MASTER THESIS

Optimisation of Airspace Sectorisation

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Resumen

Para hacer frente a los desafíos de la creciente demanda de tráfico aéreo, se necesita una mejora de la ATM europea. En este contexto, uno de los problemas que se plantea es el diseño automático de la sectorización del espacio aéreo para que sea adaptable a nuevas rutas de tráfico. En este trabajo de fin de máster se propone una herramienta de apoyo a la toma de decisiones sobre la configuración de sectores aéreos. Dicha herramienta se basa en la resolución de un modelo de Programación Lineal Entera Mixta Multicriterio. El objetivo principal de nuestro estudio es obtener sectorizaciones óptimas, en las que los conflictos entre sectores se distribuyan uniformemente y se minimicen las transferencias de aviones entre los mismos. Introducimos una fase de preprocesamiento en la que el espacio aéreo se presenta como un grafo, utilizando como entrada una muestra de datos de tráfico real y sus conflictos, así como los volúmenes elementales existentes. Estos se combinan para formar sectores optimizándose los principales objetivos de nuestro modelo. Nuestra herramienta es de código abierto y, para su validación, se han utilizado datos reales de tráfico aéreo, específicamente se presentan experimentos computacionales para la Región Sur de España.

Abstract

To face the challenges of the increasing air traffic demand, an improvement of European ATM is needed. One of the existing problems is the design of the airspace sectorisation in order to be adaptable to new traffic routes. In this Master's thesis we aim to create a Sector Configuration Decision Support Tool based on the resolution of a Multicriteria Mixed Integer Linear Programming model. The main objective in our study is to obtain optimal sectorisations, where the conflicts between sectors are evenly distributed while aircraft transfers are also minimised. We introduce a pre-processing phase where the airspace is presented as a graph model, using as input a sample of real traffic data and its conflicts, as well as existing elementary volumes. We will combine these to form sectors, so that the main objectives of our model are optimised. Our Tool is open source and real air traffic data has been used for its validation, specifically computational experiments for the South Region of Spain are presented.

Chapter 1

Introduction

1.1 ATM

Air Traffic Management (ATM) is generally accepted as covering all the procedures and services involved to ensure the safe and orderly flow of air traffic. It involves the dynamic and integrated management of the air traffic and airspace- safely, economically, and efficiently, through the provision of facilities, human resources and technology. It can be divided in three distinct activities: the Air Traffic Services (ATS), Airspace Management (ASM) and Air Traffic Flow Management (ATFM). [11].

The ATFM is an activity that is primarily done before flights take place. It aims to establish a safe, orderly and a punctual flow of air traffic, by insuring that the traffic volumes are compatible with the airspace capacities, and that the airspace capacities are utilized to the maximum extent possible.

ATS performs the control of the air traffic in real time and is a process of constant exchange of information. It covers three different activities: the Flight Information Service (FIS), whose objetive is to provide advice and information useful for the safe conduct of flights; the Alerting Service (ALRS) to notify appropiate organizations regarding aircraft in need of search and rescue aid, and to assist such organizations when needed; and the Air Traffic Control service (ATC), whose aim is to prevent collisions between aircraft and obstructions on the manoeuvering area, as well as mantaining an orderly flow of air traffic. [10].

At last, ASM relates to the design and use of the available airspace, aiming at maximising its utilisation by dynamic time-sharing, and, at times, the segregation of the airspace among various categories of users based on short-term needs [15].

1.2 Airspace Sectorisation

As it has been said before, the main objective of the Air Traffic Control (ATC) is to prevent collisions between aircraft and to maintain an orderly flow of air traffic by a process of constant exchange of information between air traffic control units and controlled aircraft. An airspace with the size and traffic volume such as the Spanish one, for example, cannot be managed by a single team of controllers, therefore the airspace must be divided into sectors. Normally, a sector is part of a control area and/or a flight information region (FIR)/upper information region (UIR). Sectorisation is a fundamental feature of the Air Traffic Control system, as it allows the distribution of the control work. However, it requires the coordination of the flows between adjacent sectors, so the sectorisation cannot be randomly made, as it has to meet some requirements.

The control of the upper airspace is assigned to ATC units named Area Control Centres (ACCs). Each ACC is a facility responsible for the control of en-route aircraft in a particular airspace volume. This airspace is further subdivided into smaller volumes called elementary sectors that can be combined to form control sectors. The subdivision of an ACC's airspace into control sectors can be modified during the day, on one hand, control sector can be split when the traffic load increases, on the other hand, merged when the traffic load decreases. We call configuration as the set of control sectors at a particular time span.

Because of the existence of high air traffic density areas, the capacity of a sector is based on air traffic controller workload, i.e. the mental and physical work done by the controller to be able to control traffic [13]. Thus, the capacity of an ATC sector can be defined as "the maximum number of aircraft that are controlled in a particular ATC sector in a specified period, while still permitting an acceptable level of controller workload".

In the controller's workload, three main components are usually distinguished:

- **Monitoring workload:** in a sector, the controller must check each aircraft trajectory, and that the flight plans are correctly followed.
- **Coordination workload:** When an aircraft changes sectors, the pilots and controllers have to exchange information in order to have a safely transfer of aircraft between the sectors. The coordination workload is proportional to the flow cut by the sector borders.
- **Conflict workload:** it results from the resolution of conflicts (i.e., losses of separation minima) between aircraft. When a conflict is detected, the controller has to change the aircraft trajectories in order to ensure the safety of the flights.

Besides that, the following constraints are often included on the sectorisation problem [7]:

- **Bounded workload:** The workload of each sector cannot exceed a certain upper bound. There exists a maximum threshold called sector capacity, that specifies the maximum number of allowable aircraft in any sector at any time. Usually the monitoring workload is the one bounded.
- **Balanced workload:** The workload of each sector should be within some given imbalance factor of the average across all sectors. Any type of workload can be balanced. For example, it can be needed a balanced distribution of the conflicts points along the sectors, so that none of the controllers may be overworked with conflicts.
- **Balanced size:** The size of each sector should be within some given imbalance factor of the average across all sectors.
- **Minimum dwell time:** Every flight entering a sector, should stay within it a given minimum amount of time, therefore the controller has enough time to coordinate the flight.
- **Minimum distance:** Conflict points should be far away from sector's borders. This ensures that air traffic controllers will have enough time to manage the possible conflicts.
- **Connectedness:** A sector must be a contiguous portion of airspace and it cannot be fragmented into disconnected blocks.
- **Convexity:** The shape of the sector is probably not going to be convex, but, in the sense of trajectory-based, the situation when a flight enters the same sector twice or more times should be avoided.
- **Compactness:** The geometric shape of the sector should be easy to keep in mind.
- **Non-jagged boundaries:** A sector must have a boundary that is not too jagged.
- **Flow crossing sector borders:** Flows should cross a sector boundary as orthogonally as possible.

1.3 The future: Free Route Airspace

To face the challenges of the increasing air traffic demand, an improvement of European ATM is needed. For this purpose, the European Comission, Eurocontrol and other relevant European Airspace stakeholders founded SESAR Joint Undertaking. SESAR is the mechanism that concentrates all EU researchers to develop the ambitious Single European Sky (SES) [17]. The main objectives of the SES are:

- To restructure European airspace as a function of air traffic flows
- To create additional capacity
- To increase the overall efficiency of the ATM system

To fulfil these tasks, one of the developed concepts is the Free Route Airspace (FRA). The FRA is defined as "a specified volume of airspace in which users may freely plan a route between a defined entry and exit point, in which routeing may be possible via intermediate waypoints, without reference to the ATS route network" [6]. Instead of being restricted to fixed routes, there exists much more flexibility and it enhances the airspace possibilities. Some of the benefits of the FRA concept are:

- Reduced flight time
- Reduced CO2 emissions
- Reduced fuel waste
- A reduction of conflicts, since there would be the same number of aircraft but spread over more different routes
- Better use of the airspace

To be able to transition to a FRA concept, the sectorisation may need to be restructured, as it should be made to integrate the new traffic flows. Instead of having fixed flows of air traffic crossing at certain points, the new traffic will be spread across the whole airspace and conflict points would change. The sectorisation may need to be more flexible and ensure a good coordination between sectors. The sectorisation should take into account [1]:

- the main traffic flows and orientation
- minimising short transits through sectors
- minimising sector re-entry
- the conflict points
- positions of airspace reservations/restrictions
- civil/military coordination aspects

The sectorisation should also be unconstrained by FIU/UIR or State boundaries and be able of being redesign to meet demand.

FRA has been successfully implemented in much of northern, south-east and central south-east Europe, as well as in Portugal, which has been the first country to fully introduce FRA in 2009. FRA is expected to be implemented in most of Europe, including Spain, by the end 2022, see Figure 1.1.

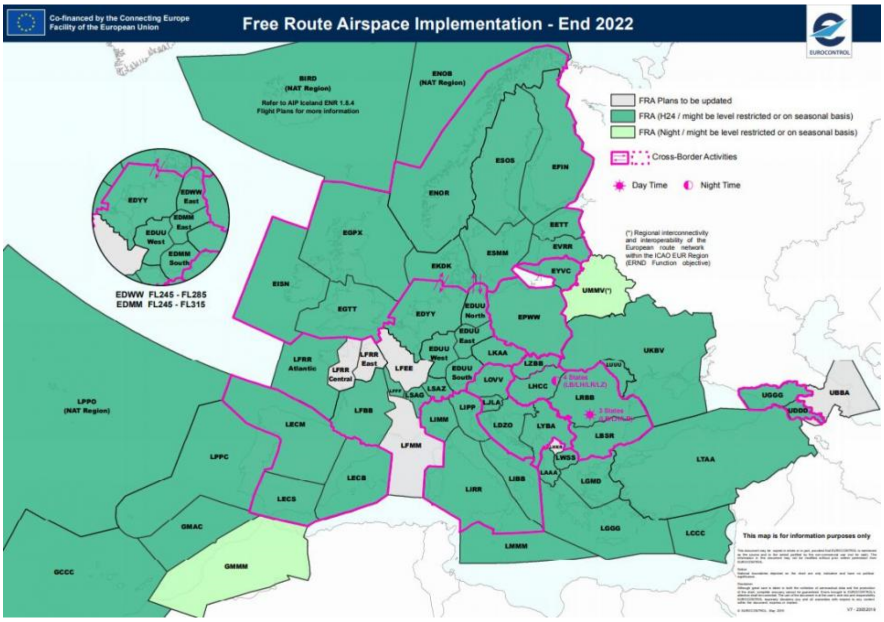


Figure 1.1: Free Route Airspace Implementation in Europe - End 2022

1.4 Objectives and contribution of this study

The main objective of this study is the creation of a Sector Configuration Decision Support Tool based on a mathematical model, specifically on the resolution of a Multiobjective Mixed Integer Linear Programming model.

In our optimisation problem different operational criteria will be considered such as: the evenly distribution of the conflicts between the sectors and the minimisation of aircraft transfers. To increase its acceptability and facilitate its implementation, a sector size restriction and design principles will be also imposed.

Using as input data a traffic sample and its conflicts, as well as, a series of existing elementary volumes, we will build sectors combining the elementary volumes, so that the different criteria are optimised.

The Sector Configuration DST is open source and real air traffic data has been used for its validation. The software has been tested for the South Region of Spain, specifically.

1.5 Thesis organization

The thesis is organised as follows. Chapter 1.5 discusses previous work related to the airspace sectorisation. In Chapter 3, the sector design problem is stated. We discuss the pre-processing phase of the sectorisation, where a weighted graph model is presented, and a mathematical formulation is proposed. The model has been applied to the South Region of Spain and the results are discussed in Chapter 4. To conclude, we present some conclusions and remarks.

Chapter 2

Description of the State of the Art

In this section we briefly review existing approaches in the literature to address the sectorisation problem. Airspace sectorisation aims to give a partition of a given airspace into a number of sectors, subject to operational constraints and minimising a certain cost function. One can structure the different airspace sectorisation around the following criteria [7]:

- Approach.
  - Graph-based model: a graph is constructed whose vertices represent the intersections of the existing trajectories and whose edges thus represent segments of the trajectories.
  - Region based: the airspace is divided into regions that are smaller than the sectors. It is a combinatorial problem.
- Dimensionality
  - 2D: the sectorisation is only defined in two dimensions (latitude and longitude)
  - 2.5D: the layers of the 3D airspace are considered to be independent so the sectorisation is only computed in two dimensions.
  - 3rd D: The input regions are in 2D and the sectorisation preserves those boundaries but can readjust them in the third dimension.
  - 3D: the sectorisation is defined in 3D dimensions
- Constraint and cost functions: the number and types of operational constraints used to define the sectorisation may vary from one method to other, same as the cost function to be minimised that depends on the goals one wants to reach. Due to the multiobjective nature of the problem, the cost function is usually a combination of objectives functions, such as the coordination cost, workload imbalanced or the number of entry points.

- Mathematical Optimization tool: there are different types of algorithms to be used to implement the sectorisation model, such as:
    - Stochastic local search
    - Exact Mathematical Programming methods
    - Evolutionary algorithms

Dividing the different problems by the approach used, we go on to explain some of the techniques that have been used as up to now to solve the sectorisation problem.

2.1 Graph-based model

In a graph-based model, the airspace is represented as a graph, and the sectorisation problem is the combinatorial problem of graph partitioning. A sector is defined by one of the sub-graph obtained, and the sector borders have to be constructed afterwards. What may differ from one method to another is the way the graph is constructed, as well as the post-processing phase.

According to [12], the vertices of the weighted graph represent the airports and waypoints, whereas the edges represent segments of air routes. The weights corresponds to the quantified air traffic flow along each segment. The sectorisation algorithm is based on a graph search algorithm. The task of partitioning the airspace is formulated as a problem of partitioning the corresponding graph. The graph is partitioned into two sub-graphs by minimising the cost function (2.1),where $\text{deg}(\mathbb{G}_i)$ corresponds to the monitoring workload and $\text{cut}(\mathbb{G})$ corresponds to the total coordination workload among the sectors. $H \in \{0,1\}^{nx2}$ where n is the number of graph vertices is an indicator, $H_{i,k} = 1$ if vertex i belongs to subgraph k. This problem is NP-Complete and its relaxed problem is solved by the normalized spectral clustering method. Additional steps are later included to ensure operational constraints, such as the connectivity of the sectors, are met.

$$min_H \left\{ J = \sum_{i=1}^{2} \frac{\text{cut}(\mathbb{G})}{\text{deg}(\mathbb{G}_i)} \right\} \tag{2.1}$$

According to [14], a network flow graph is created based on pathways and a flow pattern, as well as on an existing sectorisation. Each node represents a point on a border of the sector, through which, an aggregated flow passes. Each edge represents the connection between entering and exit points of the flow in the original sector. In order to count the aircraft, the entire airspace is discretized in grid cells and each grid cell is assigned to its nearest node, i.e. computing the Euclidean distance from the centre of each grid cell to each node and assigning it to the closest. Each grid cell has a weight associated such as the average number of aircraft in the cell, and in turn, the weight of each node is the sum of all the associated grid weights. The sectorisation problem is the flow graph partitioning problem, which is resolved using an heuristic approach based on spectral bisection.

According to [3], an undirected graph is constructed for the given airspace, where the vertices represent key points such as airports and waypoints and the edges represent the air routes. The vertices are used to construct Voronoi cells, which divide the airspace. Then, aircraft counts of each cell and each air-route are computed and used later as the corresponding weights of the vertices and edges. The algorithm used is a mixture of the general weighted graph cut algorithm, a dynamic load balancing algorithm and a heuristic algorithm. The sectors borders are build using Voronoi diagram, combining the initial cells in each sub-graph created.

According to [2] large airspace blocks are used. These are smaller than current elementary sectors, and already exist. A graph is created, where the set of vertices is the set of building blocks and the set of edges is such that (u, v) belongs to the set only if there can be a direct trajectory from u to v. The weight of each vertex is computed as the sum of the time spent by each aircraft in the block and the weight of each edge depends on the number of aircraft flying from a block to another. To build the partition of the airspace different objectives are considered: minimising the workload distribution, minimising the total number of transfers and ensuring that resulting sectors have acceptable geometric shapes. To solve the sectorisation problem, a stochastic algorithm is used.

The strength of the graph-based model is that it is based on the flow structure and the topological structure of the airspace. The main aspects of airspace are, therefore, considered, like airports, waypoints or conflict points. However, the main weakness of these models is the difficulty of accommodating all the operational constraints of interest. Also, it is hard to adapt the model to a 3D sector design.

2.2 Region-based model

In the region-based model, the airspace is partitioned into smaller elements and the sectorisation problem is then a combinatorial problem of grouping these regions. The way the airspace is represented and the optimization algorithms used may differ in different methods.

According to [4], the airspace is tiled by a tessellation of hexagonal cells, and for each cell the workload metric and connectivity metric are computed. To address the sectorisation problem, the authors use a Mixed-Integer Programming optimisation method, minimising the workload flow between cells. Certain cells, called "seed", can become "sinks" that absorb the flow. The number of sinks is constrained to be equal to the number of desired sectors. A sector is formed as a cluster of cells that feed into one sink. With the same idea behind, in [5], the problem of combining sectors is formulated as a MILP model, where the objective in this case is to minimise the number of sectors clusters or number of sectors that become sinks.

A different method is used in [18] where Voronoi diagrams are used in order to partition the airspace. In this case, N points are randomly generated, and Voronoi Diagrams are applied to these points to generate boundaries of N sub-divisions. The generating points are moved using a GA optimisation algorithm, where various objective functions are minimised. In order to count the number of aircraft, the airspace has to be divided into small rectangular grids anyway.

The strengths of the region-based model is it adaptability to satisfy all the operational constraints and also its adaptability to a 3D sector design. However, the main weakness of these regions is that, most of the time, the grouping of small cells does not give satisfactory shapes of sectors.

Chapter 3

Formulation of the Sectorisation Problem

In this chapter, the Sectorisation Problem is introduced and expressed as a Mixed Integer Linear Programming (MILP) model. The results are based on the chapter 4 of the thesis presented in [16].

The input data are the elementary volumes, the state and the evolution of the air traffic at a particular time span and its corresponding conflicts. The chosen air traffic is the one given by the filed flight plans, not the actually flown trajectories, as the latest should not contain any conflicts, as they would be solved by air traffic controllers.

This chapter is organized as follows: first the sectorisation problem is stated, then the data pre-processing is described and, at last, the mathematical model is outlined.

3.1 Problem statement

The airspace is modelled as a weighted graph $G = (V, E, F)$, where V is the set of vertices, E is the set of edges and F is the set of weights. Each vertex represents an elementary volume of the airspace, which is already known. There exists an edge between every two adjacent elementary volumes and f_e represents the coordination workload from edge $e = (v_1, v_2)$, specifically the number of aircraft transfers between the elementary volumes v_1 and v_2. The number of conflicts in each elementary volume is represented as N_c^v. Our goal is to define a valid airspace sectorisation, where the elementary volumes are grouped in larger sectors in a way where the air traffic flow is well represented, minimising the coordination workload, and, most importantly, balancing the number of conflicts in each sector. The number of final sectors n is also an input data, as we aim to have the same number of sectors as in current sectorisations. In order to not obtain sectors too big and thus not operationally useful, a sector size constraint is also imposed. A distance matrix $D = (d_{ij})$ is defined where d_{ij} is the maximum distance between elementary volumes i and j.

Recall that the sectorisation must satisfy a series of safety constraints. There can not be any conflict near the border of any sector, and the flows should cross the sectors boundaries as orthogonal as possible. All these constraints are already satisfied by the existing elementary volumes, thus no further analysis is required in our model. The connectivity constraints are implied in the minimisation of the coordination workload, but are not explicitly imposed.

The two main objectives for minimisation are: the coordination workload and the balance of the number of conflicts. The maximum conflict difference between sectors, can also be imposed as an operational constraint. Doing this, we will compare different optimal solutions, considering the minimisation of the coordination workload as our objective and different bounds for the conflict balance as the main constraint.

3.2 Data pre-processing

We are given a set V of elementary volumes. The following parameters will be needed, and calculated in a pre-processing stage:

1. A matrix F of dimension VxV, which gives the coordination workload (the air traffic flows) between each elementary volume.

2. A vector N_c with V rows, whose components are the number of conflicts in each volume.

3. A matrix D of dimension VxV, which gives the maximum distance between elementary volumes.

Air traffic flows

There are different ways in which the coordination workload can be defined. It is completely correlated with the aircraft transfers between sectors, as the more aircraft cross one border, the harder for the controller the coordination is. Knowing this, we define the weight each edge has in a simple way. For every aircraft transfer between two sectors, the corresponding weight for its edge increases by one, that is, the weight is simply the number of aircraft that cross each border. We have proceeded as follows.

First, it is noted, that we consider each trajectory divided into segments. These are defined by the navigation points each aircraft flies over. Navigation points are fixed two dimensional points in the airspace, and each flight trajectory must be defined only as a sequence of these. Navigation points are also important, as they are used as reference to characterise conflicts.

Secondly, the airspace will be sectorised in two dimensional elementary volumes, previously known. These are specified by the latitude and longitude of each vertex that defines each polygon.

Finally, to measure the coordination workload, the intersections between each trajectory segment and each polygon are calculated, creating for each volume a list consisting of the coordinates of each intersection. The number of intersections for each border is counted, and this number is defined as the coordination workload between the corresponding two elementary volumes.

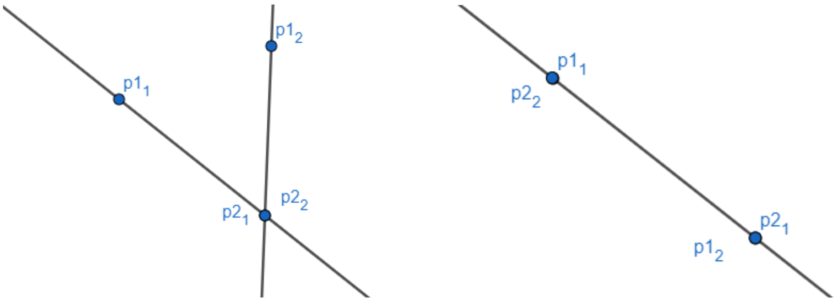
However, we have to take into account some exceptions. There are flights that, even though they cross the elementary volumes borders, and thus one would count them as an increase in the coordination workload, their stay in that particular volume is minimal, and the aircraft is not even transferred to its corresponding sector. That is why, to be more precise in our coordination workload computations, the transfers from aircraft whose flight time inside a volume is less than 2 min, will not be counted.

Conflicts

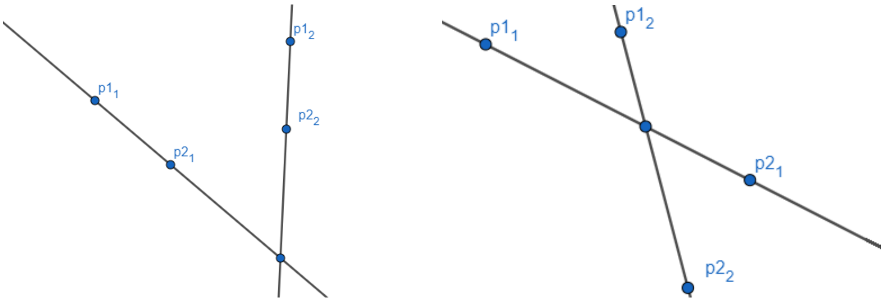
It is considered that two aircraft are in conflict when they simultaneously violate a set of vertical and horizontal separation minima. In this work, these are considered as 1000 ft and 10 nm respectively. The conflicts will be computed by the NEST tool, a simulation software for network capacity planning and airspace design by EUROCONTROL. As the output we receive the two navigation points between which each of the two planes is located when the conflict begins. The conflict should be assigned to one or at most two navigation points. We consider the following cases, depicted in Figure 3.1:

1. Case 1. If two of the navigation points coincide, then this means that the two aircraft converge or diverge from the same point, to which the conflict is assigned.
2. Case 2. If the two flights are between the same two navigation points then half a conflict is assigned to each of them.
3. Case 3. If the four points are different, but the trajectories intersect in the middle, then, half a conflict will be assigned to the two nearest points to the intersection, each belonging to a trajectory.
4. Case 4. If the four points are different and the trajectories intersects in a point which is beyond the segments in which the loss of separation begins, then, half

a conflict is assigned to the two nearest points to the intersection, that is, the two points that are the closest to each other.



(a) Case 1 (b) Case 2



(c) Case 3 (d) Case 4

Figure 3.1: Conflicts between two aircraft

Distance matrix

The only parameter that remains to be calculated is the distance matrix D. There aren't any restrictions to the size of the resulting sectors, and, in situations with a lot of air traffic and thus a considerable amount of conflicts that need to be resolved, the size of the sector has to be such that it can be seen completely in the screen by the controller. That is why we add a maximum size constraint to our formulation. We define the maximum distance between two elementary volumes as the distance between their two most distant vertices. The diameter of a sector is thus defined as the maximum distance between all the elementary volumes that form it.

3.3 Model

To model the sectorisation problem, the following variables are defined:

- Binary variable $X_{v,g}$, which is equal to 1 if vertex v (i.e volume sector represented by v) belongs to component g (i.e. sector g), and 0 otherwise.
- Binary variable $W_{v,g}$, which is equal to 1 if vertex v represents sector g, and 0 otherwise.
- Binary variable $Y_{e,g}$, which takes the value 1 if the extremities of e belong to different sectors, i.e. $e = (v_1, v_2)$, $v_1 \in g$ and $v_2 \notin g$ or $v_1 \notin g$ and $v_2 \in g$, and 0 otherwise.
- difconf: maximum conflict difference between sectors
- N_c^g: number of conflicts in sector g
- D_g: diameter of sector g.

Additionally, the following parameters are used:

- $F = (f_{ij})$: the flow matrix, where f_e indicates the number of aircraft crossing the edge $e = (i, j)$.
- N_c^v: the conflict vector, where $N_c^v(i)$ are the number of conflicts in elementary volume i.
- MaxDif: maximum difference conflict allowed
- $D = (d_{ij})$: matrix of maximum distance between each elementary volume
- D_{max}: maximum diameter allowed

The sectorisation problem can be formulated as follows:

$$\min \sum_{e\in E}\sum_{g} f_e Y_{e,g} \tag{3.1}$$

s.t.

$$\sum_{g} X_{v,g} = 1 \quad \forall v \in V \tag{3.2}$$

$$\sum_{v\in V} W_{v,g} = 1 \quad \forall g \tag{3.3}$$

$$X_{v,g} - W_{v,g} \geq 0 \quad \forall v \in V, \quad \forall g \tag{3.4}$$

$$Y_{e=(v_1,v_2),g} + X_{v_1,g} + X_{v_2,g} \leq 2 \quad \forall g \forall e \in E \tag{3.5}$$

$$Y_{e=(v_1,v_2),g} - X_{v_1,g} - X_{v_2,g} \leq 0 \quad \forall g \forall e \in E \tag{3.6}$$

$$Y_{e=(v_1,v_2),g} - X_{v_1,g} + X_{v_2,g} \geq 0 \quad \forall g \forall e \in E \tag{3.7}$$

$$Y_{e=(v_1,v_2),g} + X_{v_1,g} - X_{v_2,g} \geq 0 \quad \forall g \forall e \in E \tag{3.8}$$

$$N_c^g - \sum_{v\in V} N_c^v X_{v,g} = 0 \quad \forall g \tag{3.9}$$

$$\text{difconf} - N_c^{g_1} + N_c^{g_2} \geq 0 \quad \forall g_1, g_2 \tag{3.10}$$

$$\text{difconf} \leq \text{MaxDif} \tag{3.11}$$

$$D_g \geq X_{v_1 g} X_{v_2 g} d_{v_1 v_2} \quad \forall g \forall v_1, v_2 \in V \tag{3.12}$$

$$D_g \leq D_{max} \quad \forall g \tag{3.13}$$

$$X_{v,g}, Y_{e,g}, W_{v,g} \in \{0,1\} \tag{3.14}$$

The objective function (3.1) minimises the number of transfers between sectors. It is noted that we minimise the product $f_e Y_{e,g}$ which in reality is a way of demanding the connectivity of the sectors, as, if two elementary volumes are not adjacent, then their corresponding f_e would be 0, and $Y_{e,g}$ could be 1 (that is the elementary volumes may be in different sectors), but if there is a lot of air traffic between two volumes, and therefore f_e has a high value, then Y_e would tend to 0 in order to minimise the objective value. In addition, we try to distribute the number of conflicts between the sectors by imposing a maximum allowable conflict difference with constraint (3.11). Constraint (3.2) ensures that each elementary volume belongs to exactly one sector, while constraint (3.3) ensures that only one elementary volume represents each sector. Constraint (3.4) indicates that a elementary volume can represent a sector only if it belongs to it. With constraints (3.5)-(3.8) the variable $Y_{e,g}$ is defined, it imposes that $Y_{e=(v_1,v_2),g} == 1 \Leftrightarrow X_{v_1,g} \neq X_{v_2,g}$. Constraint (3.9) computes the number of conflicts in each sector, while (3.10) defines the maximum conflict difference between two sectors. Finally, constraint (3.12) computes the diameter of the sector, that is, the distance between its two furthest elementary volumes, and with (3.13) a maximum sector size is imposed.

The previous model is linear except for constraint (3.12), in which binary variables are multiplied. However, it is not difficult to linearise this constraint and thus to obtain the Mixed Integer Linear Problem that we want.

Using Fortet's linearisation, we introduce a binary variable $Z_{ijk} \in \{0, 1\}$, such that, $Z_{ijk} == 1 \Leftrightarrow Xik = Xjk = 1$. This yields:

$$Z_{ijk} \leq X_{ik} \tag{3.15}$$

$$Z_{ijk} \leq X_{jk} \tag{3.16}$$

$$Z_{ijk} \geq X_{ik} + X_{jk} - 1 \tag{3.17}$$

The final constraint added to our model would be just:

$$D_g \geq Z_{v_1 v_2 g} d_{v_1 v_2} \quad \forall g$$

Chapter 4

Computational Experiments

4.1 Pre-processing

The airspace of the South region of Spain is composed of 21 elementary volumes, but as we are only evaluating the upper airspace, approach volumes will not be considered for this first model. We will, essentially, be working with a two dimensional sectorisation, the elementary volumes would not be divided into two levels. However, in order to represent better the airspace, around the area of Málaga, the aircraft would be counted from flight level 145 (14500 ft), while in the rest of the airspace, the aircraft trajectories would be considered from flight level 195 (19500 ft).

Taking into consideration all these factors, our model will start with 12 elementary volumes, as it can be seen in figure 4.1.

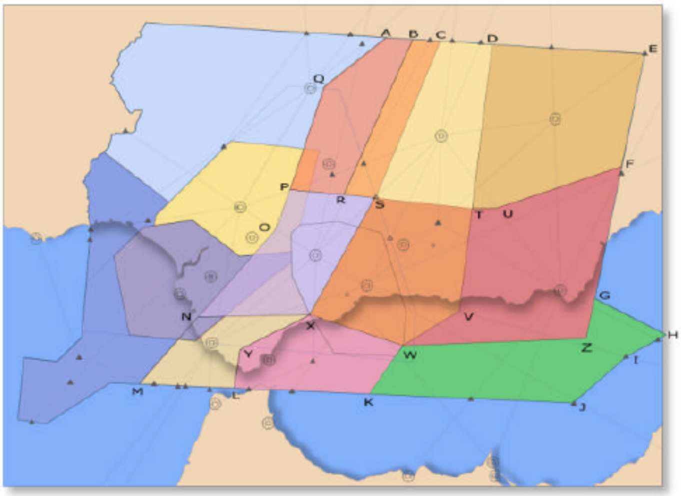


Figure 4.1: 2D Elementary volumes in the south region of Spain

The full set of planned trajectories for 18/08/2019 from 8AM to 3PM (UTC) was used. The airspace state can be seen in figure 4.2. The corresponding conflicts were calculated by NEST. To define the conflicts, 1000ft and 10NM have been considered as the minimum allowed distances.

Once the intersections between the traffic flows and the elementary volumes, as well as the conflicts, have been calculated, the corresponding graph can be seen in figure 4.3.

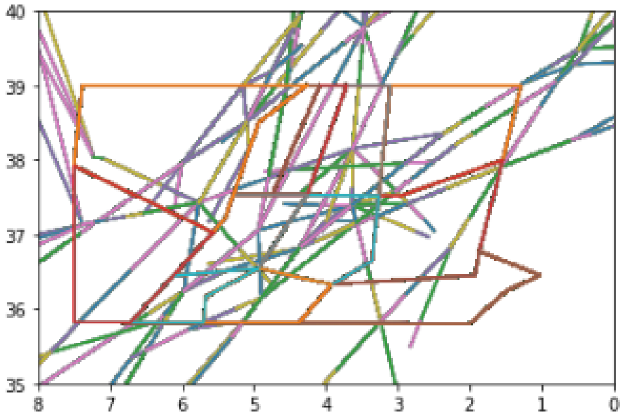


Figure 4.2: Airspace State. Trajectories and elementary volumes.

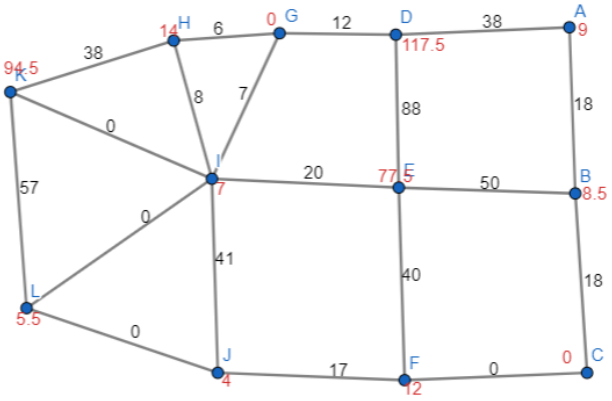


Figure 4.3: Weighted graph representing the airspace state

The weights of the edges are the number of aircraft crossing the borders of each elementary volume, and the weights of the nodes are the number of conflicts per volume. The maximum distance between each elementary volume, has been calculated as well. This yields the following matrices and vectors to be used as parameters for our optimisation model.

$$F = \begin{pmatrix} 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 63 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 15 & 0 & 0 & 0 & 143 & 0 & 32 & 0 & 0 & 0 & 0 & 0 \\ 0 & 63 & 0 & 143 & 0 & 73 & 0 & 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 73 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 32 & 0 & 0 & 0 & 12 & 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 62 & 0 & 40 & 0 \\ 0 & 0 & 0 & 0 & 30 & 0 & 13 & 62 & 0 & 24 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 9 & 0 & 0 & 102 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 102 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0.0 & 416.6 & 492.7 & 368.4 & 488.7 & 604.8 & 417.5 & 473.6 & 607.5 & 699.7 & 701.4 & 775.7 \\ 416.6 & 0.0 & 385.6 & 345.4 & 396.9 & 509.1 & 370.8 & 405.0 & 523.5 & 611.0 & 669.7 & 691.5 \\ 492.7 & 385.6 & 0.0 & 411.1 & 434.7 & 524.7 & 442.2 & 490.5 & 566.3 & 637.0 & 761.7 & 738.2 \\ 368.4 & 345.4 & 411.1 & 0.0 & 340.1 & 456.1 & 244.5 & 292.2 & 438.9 & 534.8 & 503.6 & 602.5 \\ 488.7 & 396.9 & 434.7 & 340.1 & 0.0 & 330.2 & 304.4 & 298.8 & 338.9 & 427.9 & 518.6 & 507.2 \\ 604.8 & 509.1 & 524.7 & 456.1 & 330.2 & 0.0 & 416.5 & 395.9 & 248.9 & 316.8 & 486.2 & 435.8 \\ 417.5 & 370.8 & 442.2 & 244.5 & 304.4 & 416.5 & 0.0 & 239.0 & 389.7 & 486.0 & 438.2 & 549.0 \\ 473.6 & 405.0 & 490.5 & 292.2 & 298.8 & 395.9 & 239.0 & 0.0 & 362.3 & 458.3 & 398.4 & 517.7 \\ 607.5 & 523.5 & 566.3 & 438.9 & 338.9 & 248.9 & 389.7 & 362.3 & 0.0 & 333.2 & 384.9 & 407.4 \\ 699.7 & 611.0 & 637.0 & 534.8 & 427.9 & 316.8 & 486.0 & 458.3 & 333.2 & 0.0 & 445.3 & 323.5 \\ 701.4 & 669.7 & 761.7 & 503.6 & 518.6 & 486.2 & 438.2 & 398.4 & 384.9 & 445.3 & 0.0 & 502.7 \\ 775.7 & 691.5 & 738.2 & 602.5 & 507.2 & 435.8 & 549.0 & 517.7 & 407.4 & 323.5 & 502.7 & 0.0 \end{pmatrix}$$

$$N_c = \begin{pmatrix} 9.0 & 8.5 & 0.0 & 117.5 & 77.5 & 12.0 & 0.0 & 14.0 & 7.0 & 4.0 & 94.5 & 5.5 \end{pmatrix}$$

4.2 Optimisation

The optimisation model has been solved in python with pyomo as a modelling language. The solver glpk, which is open source, has been used to solve all models. The calculation times have been short, around 4/5 minutes for a sectorisation of 4 sectors, and around 30 minutes for a 5-sectorisation.

4.2.1 The problem with 4 sectors

As a first step, the minimum conflict difference possible between sectors has been calculated, in order to use it later as a lower bound. It has been computed minimising the variable difconf and without taking into account the intersectoral flow. For a configuration of 4 sectors, the minimum conflict difference possible is 75.

6 Pareto optimal solutions have been computed changing the upper bound for the conflict difference. It is noted that, the higher the maximum conflict difference between sectors is, the lower the intersectoral flow is, i.e. the better the traffic flow is represented. The Pareto optimal curve can be seen in figure 4.4.

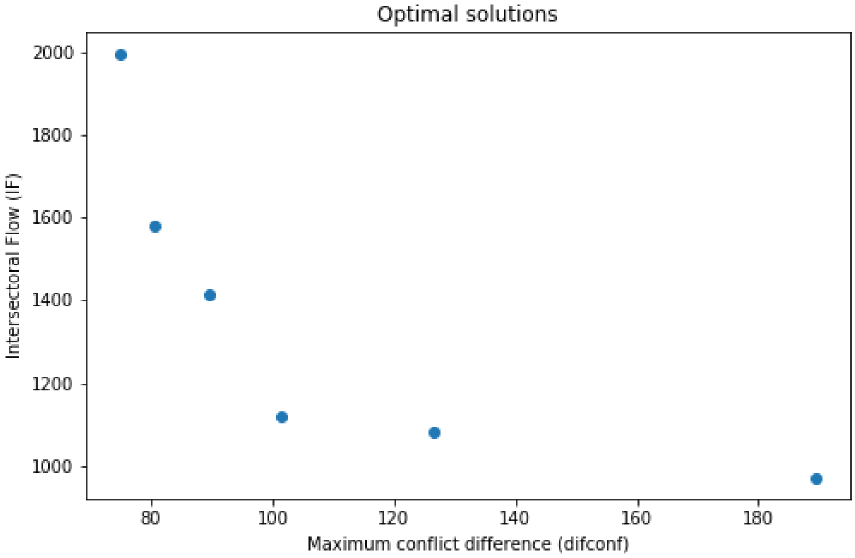


Figure 4.4: Pareto Optimal Curve for a configuration of 4 sectors in the South Region of Spain

The first optimal solution, which has been obtained for values of MaxDif from 75 to 80, can be seen in figure 4.5. In table 4.1 the number of conflicts, the intersectoral flow and the diameter of each sector is also stated.

	Conflicts	Flow	Diameter
1	117.5	506.0	244.5
2	42.5	544.0	517.7
3	95.0	660.0	492.7
4	94.5	286.0	378.8

Table 4.1: Characteristics of each sector for MaxDif=75-80

It has to be pointed out, that, firstly, a connected sectorisation has been obtained.

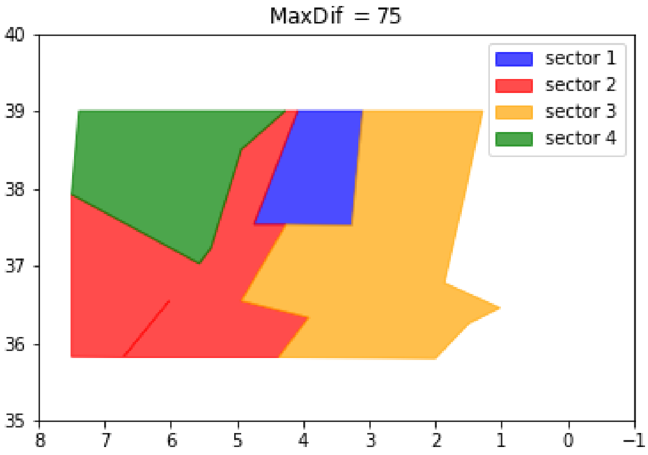


Figure 4.5: 4-sector configuration for MaxDif=75-80

Even though the maximum difference conflict allowed is the minimum possible, by minimising the aircraft transfers the connectivity is reached. Secondly, some of the sectors that had been obtained, are used or have been used in real configurations, like the first and fourth sector ("Bailén" and "Sevilla Norte", respectively).

Relaxing the maximum conflict difference constraint and allowing a value between 81 and 89, another configuration is obtained. It is shown in figure 4.6, as well as its characteristics in table 4.2.

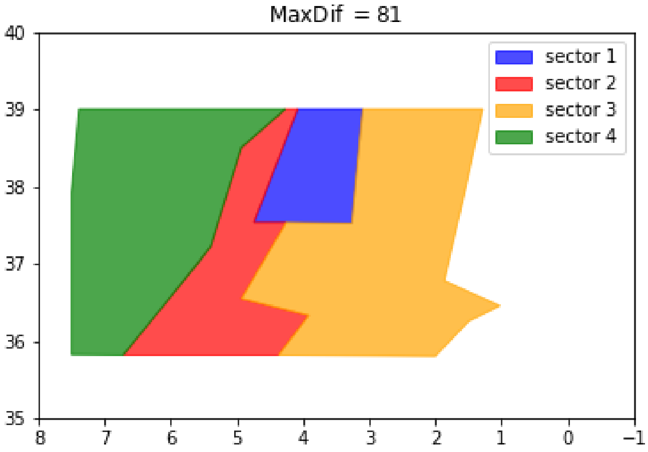


Figure 4.6: 4-sector configuration for MaxDif=81-89

It is noted, that the changes between the two configurations are not drastic. In this

	Conflicts	Flow	Diameter
1	95.0	660.0	492.7
2	37.0	336.0	458.3
3	117.5	506.0	244.5
4	100.0	78.0	502.7

Table 4.2: Characteristics of each sector for MaxDif=81-89

case, the fourth sector increases its size, and its result is another known sector called "Sevilla".

Different configurations are obtained when we allow a maximum difference conflict of 90-101 or 102-126, shown in 4.7 and 4.8 respectively. Tables 4.3 and 4.4 present their respective characteristics.

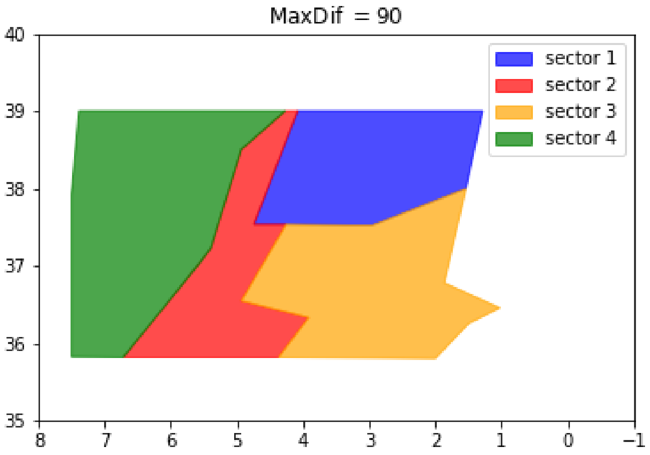


Figure 4.7: 4-sector configuration for MaxDif=90-101

	Conflicts	Flow	Diameter
1	126.5	422.0	417.5
2	86.0	576.0	434.7
3	37.0	336.0	458.3
4	100	78.0	502.7

Table 4.3: Characteristics of each sector for MaxDif=90-101

These two sectorisations are very similar, the changes between them being quite smooth. It has to be highlighted that the second one is in fact an actual sectorisation used in the South Region of Spain. Indeed, the four sectors obtained are known as

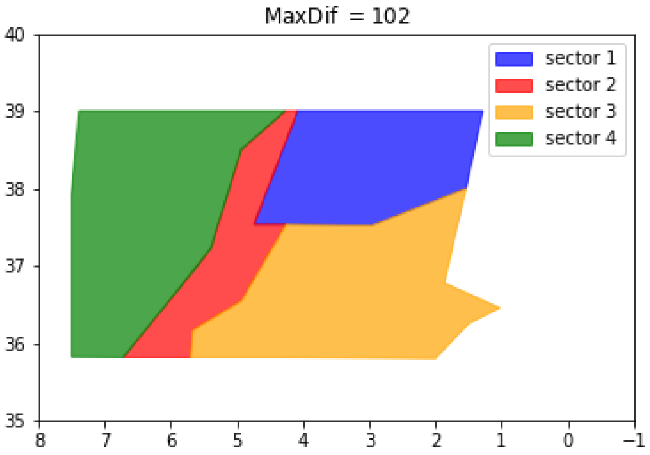


Figure 4.8: 4-sector configuration for MaxDif=102-126

	Conflicts	Flow	Diameter
1	25.0	190.0	458.3
2	126.5	422.0	417.5
3	100.0	78.0	502.7
4	98.0	430.0	524.7

Table 4.4: Characteristics of each sector for MaxDif=102-126

"Norte", "Martín", "Central Sur" and "Sevilla", respectively.

4.2.2 The problem with 5 sectors

The same experiments as in the section before had been made for a configuration of 5 sectors. It is noted, that at this stage in the current sectorisation, the third dimension starts to be considered, i.e., in the actual sectorisation, the sectors start to be divided into different levels instead of being a single block, so their definition in three dimensions becomes essential. That is why, with our "2.5" dimensional model, to produce known solutions would be harder that in the previous case.

Minimising the variable difconf, a value of 89 has been determined to be the lower bound for the maximum difference conflict between sectors.

8 different configurations have been obtained. In figure 4.9 the optimal pareto solutions can be seen. As before, it is noted how the two main objectives are related.

It is noted, that there are some solutions that are much more stable than others, where even though one relaxes the maximum difference conflict constraint, the solution stays the same.

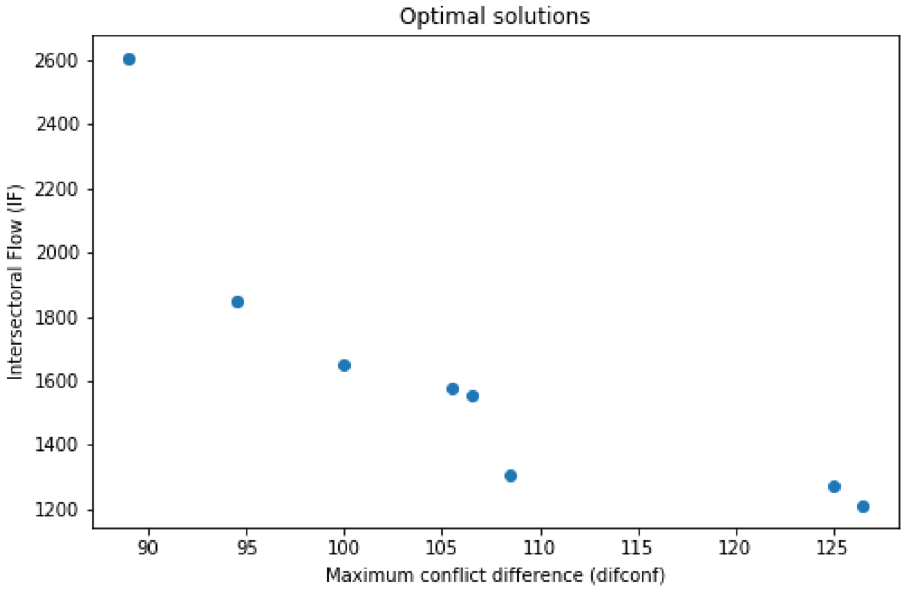


Figure 4.9: Pareto Optimal Curve for a configuration of 5 sectors in the South Region of Spain

We will show, next, four out of the eight configurations. First, it is shown in figure 4.10 and table 4.5, the sectorisation for the minimum conflict difference allowed. As we can see, this configuration does not satisfy all the operational constraints, as we obtain an unconnected sector. The connectedness is reached in our formulation by minimising the transfers flows, but this may not be entirely possible if the constraint on the maximum conflict is too tight. In this case, one should relax this condition and search for another pareto optimal configuration.

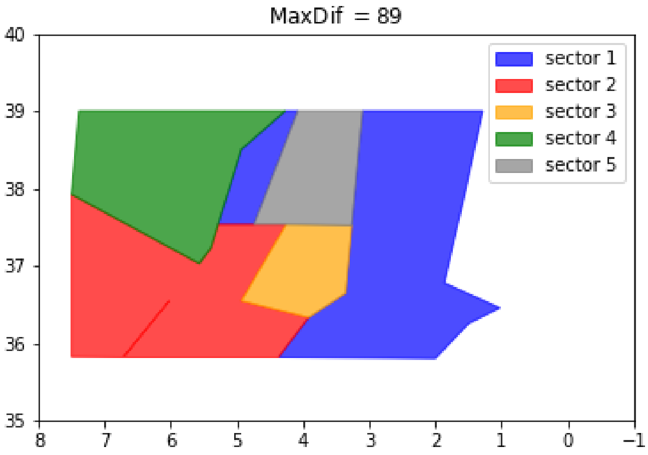


Figure 4.10: 5-sector configuration for MaxDif=89-94

	Conflicts	Flow	Diameter
1	31.5	484.0	492.7
2	77.5	748.0	0.0
3	117.5	506.0	244.5
4	94.5	286.0	0.0
5	28.5	584.0	435.8

Table 4.5: Characteristics of each sector for MaxDif=89-94

Relaxing the conflicts constraint, we obtain the first connected configuration with MaxDif=100, as it can be seen in figure 4.11. What's interesting about this sectorisation is that we obtain sectors that are or have been used in the current configurations. The first sector is known as "YESTE", and the second, forth and fifth sectors have already been presented in the different configurations for 4 sectors, as "Martin", "Sevilla" and "Bailen" respectively.

	Conflicts	Flow	Diameter
1	25.0	190.0	458.3
2	117.5	506.0	244.5
3	100.0	78.0	502.7
4	89.5	602.0	330.2
5	17.5	276.0	492.7

Table 4.6: Characteristics of each sector for MaxDif=100-105

There are some configurations that are characterized as unstable, in the sense that they only appear under very specific conditions. This is the case for the sectorisation

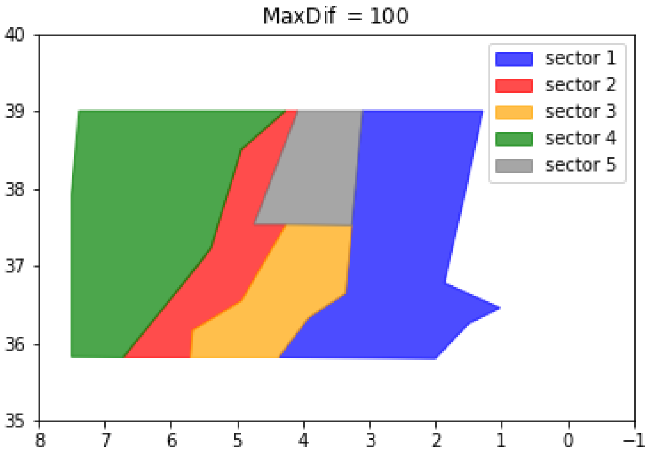


Figure 4.11: 4-sector configuration for MaxDif=100-105

shown in figure 4.12 and table 4.7, which only appears when the parameter MaxDif is fixed at 106.

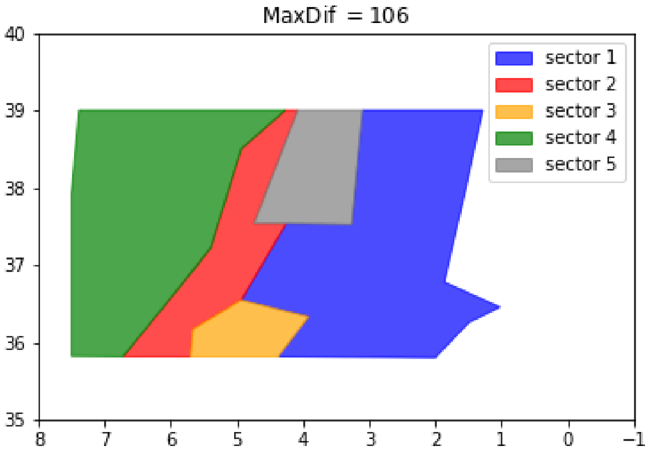


Figure 4.12: 5-sector configuration for MaxDif=106

At last, it is shown one of the most stable configurations for 5 sectors, the sectorisation displayed in figure 4.13 and table 4.8 is maintained for values of MaxDif from 109 to 125. It is, also, the configuration that comes closest to the current sectorisation.

	Conflicts	Flow	Diameter
1	95.0	660.0	492.7
2	117.5	506.0	244.5
3	25.0	190.0	458.3
4	100.0	78.0	502.7
5	12.0	146.0	430.4

Table 4.7: Characteristics of each sector for MaxDif=106

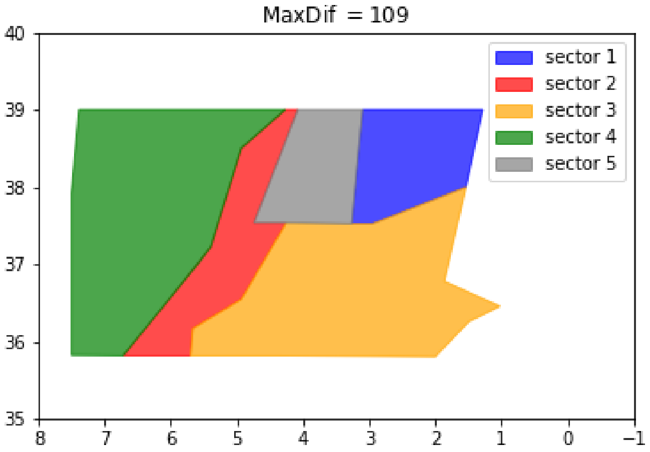


Figure 4.13: 5-sector configuration for MaxDif=109-125

	Conflicts	Flow	Diameter
1	98.0	430.0	524.7
2	9.0	104.0	0.0
3	25.0	190.0	458.3
4	100.0	78.0	502.7
5	117.5	506.0	244.5

Table 4.8: Characteristics of each sector for MaxDif=109-125

Chapter 5

Concluding Remarks and Extensions

In this Master's thesis a Sector Configuration Decision Support Tool has been created. This tool is based on a Mathematical Optimisation model, specifically on the resolution of a biobjective Mixed Integer Linear Programming problem, solved as a series of Mixed Integer Linear problems since one of the objectives is expressed as a constraint. The main objectives of our model are the evenly distribution of conflicts between the sectors and the minimisation of the number of aircraft transfers. There are some properties of our model that should be highlighted.

First, not only is the model able to reproduce some actually implemented solutions, but it also provides some seemingly unexplored configurations, depending on the operational objectives and constraints used. In addition, the changes between the different configurations do not have drastic variations, and instead they have a soft border evolution. This means that if different configurations are to be used throughout the day, it could be possible to find smooth transitions between them. To be able to find the best sectorisation for a certain part of the day, the number of sectors should be also considered as a parameter, that could be modified.

Secondly, our model could be applicable to a FRA scenario. In a FRA, there are not fixed routes, and the airspace is redefined and adapted to the aircraft flows, which would be freely defined, the entry and exit points being the only fixed points of the trajectory. If the geometry of the elementary borders are properly adapted to fit the new traffic flows, the developed model can be straightforwardly applied. Actually, the elementary volumes could be easily changed, using instead a partition of them or maybe new ones that are adapted to a FRA scenario. Additionally, our model is based on flow adaptation, so considering a FRA scenario would be the next natural step.

Thirdly, the flexibility of our model should be mentioned. The objective function and the constraints can be completely adapted to minimise any complexity criteria, and any ERNIP recommendation could be easily included as a constraint, as long as it could be expressed by means of (linear) inequalities. Besides that, our tool is fully applicable to any airspace scenario, maybe to Europe or other regions of Spain.

Fourthly, some improvements could be done in the future. It has been considered the problem in "2.5" dimensions. The next step would be extending it to 3 dimensions, considering, instead of polygons, volumes as the elementary volumes. This extension would allow the methodology to be applied, not only to en-route control sectors, but also to approach sectors.

Last but not least, the configuration problem over time should be taken into account. During the course of a day, the workload fluctuates and as the traffic in the airspace is changing with time. The developed model should be extended to consider the possibility that different configurations can be used along the day, including as decision variables the transition times between configurations and the number of sectors at each time.

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