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An Application of the Logarithmic Mean Divisia Index Method for Predictive Control Schemes to a Power Flow Network*

J. M. Maestre†, P. Velarde‡ and F. J. Muros†

Abstract— **In this paper, a method typically used in economics is applied to distributed decision making problems solved by model predictive control. The main purpose is to analyze the changes in certain aggregate indicators under study, which are decomposed among a number of factors. In particular, the logarithmic mean divisia index (LMDI) method is used to study how controllers contribute to the performance and how disturbances are generated. Finally, a flow network, which is a system structure that appears in many real problems such as power grids, is used as an academic case study to illustrate the proposed method.**

I. INTRODUCTION

Model predictive control (MPC) is a computer-based control approach widely used in the process industry [1], [2]. The key to its success is the versatility of its framework. MPC uses a mathematical model of a system to predict its future evolution as a function of the sequence of present and future inputs applied. The model is used to formulate an optimization problem in which the evolution of the system is steered according to a certain objective, e.g., regulation towards a reference. The first element of the input sequence, i.e., that corresponding to the current time instant, is applied to the system and the rest of the elements are discarded, although they provide valuable information regarding the expected future. At the next time instant, the optimization problem is solved again using the most recent information available and once again only the calculated actions for the present time are implemented. This procedure is repeated in a receding horizon fashion. Issues such as constraints in the system variables, delays, and multiple objectives, can be explicitly included in the formulation of the optimization problem.

As a computer-based approach, the applications of MPC have grown enormously with the great advances in information and communication technologies of the last decades. One of the blooming fields in the research of this type of controllers has been that of distributed MPC (DMPC). Under this paradigm, the overall control problem is partitioned into a set of less complex control problems that are assigned to local controllers or *agents*. The performance of the overall system depends on the coupling and interaction between the local controllers, which can exchange information to coordinate their actions. Likewise, it is also possible to use the DMPC framework to improve the coordination in situations in which different independent entities with possibly different objectives interact with each other. See [3] for surveys on the topic.

In most DMPC schemes, local controllers cooperate in static groups. More specifically, each controller coordinates its control actions with any other agent it is coupled to, which means that there is permanent communication between coupled controllers. While communication is essential for the sake of coordination, it may not always be strictly necessary. The coupling degree between subsystems may evolve with time and eventually the side effects derived from the evolution of the subsystems can be low so that the disturbances generated can be neglected. In this situation, it may be preferable to switch to a decentralized behavior in which there is no communication between the corresponding controllers. This is the rationale behind coalitional control schemes, in which the local controllers are grouped dynamically into cooperating sets called *coalitions* [4].

Several applications have been developed under the coalitional MPC approach: as [5], where a hierarchical coalitional control scheme of an irrigation canal is presented; [6], where cooperative game theory tools are considered to include constraints on the links and the agents of networked control systems; or [7], which apply a non-centralized time-varying scheme to large-scale systems so that each controller can operate under a decentralized, distributed, and/or hierarchical fashion over the time. Another related work is presented in [8], where a DMPC framework is proposed for tracking by considering a time-varying communication topology.

In this work, we deal with the problem of estimating the impact of individual local controllers and coalitions from an economic viewpoint. In particular, we propose to use the logarithmic mean divisia indices (LMDI) to decompose the influence of the controllers and coalitions, hence gaining a valuable insight into their contribution to the overall performance. To the best of our knowledge, this is the first time that these indices are considered in a control context. The application of this method is performed analogously as it is used in economics to analyze the changes in certain aggregate indicators under study, which are decomposed among a number of factors. A gentle introduction to this method is given in [9]. Likewise, a survey with applications of this approach in energy and environmental studies can be

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found in [10]. Notice that there are methods with similar goals that have been applied in both energy economics and control among many other fields. This is the case of the Shapley value [11], which is used as a decomposition technique for carbon emissions in [12]; as a means to calculate the relevance of controllers and links in networked systems in [4], [6]; or as an alternative tool to perform system partitioning in [13]. Also, measures to determine the relevance of the entities in a graph have been used already in social networks [14] and shareholder influence attribution in companies [15]. Likewise, the calculation of link measures from node measures has been present in other works for decades [16].

The rest of this article is structured as follows. In Section II, the distributed and coalitional control problem is introduced. Section III presents the LMDI method and how it can be applied in the context of distributed control systems. Section IV presents a simulation example to illustrate a power flow network application of the LMDI method in this context. Final conclusions and remarks are given in Section V.

II. Problem setting

In this section, we briefly introduce how the MPC problem is formulated in a coalitional control framework. For the sake of simplicity, we will present the problem in a pure coalitional setting. As it will be seen, this is the most general way to proceed.

We consider that the system is composed of a set of $\mathcal{N}=\{1,2,\ldots,N\}$ subsystems. Each of these subsystems is governed by a local controller or agent. At each time step k, the agents can communicate with each other by using a communication network described by a graph $\mathcal{G}(k)=(\mathcal{N},\mathcal{E}(k))$, where $\mathcal{E}(k)$ is the set of edges or communication links that are active at time step k. Notice the dependence on time of both $\mathcal{G}(k)$ and $\mathcal{E}(k)$. As a result of the communication network, the set of agents is partitioned into a set of disjoint communication components or coalitions $\mathcal{N}/\mathcal{G}(k)$[1]. Given a coalition $\mathcal{C}\in\mathcal{N}/\mathcal{G}(k)$, any agents $i,j\in\mathcal{C}$ can communicate with each other because there exists at least an indirect path of links in $\mathcal{E}(k)$ that allows the communication. Likewise, there is no communication between different coalitions, i.e., while there is full communication inside coalitions, they work in a decentralized fashion.

Notice that in the coalitional control framework, a centralized control problem computed in a distributed fashion is represented simply by taking $\mathcal{C}=\mathcal{N}$. Analogously, if the communication graph is empty, i.e., it has no active links, the system is partitioned into N coalitions of one agent. This case corresponds to decentralized control. In this way, it is possible to formulate all possibilities in terms of coalitions.

It will be assumed that the model of each coalition $\mathcal{C}\in\mathcal{N}/\mathcal{G}(k)$ can be described by the following discrete time equation:

$$x_{\mathcal{C}}(k+1)=A_{\mathcal{C}}x_{\mathcal{C}}(k)+B_{\mathcal{C}}u_{\mathcal{C}}(k)+E_{\mathcal{C}}w_{\mathcal{C}}(k),\qquad(1)$$

with $x_{\mathcal{C}}=[x_i]_{i\in\mathcal{C}}$ being the state of the corresponding subsystem, where $u_{\mathcal{C}}=[u_i]_{i\in\mathcal{C}}$ is the vector of manipulated variables, and with $w_{\mathcal{C}}=[w_i]_{i\in\mathcal{C}}$ being a vector of measurable disturbances. $A_{\mathcal{C}},B_{\mathcal{C}}$, and $E_{\mathcal{C}}$ are matrices of proper dimensions. Likewise, we consider the following linear constraints in the states and the inputs:

$$x_{\mathcal{C}}\in\mathcal{X}_{\mathcal{C}},\quad u_{\mathcal{C}}\in\mathcal{U}_{\mathcal{C}},\qquad(2)$$

where $\mathcal{X}_{\mathcal{C}}$ and $\mathcal{U}_{\mathcal{C}}$ define a closed set by means of a system of linear inequalities, where the constraints of the coalition are formed as the cartesian product of those in the corresponding subsystems, i.e., $\mathcal{X}_{\mathcal{C}}=\prod_{i\in\mathcal{C}}\mathcal{X}_i$ and $\mathcal{U}_{\mathcal{C}}=\prod_{i\in\mathcal{C}}\mathcal{U}_i$.

In this work, it is assumed that the control objective of each coalition $\mathcal{C}$ is the regulation of the corresponding state vector towards the origin. This is expressed mathematically by the following performance index:

$$\begin{aligned}&J(u_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1),x_k)=\\&\sum_{l=k}^{N_{\mathrm{p}}-1}\big[(x_{\mathcal{C}}(l+1)-x_{\mathrm{ref}})^TQ_{\mathcal{C}}(x_{\mathcal{C}}(l+1)-x_{\mathrm{ref}})\\&\quad+u_{\mathcal{C}}^T(l)R_{\mathcal{C}}u_{\mathcal{C}}(l)\big],\end{aligned}\qquad(3)$$

where $Q_{\mathcal{C}}=diag(Q_i)_{i\in\mathcal{C}}$, and $R_{\mathcal{C}}=diag(R_i)_{i\in\mathcal{C}}$ are constant weighting matrices of the proper size, N_{p} is the prediction horizon, and x_{ref} is the desired state reference. According to (1), J depends on the control input sequence $u_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1)=(u_{\mathcal{C}}(k),\ldots,u_{\mathcal{C}}(k+N_{\mathrm{p}}-1))$ and the value of the state at time step k, $x_{\mathcal{C}}(k)$.

Inside each coalition $\mathcal{C}$, a communication-based negotiation procedure is carried out to calculate the optimal control sequence that minimizes (3). Hence, each coalition solves the following problem to calculate its control actions:

$$\begin{aligned}&u_{\mathcal{C}}^*(k:k+N_{\mathrm{p}}-1)=\\&\arg\min_{u_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1)}J(u_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1),x_k)\\&\text{s.t.}\\&x_{\mathcal{C}}(l+1)=A_{\mathcal{C}}x_{\mathcal{C}}(l)+B_{\mathcal{C}}u_{\mathcal{C}}(l)+E_{\mathcal{C}}w_{\mathcal{C}}(l),\\&x_{\mathcal{C}}(l)\in\mathcal{X}_{\mathcal{C}}\ \forall l\in\{k+1,\ldots,k+N_{\mathrm{p}}\},\\&u_{\mathcal{C}}(l)\in\mathcal{U}_{\mathcal{C}}\ \forall l\in\{k,\ldots,k+N_{\mathrm{p}}-1\},\\&x_{\mathcal{C}}(k)=x_{\mathcal{C},k},\\&w_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1)=\hat{w}_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1),\end{aligned}\qquad(4)$$

where $\hat{w}_{\mathcal{C}}(k:k+N_{\mathrm{p}}-1)=(\hat{w}_i(k:k+N_{\mathrm{p}}-1))_{i\in\mathcal{C}}$ is a deterministic sequence composed by the expected values of the disturbances. From the resulting control sequence, only the values corresponding to the current time step are applied. The rest of the sequence is discarded, although it can be used to obtain information regarding the expected evolution of the decision variables and to guarantee theoretical properties

[1]This notation is used in cooperative game theory to denote that the set of players $\mathcal{N}$ is *divided* by the communication network described by $\mathcal{G}(k)$.

such as stability. This is repeated at each time step in a receding horizon strategy.

Summing up, the main idea of the coalitional based MPC is to manage the overall network by establishing a compromise between the communication burden and the control requirements. In this sense, the cooperation among the different parts of the system is carried out by adapting the control network to the varying coupling conditions at each time instant, as pointed out in [4], [5].

III. The LMDI method for distributed and coalitional systems

In this section, we present the LMDI method and how can be applied in a distributed and coalitional control context. Basically, the goal is to use simple logarithmic properties to find appropriate indices that help us to determine how the disturbances affect the performance of the system from a global and a local perspective, paying attention to factors such as the structure of the system and the weight of the coalition regarding the overall performance. To this end, we first focus on the disturbances generated by the local controllers and propose the following index that measures the sum of disturbances experienced by all subsystems:

$$w = \sum_{i\in\mathcal{N}} |w_i| = \sum_{i\in\mathcal{N}} J\frac{J_i}{J}\frac{|w_i|}{J_i} = \sum_{i\in\mathcal{N}} JS_iI_i, \tag{5}$$

where $|w_i|$ is the norm of the disturbances registered by local controller i, J_i is its corresponding cost, and J is the overall cost of the system. Two auxiliary variables, $S_i = J_i/J$ and $I_i = |w_i|/J_i$, are introduced to simplify the notation.

Notice that the increment in the overall level of disturbances between two given time instants can be explained using the following components:

$$\Delta w = w(k) - w(k_0) = \Delta w_{\text{per}} + \Delta w_{\text{str}} + \Delta w_{\text{int}}, \tag{6}$$

where Δw stands for the increment of the overall disturbance level between the time instants k and k_0, which is set as the reference instant. Then, by taking the logarithm in Eq. (5), it is possible to link the overall increment of disturbances Δw to the increment of the overall performance Δw_{per}, the structure of the system regarding the distribution of the overall cost between the local subsystems Δw_{str}, and the intensity of the disturbances with respect to the local cost registered by each subsystem Δw_{int}. These factors will be named from now on as LMDI indices and can be calculated in the following way:

$$\begin{aligned}\Delta w_{\text{per}} &= \textstyle\sum_{i\in\mathcal{N}} \alpha_i \log\left(\frac{J(k)}{J(k_0)}\right),\\ \Delta w_{\text{str}} &= \textstyle\sum_{i\in\mathcal{N}} \alpha_i \log\left(\frac{S_i(k)}{S_i(k_0)}\right),\\ \Delta w_{\text{int}} &= \textstyle\sum_{i\in\mathcal{N}} \alpha_i \log\left(\frac{I_i(k)}{I_i(k_0)}\right),\end{aligned} \tag{7}$$

where the weights α_i are in turn calculated as

$$\alpha_i = \frac{|w_i(k)| - |w_i(k_0)|}{\log(|w_i(k)|) - \log(|w_i(k_0)|)}. \tag{8}$$

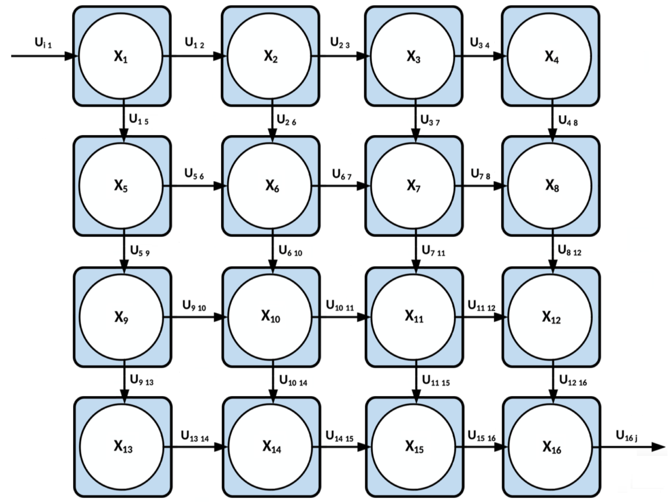

Fig. 1: Diagram of the benchmark considered in the case study. The arrows represent the direction of the flows.

Note that an alternative to built the LMDI indices would be to define the incremental variables, given by (7), (8), by taking into account consecutive time instants, i.e., the current time step k and the last time instant $k-1$ instead of k_0. In any case, both options establish performance metrics that achieve the goal of identifying how and where some changes have arisen in the control structure.

IV. Case study

In this work, a benchmark composed of 16 interconnected batteries is used to test the proposed method. This benchmark has been obtained by modifying that of [17], which was designed to test distributed and coalitional control systems with a water flow network. To this end, batteries and cables replaced tanks and pipes, respectively. The modified benchmark will be used to illustrate some preliminary results of the application of LMDI in this context.

The case study consists of 16 batteries arranged in a 4×4 matrix. Each battery is connected with its direct neighbors by means of cables that allow lossless power flow. For simplicity, it is considered that there is neither power generation nor consumption so that we can focus exclusively on the coordination between the different local controllers to regulate their states. A diagram of the benchmark can be seen in Fig. 1. The following dynamics are assumed for each battery:

$$x_i(k+1) = x_i(k) + T_{\text{s}}\frac{1}{b_i}\sum_{j\in\mathcal{N}_i} u_{ij}(k), \tag{9}$$

where the state variable $x_i(k)$ is the state of energy (SoE) of battery i and b_i is the maximum capacity of storage; T_{s} is the time step length; $u_{ij}(k)$ is the power flow through the cable that connects the batteries i and j; and $\mathcal{N}_i$ is the set of batteries connected to battery i. Note also that the SoE is expressed relatively to maximum capacity so that $x_i(k) \in [0, 1]$.

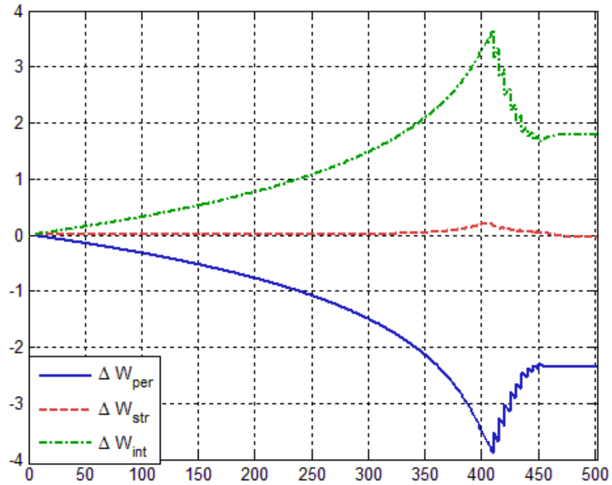

Fig. 2: LMDI for the centralized control approach.

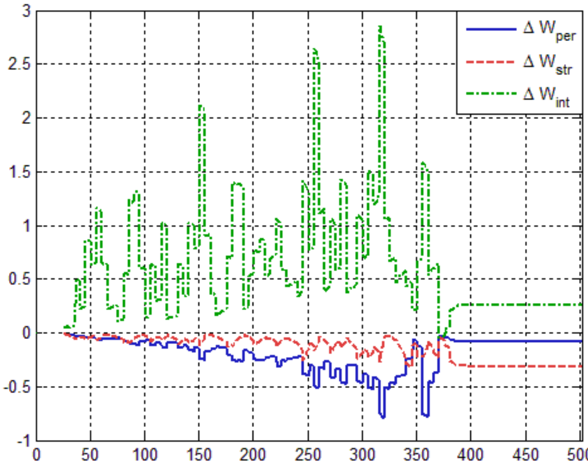

Fig. 3: LMDI for the individually rational coalitional control approach.

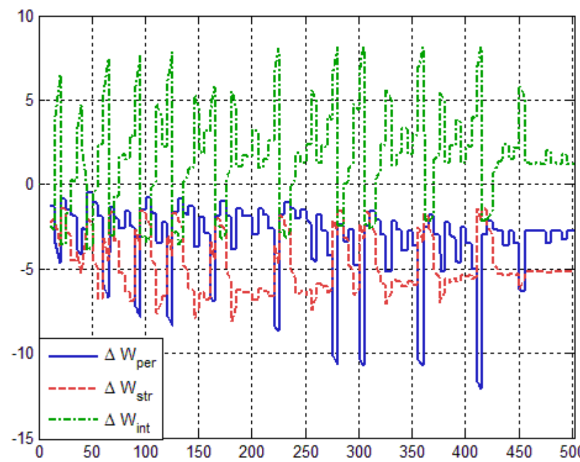

Fig. 4: LMDI for the coalitional control approach based on PageRank.

The objective of the control architecture is to minimize a cost function that includes both communication costs and performance. To this end, each subsystem is governed by a local controller that can manipulate the flow of all variables covered by the corresponding shadow in Fig. 1. There are constraints that limit the amount of power flow transmitted between batteries, which has to be lower than 10 kW. The numerical values of the parameters used for the simulation are setting as follows: the initial value for each battery is $x_i(0) = 0$, the reference level for all batteries is $x_{\rm ref} = 0.5$, the prediction horizon is $N_{\rm p} = 5$, and the weights Q_i and R_i are equal to 100 and 1, respectively. The simulations are performed over 500 s.

A. *LMDI to identify strategies*

It is possible to use the LMDI indices to identify the control strategy that it is applied in a certain scenario. This way, it is noteworthy the evolution of the previously introduced LMDI indices with time, which have been computed using the first time instant of the simulation as a reference, i.e., $k_0 = 1$. These values have been computed for three different control strategies, namely:

- Centralized control strategy: the actuation of all local controllers is coordinated using centralized planning. The evolution of the different LMDI indices can be seen in Fig. 2.
- Individually rational coalitional control strategy [17]: the local controllers can negotiate and form coalitions. In particular, coalitions are formed whenever there is a reduction of cost for the parties involved in the coalition. The evolution of the LMDI indices is represented in Fig. 3.
- Coalitional control strategy based on PageRank: it is another coalitional control strategy recently developed in [18]. In this case, the local controllers can send aid requests to their neighbors. As a consequence, a communication network is formed and the PageRank of the subsystems in the network is used as a criterion to activate the control links. The evolution of the LMDI indices is depicted in Fig. 4.

As can be seen in the aforementioned figures, each strategy presents a different LMDI footprint. In the centralized case, the control architecture is constant and this is clearly translated in the evolution of $\Delta w_{\rm str}$. Likewise, the improvement in the performance mitigates the total disturbances by means of $\Delta w_{\rm per}$. Finally, the relationship between disturbances generated and cost is worse towards the end of the simulation and for this reason $\Delta w_{\rm int}$ grows.

In the individually rational coalitional control approach, the structure evolves and hence changes are appreciated in the evolution of $\Delta w_{\rm str}$. The same holds for the PageRank strategy, although in this case the network can change more abruptly and as a consequence there are greater changes in the structure. These changes are translated in reductions of the disturbances due to the improvement of the performance.

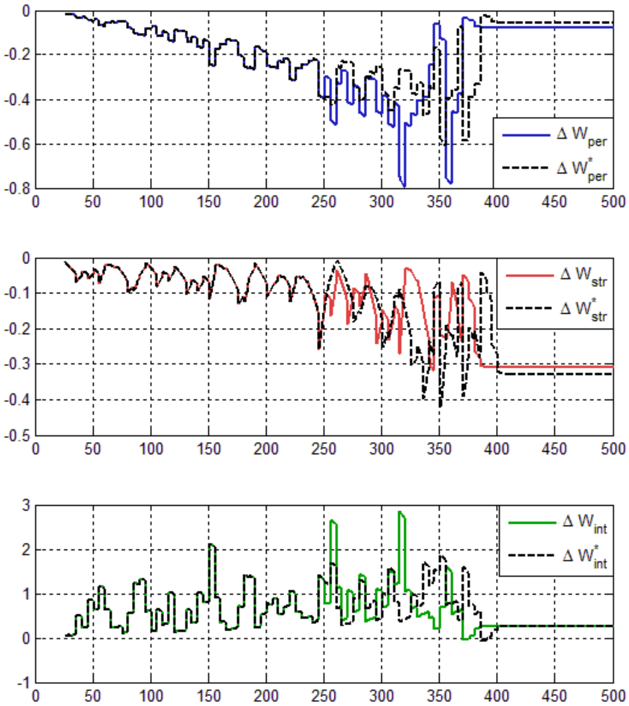

Fig. 5: A comparison of the LMDI evolution for the individually rational coalitional control approach, where the superindex * denotes a change of the reference level.

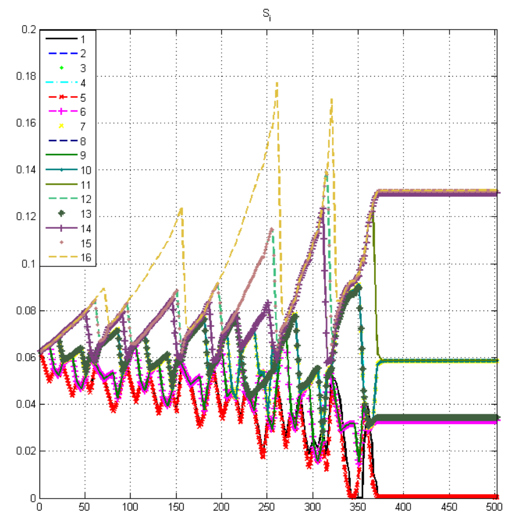

Fig. 6: Evolution of S_i for the individually rational coalitional control approach.

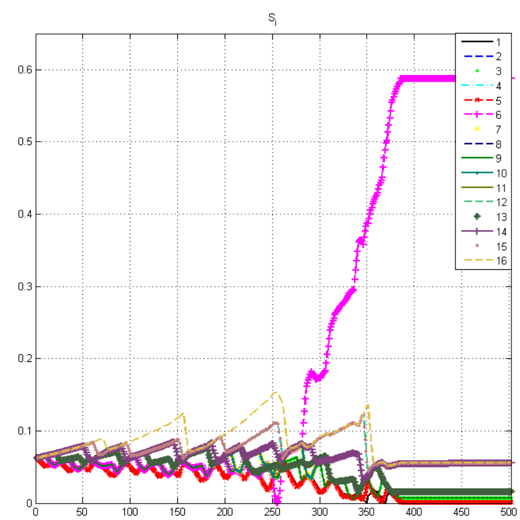

Fig. 7: Evolution of S_i for the individually rational coalitional control approach with a change of the reference level for battery 6.

In general, as the local costs fall, the relevance of the disturbances generated locally grow, specially after a switching in the control topology is implemented, which increases again the disturbances because less coordination is used. All in all, the LMDI indices can provide information about the behavior of the control system and the effect of mutual disturbances in performance.

B. LMDI detection capabilities

It is also possible to apply the LMDI method to detect modifications regarding the problem setting, e.g., reference levels, constraints, changes in the cost function, among others. This is relevant for instance from the viewpoint of cyber-security, for agents might have incentives to introduce changes in their optimization problems to take advantage of the rest of the network. To this end, we change the reference level of battery 6 at the time instant $k = 250$. In particular, the reference changes from 0.5 to 0.75 for the individually rational coalitional control approach. The LMDI footprint presents a different evolution from the time instant in which the reference was changed, as shown in Fig. 5. The dashed lines indicate the LMDI footprint by introducing the aforementioned modification in the reference level. Therefore, by pointing out when a change emerges in the problem setting, the LMDI indices show sensitivity to detect modifications in the optimization problem of an agent.

Finally, we pay attention to the disaggregate indices $S_i = J_i/J, \forall i \in \mathcal{N}$, which are the variables that show more clearly the aforementioned reference change. Fig. 6 indicates the evolution of S_i for each of the 16 batteries, before changing the reference level. A different behavior, shown in Fig. 7, arises due to this modification, particularly for battery 6. It means that significant changes are taking place, especially in the cost function of this battery compared with the remaining agents. In this manner, index S_i allows for detecting the agent that presents a change in its optimization problem, by means of the different LMDI footprint that emerges from this change in the reference. Finally, Fig. 8 shows the contribution to the LMDI indices of battery 6 (which presents a modification in its optimization problem) and for a standard battery, in this case battery 10. In particular, $\Delta w_{\rm str}$ also shows differences due to change in structure: it turns positive after the modification of the reference.

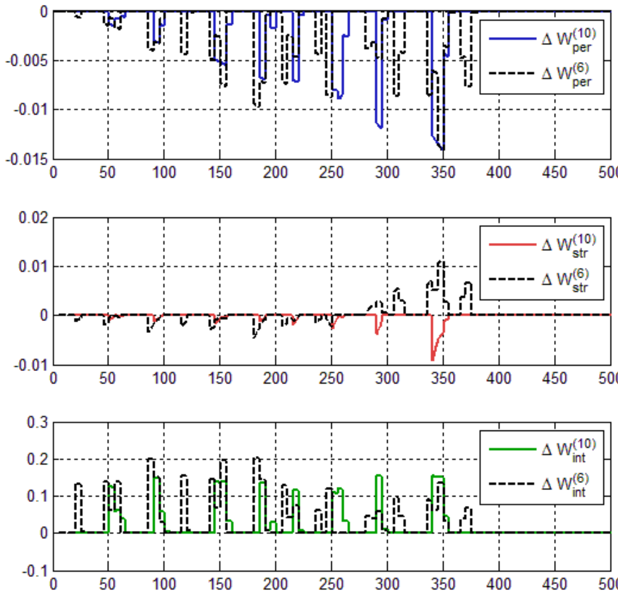

Fig. 8: LMDI for batteries 6 and 10 for the individually rational coalitional control approach.

V. Conclusions

We have transposed a method used in economics to distributed control. The LMDI method is oriented to analyze changes when different time instants are compared. Therefore, the proposed method provides a different perspective, which is specially interesting for coalitional control systems, where the detection of changes in the coupling plays an essential role.

Interestingly, a certain footprint has been detected for the type of control strategy and the evolution of the LMDI indices. This connection will be subject of further research. Likewise, it will be analyzed whether these indices can be used as key performance indicators for the assessment of control strategies and, moreover, as a way to determine the evolution of the topology of the control architecture.

Finally, modifications in the local control problem setting can be identified. Hence, changes due to malicious or malfunctioning controllers could be detected by using this method. For this reason, applications of LMDI to cybersecurity and fault detection real case problems will be object of further study. Likewise, ways to design the optimal topologies via the LMDI indices will also be considered as future research.

References

[1] E. F. Camacho and C. Bordons. *Model predictive control in the process industry*. Springer Science & Business Media, 2012.

[2] S. J. Qin and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7):733–764, July 2003.

[3] R. Scattolini. Architectures for distributed and hierarchical model predictive control – A review. *Journal of Process Control*, 19(5):723–731, May 2009.

[4] J. M. Maestre, D. Muñoz de la Peña, A. Jiménez Losada, E. Algaba, and E. F. Camacho. A coalitional control scheme with applications to cooperative game theory. *Optimal Control Applications and Methods*, 35(5):592–608, September/October 2014.

[5] F. Fele, J. M. Maestre, M. Hashemy, D. Muñoz de la Peña, and E. F. Camacho. Coalitional model predictive control of an irrigation canal. *Journal of Process Control*, 24(4):314–325, April 2014.

[6] F. J. Muros, J. M. Maestre, E. Algaba, T. Alamo, and E. F. Camacho. Networked control design for coalitional schemes using game-theoretic methods. *Automatica*, 78:320–332, April 2017.

[7] A. Núñez, C. Ocampo-Martinez, J. M. Maestre, and B. De Schutter. Time-varying scheme for noncentralized model predictive control of large-scale systems. *Mathematical Problems in Engineering*, 2015.

[8] B. Ding, L. Ge, H. Pan, and P. Wang. Distributed MPC for tracking and formation of homogeneous multi-agent system with time-varying communication topology. *Asian Journal of Control*, 2015.

[9] B. W. Ang. The LMDI approach to decomposition analysis: a practical guide. *Energy policy*, 33(7):867–871, May 2005.

[10] B. W. Ang and F. Q. Zhang. A survey of index decomposition analysis in energy and environmental studies. *Energy*, 25(12):1149–1176, December 2000.

[11] L. S. Shapley. A value for n-person games. *Contributions to the Theory of Games II. Edited by H. W. Kuhn and A. W. Tucker. Annals of Mathematics Studies*, 28:307–317, 1953.

[12] J. Albrecht, D. François, and K. Schoors. A shapley decomposition of carbon emissions without residuals. *Energy Policy*, 30(9):727–736, July 2002.

[13] F. J. Muros, J. M. Maestre, C. Ocampo-Martinez, E. Algaba, and E. F. Camacho. A game theoretical randomized method for large-scale systems partitioning. *IEEE Access*, 6(1):42245–42263, December 2018.

[14] R. Narayanam and Y. Narahari. A Shapley value-based approach to discover influential nodes in social networks. *IEEE Transactions on Automation Science and Engineering*, 8(1):1–18, July 2010.

[15] B. Maury and A. Pajuste. Multiple large shareholders and firm value. *Journal of Banking & Finance*, 29(7):1813–1834, 2005.

[16] P. Borm, G. Owen, and S. Tijs. On the position value for communication situations. *SIAM Journal on Discrete Mathematics*, 5(3):305–320, August 1992.

[17] J. M. Maestre, F. J. Muros, F. Fele, and E. F. Camacho. An assessment of coalitional control in water systems. In *Proceedings of 2015 European Control Conference*, pages 3291–3296, Linz, Austria, July 2015.

[18] J. M. Maestre and H. Ishii. A PageRank based coalitional control scheme. *International Journal of Control, Automation and Systems*, 15:1983–1990, October 2017.