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Control of the longitudinal flight dynamics of an UAV using adaptive backstepping

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Abstract: An adaptive backstepping approach is used to control the longitudinal dynamics of an Unmanned Air Vehicle (UAV). The nonlinear controller designed makes the system follow references in the aerodynamic velocity and flight path angle, using the elevator deflections and the thrust as actuators. Moreover, the (global) solution is valid for all the flight envelope, since it is based on a general nonlinear model. The adaptation scheme proposed allowed us to design an explicit controller with a minimal knowledge of the aircraft aerodynamics. Simulations are included for a realistic UAV model that includes actuator saturation.

Keywords: Adaptive control, Backstepping, Aircraft control, Autonomous vehicles, Nonlinear control, Parameter estimation, Lyapunov stability.

1. INTRODUCTION

In recent years, the interest in unmanned air vehicles (UAVs) has increased considerably. Not having a pilot makes aircraft lighter, cheaper and more efficient for missions such as surveillance or reconnaissance. The absence of a pilot implies that the automatic flight control system has an important role in the UAV design process. Consequently, many approaches have appeared in the literature.

Traditionally, flight controllers have been designed based on a linearized aircraft model for a selected operating point. Several control techniques are then applied, usually with excellent results (McLean (1990)). However, when the flight condition is changed, the model is no longer valid and the controller performance can be reduced.

Gain scheduling methods have been successfully developed to deal with different operating points (Nichols et al. (1993)). However, these methods have the disadvantages of having to compute different controllers for different operating points, and needing to estimate the aircraft stability derivatives on a wide range of flight conditions (which can be a very difficult task).

Nonlinear control techniques have been considered to overcome these difficulties. For instance, feedback linearization (Ochi and Kanai (1991)) has been used to handle the nonlinear equations of motion, generating controllers suitable for all the flight envelope, if a precise knowledge of the aircraft model exists. However accurate aerodynamic and propulsive models are often not available.

Backstepping is another nonlinear control technique which can handle the nonlinear equations of motion, if the system has a cascade structure (Krstić et al. (1995)). If adaptation laws are also included, the adaptive backstepping control technique can deal with systems in which parametric uncertainties are present. This would be very useful for a flight control system, since aerodynamic and propulsive models are not known accurately. Thus, model errors can be explicitly taken into account in the controller design.

Several examples of backstepping applied to flight control can be found in the literature. For instance, Härkegård (2003) develops some aircraft flight controllers which use this technique; the aerodynamic moments are used as virtual control signals in the backstepping design, and a control allocation scheme is used to find the aerodynamic surface deflections.

In Farrell et al. (2005) an adaptive backstepping flight controller for a high-performance UAV is developed, guaranteeing the Lyapunov stability and considering the presence of physical constraints in the control system such as saturations, bandwidth limitations or rate limits. A linear aerodynamic model is used, and adaptation laws are implemented to estimate online the stability derivatives in the model.

A similar approach is described in Sonneveldt et al. (2007). In that work a constrained adaptive backstepping controller is designed for the F-16/MATV simulation model, using neural networks to model its aerodynamics, whose parameters are estimated through adaptation laws.

The goal of this work is to design a control law able to deal with the aircraft longitudinal dynamics, for all the normal operating regimes of the aircraft, with a minimal information of the aerodynamic model. The controller must be able to make the system seek the references in

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the aerodynamic velocity and flight path angle, using as actuators the elevator deflections and the thrust level.

An adaptive backstepping strategy is proposed, which exploits the structure of the system and general properties from aerodynamics. The nonlinear longitudinal aircraft model is used, and since only some specific properties of the aerodynamic coefficients are known, an adaptation law is designed for their online estimation. The resulting control laws are explicit and simpler than those produced by the previously cited works, and do not require much computational power on board.

Simulations are included for a realistic UAV model that includes actuator saturation and nonlinear aerodynamics. The model is an accurate description of the Cefiro aircraft (Bernal et al. (2009)), an UAV recently designed and constructed in the University of Seville.

The paper is structured as follows: First, in Section 2 aircraft model used in this work is presented. The controller design is detailed in Section 3, which begins with the velocity controller (3.1) and follows with the flight path angle controller (3.2). Simulation results are shown in Section 4. Section 5 closes the paper with some concluding remarks.

2. AIRCRAFT MODEL

Let $(V_a, \gamma, \theta, q) \in \mathbb{R}^4$ be the state vector where V_a is the aerodynamic velocity, γ is the flight path angle, θ is the pitch angle, q is the pitch angular velocity and, let $(F_T, \delta_e) \in \mathbb{R}^2$ be the control input vector where F_T is the engine thrust and δ_e the elevator angle. The equations of motion of the aircraft longitudinal dynamics from (Stevens and Lewis (2003)) read

$$\dot{V}_a = \frac{1}{m}\left(-D + F_T \cos\alpha - mg\sin\gamma\right), \tag{1}$$

$$\dot{\gamma} = \frac{1}{mV_a}\left(L + F_T \sin\alpha - mg\cos\gamma\right), \tag{2}$$

$$\dot{\theta} = q, \tag{3}$$

$$\dot{q} = \frac{M(\delta_e)}{I_y}, \tag{4}$$

where m and I_y are the mass and the inertia; V_a is the aerodynamic velocity; γ is the flight path angle; θ is the pitch angle; q is the pitch angular velocity; F_T is the engine thrust and, finally, L, D and $M(\delta_e)$ are the aerodynamics forces lift, drag and pitching moment, respectively. In Fig. 1 a detailed definition of the forces, moments, and velocities are shown. Note that $\alpha = \theta - \gamma$, where α is the angle of attack.

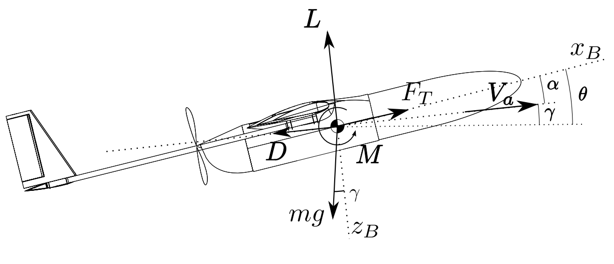

Fig. 1. Definition of forces, moments and angles.

As usual in aerodynamic modeling, the aerodynamic forces and moments are computed through their non-dimensional coefficients, as follows:

$$L = \frac{1}{2}\rho V_a^2 S C_L,\ D = \frac{1}{2}\rho V_a^2 S C_D,\ M = \frac{1}{2}\rho V_a^2 S\bar{c} C_m, \tag{5}$$

where ρ is the air density, S is the reference wing surface, $\bar{c}$ is the mean chord and C_L, C_D and C_m are the lift, drag and pitching moment coefficients. Moreover, we consider the following models for the drag and moment coefficients (see for instance Etkin and Reid (1996); Pamadi (2004) and Schmidt (1998)):

$$C_D = C_{D_0} + k_1 C_L + k_2 C_L^2, \tag{6}$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_q} q + C_{m_{\delta_e}}\delta_e, \tag{7}$$

where C_{D_0}, k_1, k_2, C_{m_0}, C_{m_α}, C_{m_q} and $C_{m_{\delta_e}}$ are aircraft aerodynamic coefficients, and δ_e is the elevator angle. In this work, C_{D_0}, k_1, k_2 , C_{m_0}, C_{m_α} and C_{m_q} are considered to be unknown parameters, while $C_{m_{\delta_e}}$ is known.

Regarding the lift coefficient model, the following assumption is done, which it is satisfied by conventional airplanes in the non-stalled regime [1] .

Assumption 1. The lift coefficient C_L is only a function of α. The reference axis x_B is chosen so that $C_L(0) = 0$, i.e. x_B is parallel to the aircraft zero-lift line. Then, the property $x \cdot C_L(x) \geq 0$ is satisfied for all $x \in \mathbb{R}$.

3. CONTROLLER DESIGN

From a control viewpoint, $(V_a, \gamma, \theta, q) \in \mathbb{R}^4$ is the state vector and $(F_T, \delta_e) \in \mathbb{R}^2$ is the control input vector. Thus, the control objective is to make the system seek known references in velocity and flight path angle using the elevator and thrust as control signals.

In order to simplify the controller design, we first consider velocity dynamics, given by Equation (1), and then the pitch dynamics given by (2)–(4). Thus, two different controllers are designed: the aerodynamic velocity is controlled using only thrust (F_T) and the flight path angle (pitch dynamics) is controlled with the elevator angle (δ_e).

3.1 Control of aerodynamic velocity

Substituting the moment model D from (5) into (1), the velocity dynamics reads

$$\dot{V}_a = \frac{1}{m}\left(-\frac{1}{2}\rho V_a^2 S C_D + F_T\cos\alpha - mg\sin\gamma\right). \tag{8}$$

The engine thrust F_T is the control input, while the α and γ are considered to be measurable. In addition, as is shown in (6), the following drag model is considered:

$$C_D = C_{D_0} + k_1\alpha + k_2\alpha^2, \tag{9}$$

where C_{D_0}, k_1 and k_2 are unknown parameters. Denote V_r to the reference velocity and define the error $z_V := V_a - V_r$. Thus, the evolution of the error from (8) becomes

[1] See for instance Abot and Von Doenhoff (1959), where an extensive compendium of lift curves with this property can be found.

$$\begin{aligned}\dot{z}_V =& -\frac{1}{2m}\rho\,(z_V+V_r)^2\,S\boldsymbol{\varphi}_V(\alpha)^T\cdot\boldsymbol{\theta}_V + F_T\frac{\cos\alpha}{m}\\ &-g\sin\gamma-\dot{V}_r\\ =& -\beta_1\left(z_V^2+V_r^2+2z_VV_r\right)\boldsymbol{\varphi}_V(\alpha)^T\cdot\boldsymbol{\theta}_V\\ &+F_T\frac{\cos\alpha}{m}-g\sin\gamma-\dot{V}_r,\end{aligned} \tag{10}$$

where we have defined

$$\boldsymbol{\varphi}_V(\alpha) := \left[1\ \alpha\ \alpha^2\right]^T, \boldsymbol{\theta}_V := [C_{D_0}\ k_1\ k_2]^T, \beta_1 := \frac{\rho S}{2m}, \tag{11}$$

with $\boldsymbol{\theta}_V \in \mathbb{R}^3$ the unknown parameters vector, the vector $\boldsymbol{\varphi}_V \in \mathbb{R}^3$ defined through the drag model (9) as $C_D = \boldsymbol{\varphi}_V(\alpha)^T\cdot\boldsymbol{\theta}_V > 0$ and, the scalar parameter $\beta_1 > 0$. Now we are in position to state our first result.

Proposition 1. Consider the system (10). Let $\hat{\boldsymbol{\theta}}_V$ be the estimate of $\boldsymbol{\theta}_V$ defined in (11), then the *adaptative-state feedback* given by

$$\begin{aligned}F_T =& \frac{m}{\cos\alpha}\Big(g\sin\gamma+\dot{V}_r+\beta_1(z_V^2+V_r^2)\boldsymbol{\varphi}(\alpha)^T\cdot\hat{\boldsymbol{\theta}}_V\\ &-\kappa_{V_1}z_V\Big),\end{aligned} \tag{12}$$

$$\dot{\hat{\boldsymbol{\theta}}}_V = -\beta_1\left(z_V^3+z_VV_r^2\right)\boldsymbol{\Gamma_V}\boldsymbol{\varphi}_V(\alpha), \tag{13}$$

guarantees global boundedness of z_V and $\hat{\boldsymbol{\theta}}_V$ and convergence of z_V to zero.

Proof. Define the Lyapunov function as

$$W_V = \frac{1}{2}z_V^2+\frac{1}{2}\tilde{\boldsymbol{\theta}}_V^T\boldsymbol{\Gamma_V}^{-1}\tilde{\boldsymbol{\theta}}_V, \tag{14}$$

where $\tilde{\boldsymbol{\theta}}_V := \boldsymbol{\theta}_V - \hat{\boldsymbol{\theta}}_V$ is the estimation error vector and $\boldsymbol{\Gamma_V} = \boldsymbol{\Gamma_V}^T > 0$ is the adaptation gain matrix. Thus, the derivative with respect to time of (14) along the trajectories of (10) reads

$$\begin{aligned}\dot{W}_V =& z_V\Big(-\beta_1\left(z_V^2+V_r^2+2z_VV_r\right)\boldsymbol{\varphi}_V(\alpha)^T\cdot\boldsymbol{\theta}_V\\ &+F_T\frac{\cos\alpha}{m}-g\sin\gamma-\dot{V}_r\Big)+\tilde{\boldsymbol{\theta}}_V^T\boldsymbol{\Gamma_V}^{-1}\dot{\tilde{\boldsymbol{\theta}}}_V\\ =& -2\beta_1V_r\boldsymbol{\varphi}_V(\alpha)^T\cdot\boldsymbol{\theta}_Vz_V^2-\kappa_{V_1}z_V^2\\ &-\beta_1z_V(z_V^2+V_r^2)\boldsymbol{\varphi}_V(\alpha)^T\cdot\tilde{\boldsymbol{\theta}}_V+\tilde{\boldsymbol{\theta}}_V^T\boldsymbol{\Gamma_V}^{-1}\dot{\tilde{\boldsymbol{\theta}}}_V\\ =& -2\beta_1V_r\boldsymbol{\varphi}_V(\alpha)^T\cdot\boldsymbol{\theta}_Vz_V^2-\kappa_{V_1}z_V^2\\ &+\tilde{\boldsymbol{\theta}}_V^T\Big(\boldsymbol{\Gamma_V}^{-1}\dot{\tilde{\boldsymbol{\theta}}}_V-\beta_1z_V(z_V^2+V_r^2)\boldsymbol{\varphi}_V(\alpha)\Big),\end{aligned} \tag{15}$$

where we replaced F_T by (12). By construction the first and second terms of (15) are negative and, the last term is rendered zero through the adaptation law (13) and noting that $\dot{\hat{\boldsymbol{\theta}}}_V = -\dot{\tilde{\boldsymbol{\theta}}}_V$. Thus, since W_V is positive definite and radially unbounded and $\dot{W}_V \le 0$ then, by LaSalle-Yoshizawa theorem we conclude global boundedness of z_V and $\hat{\boldsymbol{\theta}}_V$ and convergence of z_V to zero.

3.2 Control of the flight path angle

The pitch dynamics are governed by equations (2)–(4), which plugging (5) become

$$\dot{\gamma} = \frac{1}{mV_a}\left(\frac{1}{2}\rho V_a^2SC_L+F_T\sin\alpha-mg\cos\gamma\right), \tag{16}$$

$$\dot{\theta} = q, \tag{17}$$

$$\dot{q} = \frac{\rho V_aS\bar{c}}{2I_y}\left(C_{m_0}+C_{m_\alpha}\alpha+C_{m_q}q+C_{m_{\delta_e}}\delta_e\right), \tag{18}$$

where the V_a and F_T are those obtained in the previous step design of Subsection 3.1.

Assumption 2. The following usual assumptions are made:

- Since $\gamma \approx \gamma_{ref}$ then it is assumed that $\cos\gamma = \cos\gamma_{ref}$, as proposed in reference Härkegård (2003).
- $\dot{\gamma}_{ref}$ is assumed to be zero.
- The aircraft engines cannot produce negative thrust. Thus, it is satisfied that $F_T \ge 0$.

Under Assumption 2 the equation (16) becomes

$$\dot{\gamma} = f(\alpha) = f(\theta-\gamma), \tag{19}$$

where the scalar function f is defined as

$$f(\alpha) := \frac{1}{mV_a}\left(\frac{1}{2}\rho V_a^2SC_L(\alpha)+F_T\sin\alpha-mg\cos\gamma_{ref}\right).$$

Property 1. Let α_0 be the trim angle of attack, that is $f(\alpha_0) = 0$ when $\gamma = \gamma_{ref}$, then, under the Assumption 1, the function $f(\alpha)$ satisfies $(\alpha-\alpha_0)f(\alpha) > 0$. This trim angle is supposed to be known.

The control objective is to make the equilibrium $(\gamma,\theta,q) = (\gamma_{ref},\theta_{ref},0)$ asymptotically stable, where γ_{ref} is given and θ_{ref} is computed from $\theta_{ref} = \gamma_{ref}+\alpha_0$. To design the controller we shift the equilibrium to zero defining the following set of error coordinates

$$z_1 = \gamma-\gamma_{ref}, \quad z_2 = \theta-\gamma_{ref}-\alpha_0 \text{ and } z_3 = q. \tag{20}$$

The equations (16)–(18) in the new set of coordinates read

$$\dot{z}_1 = \eta(z_2-z_1), \tag{21}$$

$$\dot{z}_2 = z_3, \tag{22}$$

$$\begin{aligned}\dot{z}_3 =& \beta_2\left(C_{m_0}+C_{m_\alpha}(z_2-z_1+\alpha_0)\right.\\ &\left.+C_{m_q}z_3+C_{m_{\delta_e}}\delta_e\right),\end{aligned} \tag{23}$$

where we defined $\beta_2 := \frac{\rho V_a^2S\bar{c}}{2I_y}$ and $\eta(x) := f(x+\alpha_0)$. Notice that the property 1 makes the scalar function $\eta(x)$ to satisfy $x\cdot\eta(x) \ge 0$.

Remark 1. In Härkegård (2003), a backstepping control law was designed for the cascade structure (21)–(23), using in (4) the aerodynamic moment model M as the control input and, here, we use δ_e through $M(\delta_e)$. Assuming that f satisfies property 1 and knowledge of α_0 a control allocation scheme was used to estimate the elevator deflections, using an assumed aerodynamic moment model. In this work, we use the same idea to generate a backstepping controller with no need of exact knowledge of f, and extend it using adaptive backstepping so that an aerodynamic moment model is not needed. Thus, the main difference is that $M(\delta_e)$ is given by (5) and (7) with the aerodynamic coefficients of (7) unknown.

Now the control objective is to make the origin of system (21)–(23) (globally) asymptotically stable. Thus, roughly speaking, we design a controller using minimal information about the aerodynamic model of the aircraft. To do so, we stabilize each step of the cascade explicitly using backstepping approach.

Step 1. First, equation (21) is stabilized using z_2 as a virtual control. Defining the Lyapunov function as

$$W_1 = \frac{1}{2}z_1^2,$$

the derivative reads $\dot{W}_1 = z_1\eta(z_2 - z_1)$ and then we select the control $z_2 = u_1(z_1) = -\kappa_{\gamma_1} z_1$. Thus,

$$\dot{W}_1|_{z_2=u_1(z_1)} = z_1\eta(-(1+\kappa_{\gamma_1})z_1),$$

and hence $\dot{W}_1|_{z_2=u_1(z_1)}$ is negative definite for $\kappa_{\gamma_1} > -1$.

Step 2. Defining now the error variable

$$\tilde{z}_2 := z_2 - u_1(z_1),$$

the equations (21)–(22) can be rewritten as

$$\dot{z}_1 = \eta(\xi), \tag{24}$$

$$\dot{\tilde{z}}_2 = z_3 + \kappa_{\gamma_1}\eta(\xi), \tag{25}$$

where, only for compactness, we define ξ as

$$\xi := -(1+\kappa_{\gamma_1})z_1 + \tilde{z}_2,$$

and then

$$\dot{\xi} = -\eta(\xi) + z_3.$$

The Lyapunov function for (24)–(25) is

$$W_2 = c_1 W_1 + \frac{1}{2}\tilde{z}_2^2 + F(\xi),$$

where $F(\xi)$ is a positive definite function to be defined further. Calculating $\dot{W}_2$ we get

$$\begin{aligned}\dot{W}_2 &= c_1 z_1\eta(\xi) + \tilde{z}_2(z_3 + \kappa_{\gamma_1}\eta(\xi)) + F'(\xi)(-\eta(\xi)+z_3)\\ &= (c_1 z_1 + \kappa_{\gamma_1}\tilde{z}_2 - F'(\xi))\eta(\xi) + (\tilde{z}_2 + F'(\xi))z_3.\end{aligned} \tag{26}$$

By selecting the virtual control as $z_3 = u_2(\tilde{z}_2) = -\kappa_{\gamma_2}\tilde{z}_2$, $\kappa_{\gamma_2} > 0$, and $F'(\xi) = c_2\eta(\xi)$, $c_2 > 0$, then (26) becomes

$$\dot{W}_2 = (c_1 z_1 + (\kappa_{\gamma_1} - \kappa_{\gamma_2}c_2)\tilde{z}_2)\eta(\xi) - c_2\eta^2(\xi) - \kappa_{\gamma_2}\tilde{z}_2^2,$$

which can be made negative definite if $c_1 = (1+\kappa_{\gamma_1})(\kappa_{\gamma_2}c_2 - \kappa_{\gamma_1})$, with $\kappa_{\gamma_2}c_2 > \kappa_{\gamma_1}$. Finally, $\dot{W}_2$ reads

$$\dot{W}_2 = -(\kappa_{\gamma_2}c_2 - \kappa_{\gamma_1})\xi\eta(\xi) - c_2\eta^2(\xi) - \kappa_{\gamma_2}\tilde{z}_2^2,$$

where the first term is negative definite by the property 1, and in turn F is positive definite by the properties of $\eta(\xi)$.

Remark 2. This virtual control law (Härkegård (2003)) does not need the function $f(\alpha)$, since the function $F(\xi)$ (first introduced in Krstic and Kokotovic (1995)) has been used to avoid cancellations of the terms associated to $\eta(\xi)$, which would introduce extra terms in the controller. Thus, this design leads to a robust control law.

Step 3. In this last step, we extend the backstepping design to generate the elevator deflections laws. As commented above the novelty is an adaptive scheme used to estimate online the aerodynamic moment coefficients. Moreover, the control law is designed without cancellation of terms coming from $\eta(\xi)$ in the previous step design.

Defining the error as $\tilde{z}_3 := z_3 - u_2(z_1, z_2)$ yields

$$\dot{z}_1 = \eta(\xi), \tag{27}$$

$$\dot{\tilde{z}}_2 = \tilde{z}_3 - \kappa_{\gamma_2}\tilde{z}_2 + \kappa_{\gamma_1}\eta(\xi), \tag{28}$$

$$\begin{aligned}\dot{\tilde{z}}_3 &= \beta_2\left(C_{m_0} + C_{m_\alpha}(\xi+\alpha_0) + C_{m_q}z_3 + C_{m_{\delta_e}}\delta_e\right)\\ &\quad+\kappa_{\gamma_2}(\tilde{z}_3 - \kappa_{\gamma_2}\tilde{z}_2 + \kappa_{\gamma_1}\eta(\xi)),\end{aligned} \tag{29}$$

where δ_e is the elevator deflection, which is the real control input of the aircraft, and C_{m_0}, C_{m_α}, C_{m_q} are the unknown aerodynamic coefficients. The adaptive controller proposed here is designed to stabilize equations (27)–(29) with an adaptation law to estimate these parameters.

First notice that the equation (29) can be written as

$$\dot{\tilde{z}}_3 = \beta_2\boldsymbol{\varphi}_\gamma^T\cdot\boldsymbol{\theta}_\gamma + \beta_{\delta_e}\delta_e + \kappa_{\gamma_2}(\tilde{z}_3 - \kappa_{\gamma_2}\tilde{z}_2 + \kappa_{\gamma_1}\eta(\xi)), \tag{30}$$

where $\beta_{\delta_e} := \frac{\rho V_a^2 S\bar{c}}{2I_y}C_{m_{\delta_e}}$, $\boldsymbol{\theta}_\gamma := [C_{m_0}\ C_{m_\alpha}\ C_{m_q}]^T \in \mathbb{R}^3$ is the unknown parameters vector and $\boldsymbol{\varphi}_\gamma := [1\ \xi+\alpha_0\ \tilde{z}_3 - \kappa_{\gamma 2}\tilde{z}_2]^T \in \mathbb{R}^3$. Thus, the compound Lyapunov function for this step is

$$W_3 = c_3 W_2 + \frac{1}{2}\tilde{z}_3^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_\gamma^T\boldsymbol{\Gamma_\gamma}^{-1}\tilde{\boldsymbol{\theta}}_\gamma, \tag{31}$$

where $c_3 > 0$, $\boldsymbol{\Gamma_\gamma} = \boldsymbol{\Gamma_\gamma}^T > 0$ is the adaptation gain matrix, $\hat{\boldsymbol{\theta}}_\gamma$ is the estimate of $\boldsymbol{\theta}_\gamma$ and $\tilde{\boldsymbol{\theta}}_\gamma := \boldsymbol{\theta}_\gamma - \hat{\boldsymbol{\theta}}_\gamma$ is the estimation error vector.

The Lyapunov function derivative becomes

$$\begin{aligned}\dot{W}_3 &= c_3\left[-(\kappa_{\gamma_2}c_2 - \kappa_{\gamma_1})\xi\eta(\xi) - c_2\eta^2(\xi) - \kappa_{\gamma_2}\tilde{z}_2^2\right.\\ &\quad\left.+\tilde{z}_3(\tilde{z}_2 + c_2\eta(\xi))\right] + \tilde{z}_3\left[\beta_2\boldsymbol{\varphi}_\gamma^T\cdot\boldsymbol{\theta}_\gamma + \beta_{\delta_e}\delta_e\right.\\ &\quad\left.+\kappa_{\gamma_2}(\tilde{z}_3 - \kappa_{\gamma_2}\tilde{z}_2 + \kappa_{\gamma_1}\eta(\xi))\right] + \tilde{\boldsymbol{\theta}}_\gamma^T\boldsymbol{\Gamma_\gamma}^{-1}\dot{\tilde{\boldsymbol{\theta}}}_\gamma.\end{aligned} \tag{32}$$

In the former equation, there is a cross-term $\tilde{z}_3\eta(\xi)$ whose sign is undefined. If it is cancelled, the function $\eta(\xi)$ would appear in the controller and the benefit of the controller shown in the previous backstepping step would be lost. Instead, grouping the terms $\eta(\xi)^2$, $\tilde{z}_3^2$ and $\tilde{z}_3\eta(\xi)$ and completing squares as follows

$$\begin{aligned}&-c_3c_2\eta(\xi)^2 + (c_3c_2 + \kappa_{\gamma_2}\kappa_{\gamma_1})\tilde{z}_3\eta(\xi)\\ &= -\left(\sqrt{c_3c_2}\eta(\xi) - \lambda\tilde{z}_3\right)^2 + \lambda^2\tilde{z}_3^2,\end{aligned}$$

where we have defined

$$\lambda := \frac{c_3c_2 + \kappa_{\gamma_1}\kappa_{\gamma_2}}{2\sqrt{c_3c_2}}.$$

Completing squares also in the cross-terms $\tilde{z}_3\tilde{z}_2$, we have

$$\begin{aligned}&-c_3\kappa_{\gamma_2}\tilde{z}_2^2 - \left(\kappa_{\gamma_2}^2 - c_3\right)\tilde{z}_3\tilde{z}_2\\ &= -\left(\frac{\kappa_{\gamma_2}^2 - c_3}{2\sqrt{c_3\kappa_{\gamma_2}}}\tilde{z}_3 + \sqrt{c_2\kappa_{\gamma_2}}\tilde{z}_2\right)^2 + \frac{\left(\kappa_{\gamma_2}^2 - c_3\right)^2}{4c_3\kappa_{\gamma_2}}\tilde{z}_3^2.\end{aligned}$$

Thus, (32) can be rewritten as

$$\begin{aligned}\dot{W}_3 &= -c_3(\kappa_{\gamma_2}c_2 - \kappa_{\gamma_1})\xi\eta(\xi) - \left(\sqrt{c_3c_2}\eta(\xi) - \lambda\tilde{z}_3\right)^2\\ &\quad-\left(\frac{\kappa_{\gamma_2}^2 - c_3}{2\sqrt{c_3\kappa_{\gamma_2}}}\tilde{z}_3 + \sqrt{c_2\kappa_{\gamma_2}}\tilde{z}_2\right)^2 + \tilde{z}_3\left[\beta_2\boldsymbol{\varphi}_\gamma^T\cdot\boldsymbol{\theta}_\gamma\right.\\ &\quad\left.+\beta_{\delta_e}\delta_e + \left(\lambda^2 + \frac{\left(\kappa_{\gamma_2}^2 - c_3\right)^2}{4c_3\kappa_{\gamma_2}} + \kappa_{\gamma_2}\right)\tilde{z}_3\right]\\ &\quad+\tilde{\boldsymbol{\theta}}_\gamma^T\boldsymbol{\Gamma_\gamma}^{-1}\dot{\tilde{\boldsymbol{\theta}}}_\gamma.\end{aligned} \tag{33}$$

This derivative is definite negative choosing the following control and adaptation law

$$\delta_e = \frac{1}{\beta_{\delta_e}} \left(-\kappa_{\gamma_3} \tilde{z}_3 - \beta_2 \boldsymbol{\varphi}_\gamma^T \cdot \hat{\boldsymbol{\theta}}_\gamma \right), \quad (34)$$

$$\dot{\hat{\boldsymbol{\theta}}}_\gamma = -\dot{\tilde{\boldsymbol{\theta}}}_\gamma = \beta_2 \tilde{z}_3 \boldsymbol{\Gamma_\gamma} \boldsymbol{\varphi}_\gamma, \quad (35)$$

with $\kappa_{\gamma_3} > \lambda^2 + \frac{(\kappa_{\gamma_2}^2 - c_3)^2}{4c_3\kappa_{\gamma_2}} + \kappa_{\gamma_2} > 0$.

We formally summarize the result obtained in this section in the following proposition.

Proposition 2. Consider the system (16)–(18) under Assumptions 1 and 2. Then, the *adaptative-state feedback* given by

$$\delta_e = \frac{1}{\beta_{\delta_e}} \Big(-\kappa_{\gamma_3} \left(q + \kappa_{\gamma_2} \left(\theta - \gamma_{ref} - \alpha_0 + \kappa_{\gamma_1} \left(\gamma - \gamma_{ref}\right)\right)\right) - \beta_2 \boldsymbol{\varphi}_\gamma^T \cdot \hat{\boldsymbol{\theta}}_\gamma \Big), \quad (36)$$

$$\dot{\hat{\boldsymbol{\theta}}}_\gamma = \beta_2 \Big(q + \kappa_{\gamma_2} \left(\theta - \gamma_{ref} - \alpha_0 + \kappa_{\gamma_1} \left(\gamma - \gamma_{ref}\right)\right) \cdot \boldsymbol{\Gamma_\gamma} \boldsymbol{\varphi}_\gamma \Big), \quad (37)$$

with $c_2, c_3, \kappa_{\gamma_1}, \kappa_{\gamma_2}, \kappa_{\gamma_3}$ positive and satisfying

$$\kappa_{\gamma_3} > \left(\frac{c_3 c_2 + \kappa_{\gamma_1}\kappa_{\gamma_2}}{2\sqrt{c_3 c_2}} \right)^2 + \frac{\left(\kappa_{\gamma_2}^2 - c_3\right)^2}{4c_3\kappa_{\gamma_2}} + \kappa_{\gamma_2},$$

assures that the equilibrium manifold $(\gamma, \theta, q, \hat{\boldsymbol{\theta}}_\gamma) = (\gamma_{ref}, \theta_{ref}, 0, \hat{\boldsymbol{\theta}}_\gamma^*)$ is globally asymptotically stable, for some constant $\hat{\boldsymbol{\theta}}_\gamma^*$.

Proof. First note that the closed-loop system is time-invariant. The proposed Lyapunov function (31) is positive definite and radially unbounded which, together with the *adaptative-state feedback* (36)–(37), or equivalently (34)-(35), makes $\dot{W}_3 \leq 0$ and then, by LaSalle-Yoshizawa theorem, we conclude global boundedness of $(\gamma, \theta, q, \hat{\boldsymbol{\theta}}_\gamma)$. LaSalle's invariance principle assures that all trajectories converge to the largest invariant set contained in $\{(\gamma, \theta, q, \hat{\boldsymbol{\theta}}_\gamma) \in \mathbb{R}^4 : \dot{W}_3 = 0\}$. Since $\dot{W}_3 = 0$ implies $\dot{\tilde{z}}_3 = 0$ then analyzing backwards the residual dynamics, it is straightforward to see that the equilibrium manifold $(\gamma, \theta, q, \hat{\boldsymbol{\theta}}_\gamma) = (\gamma_{ref}, \theta_{ref}, 0, \hat{\boldsymbol{\theta}}_\gamma^*)$ is globally asymptotically stable, or equivalently $(\tilde{z}_3, \tilde{z}_2, z_1, \hat{\boldsymbol{\theta}}_\gamma) = (0, 0, 0, \hat{\boldsymbol{\theta}}_\gamma^*)$.

4. SIMULATION RESULTS

In this section, simulation results of the controllers developed are shown. The simulation model is composed of Equations (1)–(4), and the aerodynamic model of Cefiro UAV, developed in the University of Seville (Bernal et al. (2009)). For a more realistic simulation, saturations in the control signals are also considered. Thus, the following limits are introduced in the thrust and elevator angle:

$$F_T \in [4.9\,\mathrm{N},\ 117.6\,\mathrm{N}]\,, \ \delta_e \in [-30^{\mathrm{o}},\ 30^{\mathrm{o}}]\,. \quad (38)$$

The tuning parameters for the velocity controller are

$$\kappa_{V_1} = 10; \quad \boldsymbol{\Gamma_V} = 0.001\boldsymbol{I}_3,$$

where $\boldsymbol{I}_3$ is the identity matrix of dimension 3. For the flight path angle controller, the parameters are

$$\kappa_{\gamma_1} = 0.2; \quad \kappa_{\gamma_2} = 0.5; \quad \kappa_{\gamma_3} = 1.5; \quad \boldsymbol{\Gamma_\gamma} = 0.001\boldsymbol{I}_3.$$

The initial estimate of the unknown parameters is

$$\hat{\boldsymbol{\theta}}_V = [\,0.05\ 0.05\ 0.05\,]^T\,,\ \hat{\boldsymbol{\theta}}_\gamma = [\,-0.1\ -1\ -10\,]^T$$

The reference maneuver selected is as follows. The velocity profile consist on three segments with constant velocity, separated by uniform acceleration and uniform deceleration segments. The flight path angle profile consist on two leveled flight segments, with a climb of 10$^{\mathrm{o}}$ between them.

Fig. 2 shows the time evolution of the aerodynamic velocity. After an initial period with some oscillations in which saturations in thrust occurs, the velocity controller achieves an excellent agreement with the reference. Fig. 4 shows the control signals. In the figure, the dashed line represent the computed control signal, whereas the solid line represents the commanded control signal (with saturations). Regarding the flight path angle controller, in Fig. 3 it can be seen that the reference seeking is achieved, but a slower response is obtained since small gains have been selected to avoid excessive oscillations. Fig. 5 shows other state variables such as the pitch and attack angles, and the pitch angular velocity, which have reasonable values throughout the maneuver. Finally, Figure 6 shows the time evolution of the estimated parameters towards certain equilibrium values.

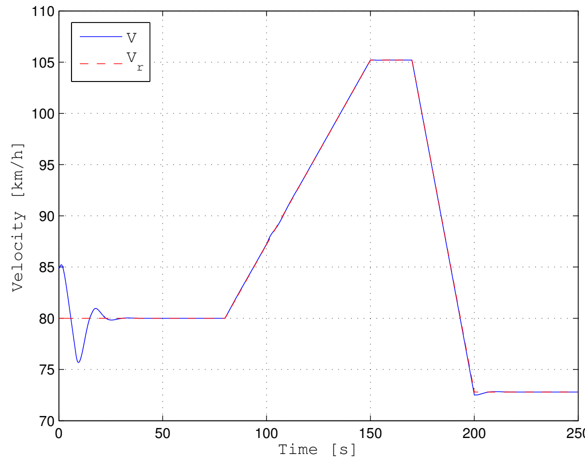

Fig. 2. Time evolution of the aerodynamic velocity (solid), compared with its reference (dashed).

5. CONCLUSION

We presented the design a simple adaptive controller for the longitudinal flight dynamics of an UAV that is able to make the aircraft follow references in velocity and flight path angle. The design is explicit, simple and easy to implement, since it does not require knowledge of the aerodynamics model and does not need much computational power. In simulations, it is shown that the controller can make the system follow the references, even in the presence of actuator saturations.

The simulations were performed using a realistic UAV model, the Cefiro aircraft developed by the University of

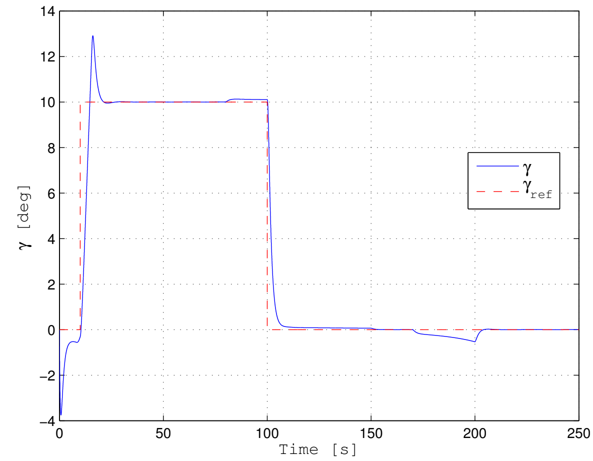

Fig. 3. Time evolution of the flight path angle (solid) compared with its reference (dashed).

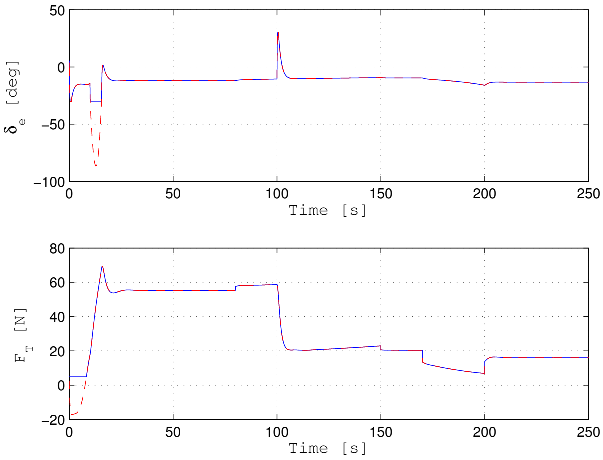

Fig. 4. Control signals: computed (dashed) and commanded (solid).

Seville. As a next step the control laws will be implemented on board the aircraft to perform experiments and further validate the results.

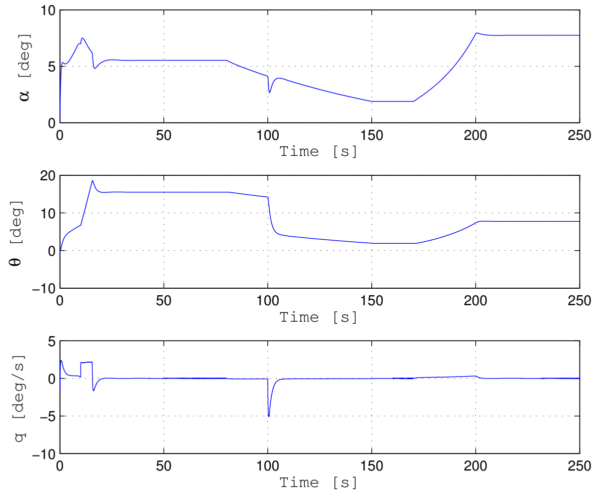

Fig. 5. Time evolution of the angles of attack α and pitch θ, and the angular velocity q.

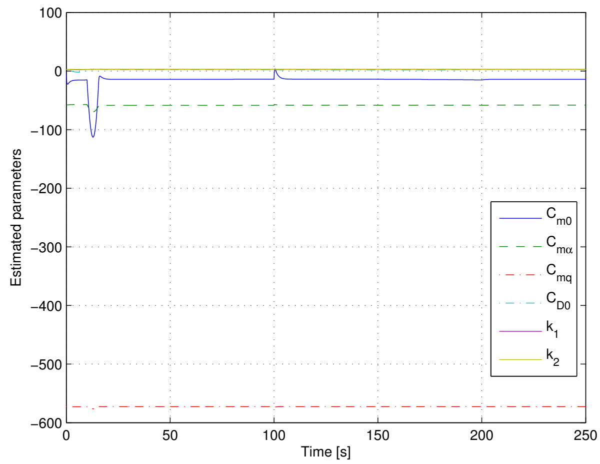

Fig. 6. Time evolution of the estimated parameters.

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