analysis algorithm [6]. The enhanced frequency domain decomposition algorithm (EFDD) was considered for this purpose. The algorithm has been implemented in the mathematical software Matlab [7]. Table 1 shows the variation of the numerical modal parameters of the laboratory structure (natural frequencies and MAC_{num} ratios [6]) associated with the damage caused. Similarly, Table 2 shows the variation of the experimental modal parameters of the structure associated with the damage caused.

Table 1. Variation of the numerical modal parameters of the laboratory structure associated with the damage caused (being f_{num}^{und} the numerical undamaged natural frequencies of the structure, f_{num}^{dan} the numerical damaged natural frequencies of the structure, Δf_{num} the relative differences between the numerical undamaged and damaged natural frequencies of the structure and MAC_{num} the numerical MAC ratios [6]).

Mode	Description	f_{num}^{und} [Hz]	f_{num}^{dam} [Hz]	Δf_{num} [%]	MAC_{num} [-]
1	1st Vertical	28.70	5.77	79.90	0.99
2	1st Lateral	12.41	11.58	6.69	1.00
3	1st Torsion	29.78	18.55	37.71	0.99
4	2nd Lateral	25.86	25.47	1.51	1.00
5	3rd Lateral	40.60	41.11	-1.26	1.00
6	4th Lateral	56.85	55.93	1.62	1.00

Table 2. Variation of the experimental modal parameters of the laboratory structure associated with the damage caused (being f_{exp}^{und} the experimental undamaged natural frequencies of the structure, f_{exp}^{dan} the experimental damaged natural frequencies of the structure, Δf_{exp} the relative differences between the experimental undamaged and damaged natural frequencies of the structure and MAC_{exp} the experimental MAC ratios [6]).

Mode	Description	f_{exp}^{und} [Hz]	f_{exp}^{dam} [Hz]	Δf_{exp} [%]	MAC_{exp} [-]
1	1st Lateral	9.75	9.23	-5.33	0.97
2	2nd Lateral	14.50	11.48	-20.83	0.85
3	3rd Lateral	16.17	14.40	-10.95	0.91
4	1st Vertical	20.14	16.55	-17.83	0.95
5	2nd Vertical	23.74	23.44	-1.26	0.91
6	1s Torsion	25.14	21.38	-14.96	0.95

The following conclusions can be obtained from the analysis of Table 1 and Table 2: (i) a clear relationship between the reduction of the natural frequencies of the structure and the occurrence of damage (reduction of stiffness associated with the occurrence of damage), (ii) a greater sensitivity of the relative differences than the MAC ratios for damage detection and (iii) the damage does not affect equally all the vibration modes.

Subsequently, the topological entropy-based method was applied to this damage detection problem. The computation of the persistent barcodes, persistent diagram and the persistent entropy was performed using the Matlab package, javaplex [8]. First, the damage detection problem was tackled numerically. Figure 4 illustrates as example the persistent barcodes of the response of the structure at one reference point (Figure 3). The persistent barcodes show the number of connected components of each signal.

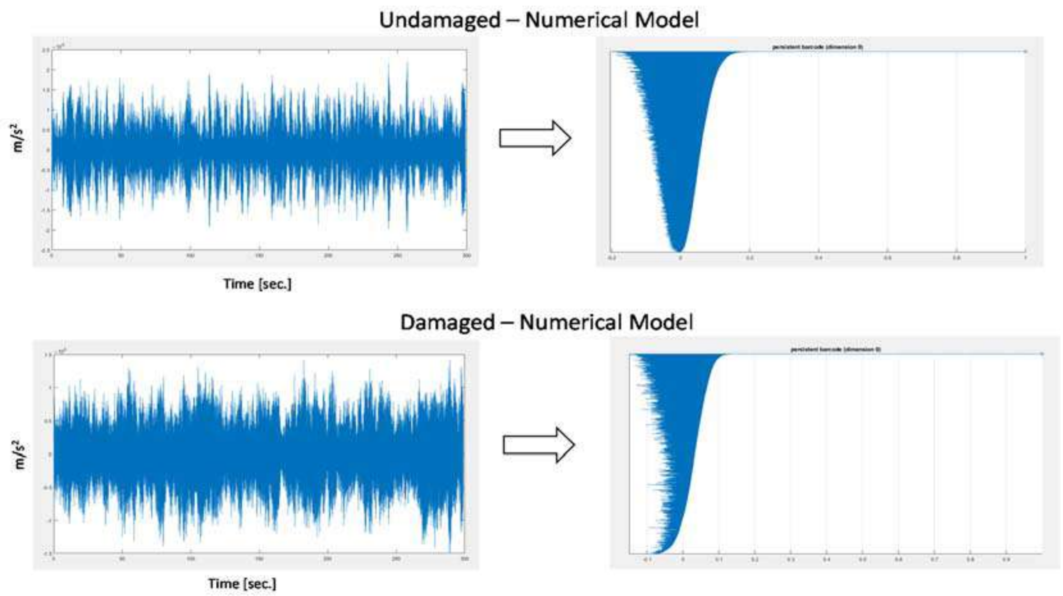

Figure 4. *Numerical response of the structure (signal) under ambient vibration and persistent barcode.*

Thus, Figure 5 illustrates the persistent entropy, H, according to the lateral (Figure 5.a) and vertical direction (Figure 5.b) of 10 different set-ups at one reference point.

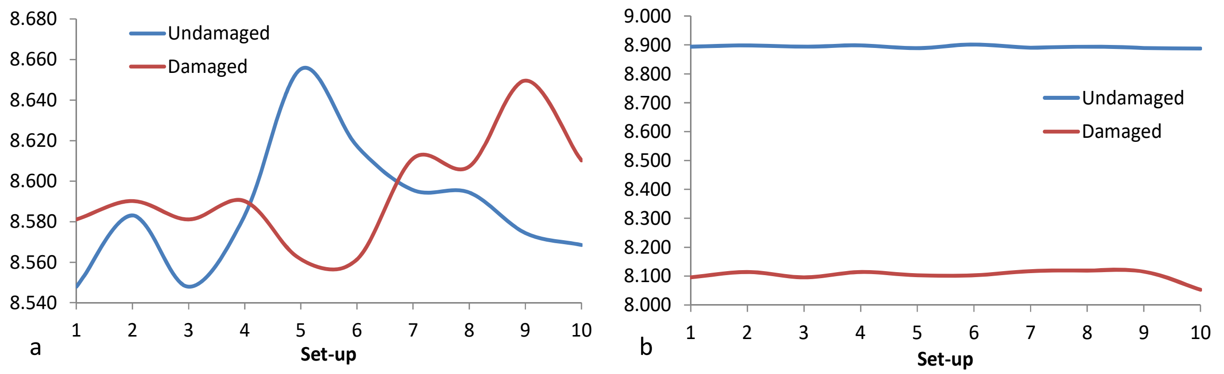

Figure 5. *Variation of the persistent entropy for the different set-ups of the numerical laboratory structure: a) lateral direction and b) vertical direction.*

As Figure 5 shows the persistent entropy is sensitive to the damage level of the structure. Additionally, Figure 5 illustrates that the variation of the persistent entropy is significant in the direction (vertical direction in this case) where the modification of the stiffness of the structure modifies more its structural behavior.

Finally the same process is repeated considering the experimental response of the structure under ambient vibration conditions. Again the two state of the structure (undamaged and damaged) were tested. The dynamic response of the structure (accelerations) at one reference point was considered in order to define the signals which are going to be compared.

In this manner, Figure 6 illustrates as example the persistent barcodes of the experimental response of the structure under ambient conditions.

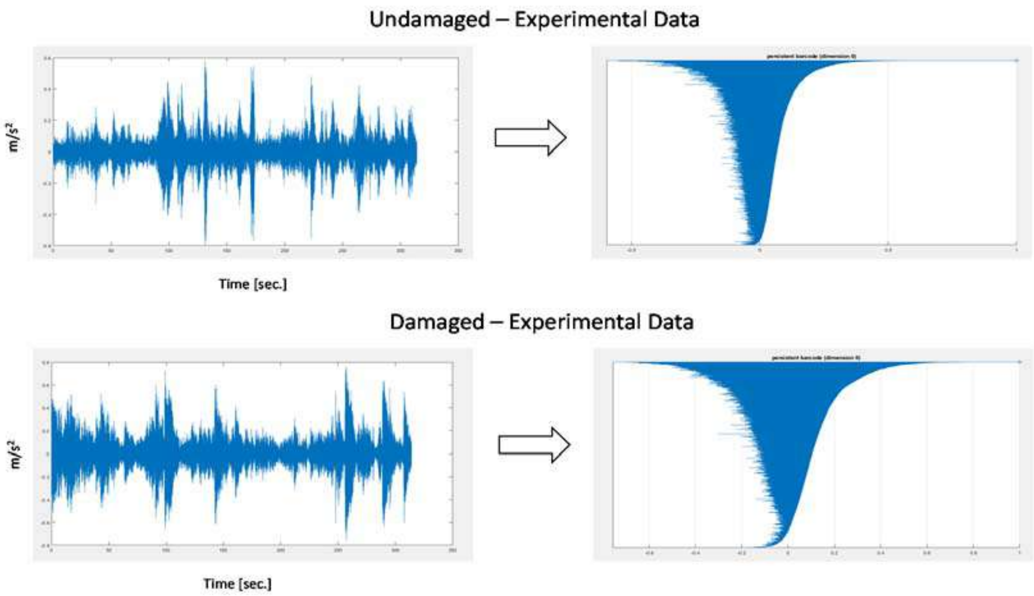

Figure 6. *Experimental response of the structure (signal) under ambient vibration and persistent barcode.*

Finally, Figure 7 illustrates the persistent entropy, H, according to the lateral (Figure 7.a) and vertical direction (Figure 7.b) of 10 different set-ups.

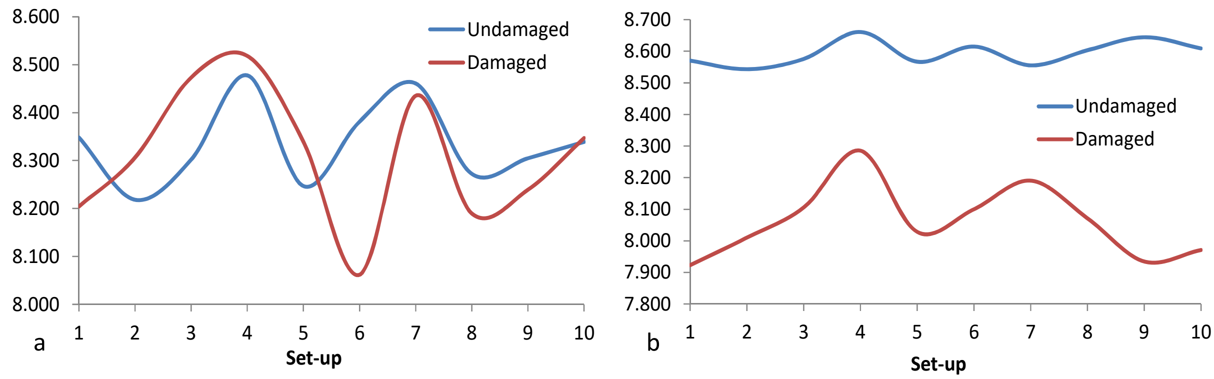

Figure 7. *Variation of the persistent entropy for the different set-ups of the experimental laboratory structure: a) lateral direction and b) vertical direction.*

As in the numerical case, the persistent entropy presents a clear variation in the vertical direction (Figure 7a) when a severe damage is applied to the structure. Hence, the method proposed may be considered as a potential tool for damage detection of civil engineering structures if a severe damage occurs in the structure. Nevertheless, additional studies are needed to analyze the effect of the change of the environmental and operational conditions on the persistent entropy of the system and to assess if the proposed method may be applied to other categories of the structural health monitoring process, as for example the damage location.

5. CONCLUSIONS

Herein a novel method, based on persistent homology and information theory, is presented and implemented for damage detection of civil engineering structures. The method is based on the comparison of the signals recorded under two different states (undamaged and damaged structure).

In order to characterize more easily the signals, these are transformed into filtered 1-dimensional simplicial complexes. The topology of these simplicial complexes is studied in terms of the persistent homology. The persistence barcodes have been obtained to characterize their shape. Subsequently, the persistent entropy has been computed and used as feature to compare the signals and consequently the damage detection of civil engineering structures. The performance of the method has been analysed comparing the results obtained between the application of the mentioned method and another conventional method (regarded as reference) on a laboratory structure. The study has been performed both numerically and experimentally. .According to the numerical analysis, the response of the structure (accelerations) has been simulated numerically, based on a finite element model. According to the experimental analysis, the response of the structure has been recorded under ambient vibration conditions. In both analyses, the response of the structure has been obtained under the two mentioned states. The results of this study show the sensitivity and effectiveness of the method for damage detection. These results highlight its potential to be used as a valuable tool for damage detection of civil engineering structures. Nevertheless, further studies are needed in order to characterize the performance of the method under environmental and operational conditions.

ACKNOWLEDGEMENTS

This work was partially funded by the Spanish Ministry for Science under research project RTI2018-094945-B-C21.

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