THE CHAMBER COMPLEX FOR THE LITTLEWOOD-RICHARDSON COEFFICIENTS OF GL_4.

EMMANUEL BRIAND, MERCEDES ROSAS, AND STEFAN TRANDAFIR

The *Littlewood-Richardson coefficients* $c^{\nu}_{\lambda,\mu}$ are a family of constants of central importance in representation theory. They are indexed by triples of integer partitions. It is known that when the size of the partitions is limited to some fixed integer N (coefficients associated to GL_N), these coefficients are given by piecewise polynomial formulas in $\lambda_1, \ldots \lambda_N, \mu_1, \ldots, \mu_N, \nu_1, \ldots, \nu_{N-1}$. More precisely, the domains of polynomiality are the big cells ("chambers") of a subdivision of a cone in $\mathbb{R}^{3N-1}$ (the "chamber complex"). In 2005, É. Rassart has calculated explicitly the chamber complex for $N = 3$, finding 18 chambers subdividing a cone in $\mathbb{R}^8$, and the corresponding polynomial formulas (in this case, linear). Using Rassart's description, E. Briand and M. Rosas recently listed exhaustively all symmetries of the chamber complex for $N = 3$, finding a group of 288 symmetries (much more that the 24 symmetries known to exist for general N). It is natural to look at what happens in the next case, $N = 4$. With this aim, we have computed (using SAGEMATH and SINGULAR) the chamber complex for $N = 4$. This object is large, since it consists in the subdivision of a cone in $\mathbb{R}^{11}$ in 67769 chambers. This computation is based on known combinatorial descriptions of the Littlewood-Richardson coefficients. For instance, the Littlewood-Richardson coefficient $c^{(26,21,14,11)}_{(16,13,5,2),(17,12,6,1)}$ counts the triangles

$$\begin{array}{ccccccccc} & & & & 36 & & & & \\ & & & 34 & & 53 & & & \\ & & 29 & & x_3 & & 65 & & \\ & 16 & & x_1 & & x_2 & & 71 & \\ 0 & & 26 & & 47 & & 61 & & 72 \end{array}$$

filled with integers subject to the inequalities $a + b \geq c + d$ in all rhombi:

$$\begin{array}{ccc} & b & & c \\ d & & a & \end{array} \qquad \begin{array}{ccc} & d & \\ a & & b \\ & c & \end{array} \qquad \begin{array}{cccc} c & & b & \\ & a & & d \end{array}$$

of the triangle. (The bottom border entries $0, 26, 47, 61, 72$ are the cumulated sums of $(26, 21, 14, 11)$, that are: $0,\ 26,\ 26{+}21,\ 26{+}21{+}14,\ 26{+}21{+}14{+}11$ and the top border entries $0, 16, 29, 34, 36, 5365, 71, 72$ are the cumulated sums of $(16, 13, 5, 2)$ and $(17, 12, 6, 1)$, that are $0,\ 16,\ 16{+}13,\ 16{+}13{+}5, \ldots, 16{+}13{+}5{+}2{+}17{+}12{+}6{+}1)$.

E. BRIAND, DEPARTAMENTO MATEMÁTICA APLICADA I, UNIVERSIDAD DE SEVILLA

M. ROSAS, DEPARTAMENTO DE ÁLGEBRA, UNIVERSIDAD DE SEVILLA

S. TRANDAFIR, SIMON FRASER UNIVERSITY.

E. Briand and M.Rosas are partially supported by MTM2016-75024-P and FEDER, and Junta de Andalucia under grant FQM333.