When your gain is also my gain. A class of strategic models with other-regarding agents

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Abstract

This paper explores the role of social preferences in a competitive framework. More precisely, we study other-regarding strategic models where agents show Rawlsian preferences and, therefore, they care about the best interest of the worst-off agent. The representation of preferences proposed is the most appropriate when the utilities of the agents are vector-valued and their components are not compensable but complementary. In these cases, the improvement of the result for each agent has to be reached by simultaneously improving all the components of the vector-valued utility. Depending on the attitude exhibited by the agents with respect to the results of the others, we distinguish different types of agents and relate them with the parameters of the Rawlsian preference function. An analysis of the sets of equilibria in terms of these parameters is presented. Particularly, in the case of two agents, the equilibria for all the values of the parameters are completely described.

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1 Introduction

In many situations, the behaviour of individuals is inconsistent with the traditional assumption reflected in competitive models where the self-interest is the main motivation of decision-makers. In fact, people often care for the well-being of others and this behaviour may have economic consequences (see Nowack, 2006; Tabibnia et al., 2008; Cooper and Kagel, 2013). The literature in psychological and behavioural game theory shows that the assumption that in strategic situations agents are rational and self-interested does not exclude that an individual may take also into account the interests of others. Some contributions on this topic are Colman et al. (2011), where the other-regarding concerns of the agents are modelled as functions of their own and their opponents' objective payoffs. They focused on Berge equilibria (Berge 1957, Abalo and Kostreva 2004, 2005) for games in which utility-maximizing agents are motivated by social attitudes defined by payoff transformations. In Monroy et al. (2017) other-regarding preferences are also incorporated into the problem of the commons by assuming that the agents take into account the utilities of all of the group. They studied the situation under different degrees of concern of each agent with respect to the utilities of the others and presented results on the equilibria for different types of agents.

In order to analyse other-regarding strategic models, the methodological framework considered in the present paper is the theory of strategic vector-valued games. We deal with non-cooperative games where the preferences of the agents are incomplete and they can be represented by vector-valued functions. In the literature, the research regarding vector-valued utilities has mainly focused on the case in which the preferences of the agents are represented by weighted additive value functions, as in Keeney and Raiffa (1976), Mármol et al. (2017) and Monroy et al. (2017). Recently, Rébillé (2019) has axiomatically characterized preferences which can be represented by pseudo-linear utility functions and also by additive separable pseudo-linear utility functions.

An important issue is to identify the contexts where other-regarding attitudes such as interpersonal altruism, fairness, reciprocity or inequity aversion describe the preferences of the agents and to what extent these other-regarding preferences have relevant economic and social effects. These attitudes are in fact significant to explain the behaviour of individuals when facing social dilemas as the sustaining of common resources (Monroy et al., 2017) or the provision of public goods (Kolstad, 2011). In many of these cases an utilitarian approach, which considers additive value functions, can capture the social nature of agents' preferences. Nevertheless, in situations where inequity aversion has to be taken into account, maxmin value functions, in the spirit of the egalitarian approach proposed by Rawls (1971), seem to be more adequate (Charness and Rabin, 2002; Engelmann and Strobel, 2004). Egalitarianism wants to improve the worst case, thereby, in a general setting, it seeks to maximize the minimum of the weighted components of the vector-valued utility functions.

Zapata et al. (2019) provided the theoretical bases for the analysis of the equilibria under the assumption that the preferences can be represented by a weighted maxmin function, also known as Rawlsian function. This representation is the most appropriate when the components of the utilities are not compensable but complementary and, therefore, the improvement of the result for each agent has to be reached by simultaneously improving all the components of the vector-valued utility.

Particularly, Rawlsian preferences are appropriate to better analyse decisions related to the consumption of goods or services with positive externalities, such as the level of education achieved in an economy. In this case, the high level acquired by a part of the population could not compensate for the low or null level achieved by another part of it. Moreover, the egalitarian approach underlies relevant economic issues such as income distribution. When other-regarding attitudes, especially fairness and inequity aversion, are taken into account, agents are not only concerned about their own income but also about how the total income generated by the society is allocated among the population. However, agents could show a distaste for low relative income of others for reasons of fairness or altruism but also for the effects in their own wellbeing. Consider, for example, the case of neighbouring districts with very different level of income within the same city where the richer district can fear the social conflicts in the poorer district. The same could be applied for countries where the richer ones fear the arrival of immigrants of the poorer ones. Likewise, agents could seek improving the worst position when their own reputation or profits depends on that of others. This would also be the case for decisions in research groups that have to obtain financial support that depends on the reputation of both each individual and the group as a whole.

In this paper we will analyse the role of Rawlsian preferences for a wide class of strategic situations which includes Cournot oligopolies (Cournot, 1838) and the problem of the commons (Hardin, 1968), among others.

We first recall some results on the links between the equilibria of vector-valued games and the equilibria of scalar weighted maxmin games. We then address the interesting case in which the agents show social preferences that are represented as Rawlsian functions. We provide results which permit the description of different types of agents depending on their attitude with respect to the gains of the others, that is, in terms of the parameters of the maxmin representation. Given the vector-valued game that describes the other-regarding strategic model, and the corresponding weights assigned by the agents, we establish the associated scalar Rawlsian game. We also provide necessary conditions and sufficient conditions on the weights of the agents which enable the identification of equilibria relying upon agents social behaviour. The key result which allows us to perform a complete analysis of the equilibria for these strategic models is the description of the best response function of the agents in terms of best responses of the classical model and of the parameters that represent the importance that each agent attaches to the outcome of the others.

The remainder of the paper is organized as follows. Section 2 includes some basic concepts and previous results on the weighted maxmin approach for the equilibria of vector-valued games. Different types of agents are defined and characterized in Section 3. Section 4 is devoted to analyse a strategic model with Rawlsian preferences. The interesting case of two agents is specifically considered. Finally, Section 5 contains some concluding remarks and the proofs are included in the Appendix.

2 Preliminaries

We consider the following notation of vector inequalities: let $x, y \in \mathbb{R}^s$, $x > y$ means $x_j > y_j$ for all j; $x \geq y$ means $x_j \geq y_j$ for all j, with $x \neq y$; and $x \geqq y$ means $x_j \geq y_j$ for all j. We denote the sets $\mathbb{R}^s_+ = \{y \in \mathbb{R}^s : y \geq 0\}$, and $\Delta^s = \{y \in \mathbb{R}^s_+ : \sum_{j=1}^s y_j = 1\}$.

A vector-valued normal-form game is represented by $G = \{(A^i, u^i)_{i\in N}\}$, where $N = \{1, \dots, n\}$ is the set of agents, A^i is the set of strategies or actions that agent $i \in N$ can adopt and the mapping $u^i : \times_{i\in N} A^i \to \mathbb{R}^{s^i}_+$ is the vector-valued utility function of agent i, $u^i := (u^i_1, \dots, u^i_{s^i})$, where s^i is the number of components of the utility function of agent i. We denote by $J^i = \{1, ..., s^i\}$ the set of indices of such components. A profile of strategies, $a = (a^1, \dots, a^n)$ with $a^i \in A^i$ can be written as $a = (a^i, a^{-i})$, where a^i is the strategy of agent i, and $a^{-i} = (a^1, \dots, a^{i-1}, a^{i+1}, \dots, a^n)$ stands for the strategy combination of all agents except agent i.

The following definitions are the extensions of the concept of Nash equilibrium

(Nash, 1951) for games with vector-valued utilities.

Definition 2.1. An action profile a^* is an equilibrium for the game with vector-valued utilities $G = \{(A^i, u^i)_{i\in N}\}$ if $\nexists i \in N$ with $a^i \in A^i$ such that $u^i(a^i, a^{*-i}) \geq u^i(a^*)$.

The set of all equilibria of game G is denoted by $\mathcal{E}(G)$.

Definition 2.2. An action profile a^* is a weak equilibrium for the game with vector-valued utilities $G = \{(A^i, u^i)_{i\in N}\}$ if $\nexists i \in N$ with $a^i \in A^i$ such that $u^i(a^i, a^{*-i}) > u^i(a^*)$.

The set of all weak equilibria of game G is denoted by $\tilde{\mathcal{E}}(G)$.

Traditionally, the preferences of the agents with vector-valued utilities have been represented by weighted additive value functions. However, in many situations, the assumption that the utilities of the agents are transferable is far from realistic, and it seems more appropriate to represent the preferences of each of the agents by adopting a weighted Rawlsian function. In this setting, the smallest weighted utility must increase in order to improve the results.

Given a vector-valued game $G = \{(A^i, u^i)_{i\in N}\}$, agent i considers the value of the action profile $a \in \times_{i=1}^n A^i$ as

$$w^i_{\gamma^i}(a) = \min_{j\in J^i} \left\{ \frac{u^i_j(a)}{\gamma^i_j} \right\},$$

where γ^i_j is the weight that indicates the importance that the agent i assigns to each component of the utility function. If $\gamma^i_j = 0$ for some $j \in J^i$, then the corresponding quotient $\frac{u^i_j(a)}{\gamma^i_j}$ is not computed in order to evaluate the minimum. Denote $\gamma^i \in \Delta^{s^i}, \gamma^i = (\gamma^i_j)_{j\in J^i}$. Now consider the scalar Rawlsian game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$, where $\gamma \in \Delta = \times_{i=1}^n \Delta^{s^i}$.

An action profile a^* is an equilibrium for the game G^γ if for all $i \in N$, $w^i_{\gamma^i}(a^i, a^{*-i}) \leq w^i_{\gamma^i}(a^*)$ for all $a^i \in A^i$. The set of equilibria for the weighted maxmin game G^γ is denoted by $E(G^\gamma)$.

The links between equilibria and weak equilibria for the vector-valued game G and the equilibria of the scalar games G^γ for all possible weights are analysed in Zapata et al. (2019). Thus, in general, the union of the equilibria sets of the scalar games G^γ is contained in the set of weak equilibria of the vector-valued game G. Moreover, the set of equilibria of G are generated when all the equilibria of the scalar games G^γ, for all γ, are determined.

Proposition 2.3. *(Zapata et al., 2019) Let* $G = \{(A^i, u^i)_{i\in N}\}$ *be a vector-valued game. Then*

$$\mathcal{E}(G) \subseteq \cup\{E(G^\gamma) : \gamma \in \Delta\} \subseteq \tilde{\mathcal{E}}(G).$$

In particular, under certain concavity conditions, which often hold in applications, the three sets coincide.

Proposition 2.4. *(Zapata et al., 2019) Let* $G = \{(A^i, u^i)_{i\in N}\}$ *be a game with vector-valued utilities such that each* A^i *is a non-empty convex subset of a finite dimensional space and for each* i*,* u^i *is strictly concave in* a^i*. Then*

$$\mathcal{E}(G) = \cup\{E(G^\gamma) : \gamma \in \Delta\} = \tilde{\mathcal{E}}(G).$$

3 Agents with Rawlsian preferences

Let $N = \{1, ..., n\}$ be a set of agents and for $j \in N$, let $u_j : \times_{i=1}^n A^i \to \mathbb{R}_+$ be agent j individual utility function. In our model, all the agents have the information about the individual utility function of the others and thus consider the same collective vector-valued utility function $u : \times_{i=1}^n A^i \to \mathbb{R}_+^n$, $u := (u_j)_{j\in N}$. This game is denoted by $G = \{(A^i, u)_{i\in N}\}$.

We assume that the preferences of each agent on the utilities of the rest of the agents and of her own utility are represented by a real-valued function on the set of vectors of utilities, which is consistent with the natural partial ordering in $\mathbb{R}^n$. For $i \in N$, $\nu^i : \mathbb{R}_+^n \to \mathbb{R}$ provides the evaluation of agent i for each vector of utilities of all the group. We adopt the term *preference function* to name such function.

In what follows the agents are classified depending on the attitude that they show with respect to the individual utilities of the other agents[1].

Definition 3.1. Let N be a set of agents with utilities $u := (u_j)_{j\in N}$. Let the preferences of agent i be represented by the preference function ν^i. Agent $i \in N$ is

a) *equanimous* if for each $u \in \mathbb{R}_+^n$ and each $\pi \in \Pi_N$, $\nu^i(u) = \nu^i(u_\pi)$.

b) *impartial* if for each $u \in \mathbb{R}_+^n$, and each $\pi \in \Pi_N$, $\nu^i(u_i, u_{-i}) = \nu^i(u_i, u_{\pi_{-i}})$.

c) *egoistic* if for all $u, \bar{u} \in \mathbb{R}_+^n$, with $u_i < \bar{u}_i$, $\nu^i(u) < \nu^i(\bar{u})$.

[1]A permutation π in the set of agents N is a bijection $\pi : N \to N$, Π_N denotes the set of permutations in N and π_{-j} denotes the corresponding permutation of the sets of agents $N \setminus j$.

d) *pro-self* if for each $u \in \mathbb{R}^n_+$, and for each $j \in N$ such that $u_i > u_j$, $\nu^i(u) \geq \nu^i(\bar{u})$, where $\bar{u} \in \mathbb{R}^n_+$ is such that $\bar{u}_i = u_j$, $\bar{u}_j = u_i$, and $\bar{u}_k = u_k$ for $k \neq i, j$.

e) *pro-social* if for each $u \in \mathbb{R}^n_+$, and for each $j \in N$ such that $u_i > u_j$, $\nu^i(u) \leq \nu^i(\bar{u})$, where $\bar{u} \in \mathbb{R}^n_+$ is such that $\bar{u}_i = u_j$, $\bar{u}_j = u_i$, and $\bar{u}_k = u_k$ for $k \neq i, j$.

The definition of pro-self agent wants to capture the idea of an agent who cares more for her utility than for that of the others, and thus, when comparing a situation with another situation in which she switches her utility with another agent, she (weakly) prefers the one in which she obtains the highest value. Note that the extreme case of a pro-self agent is an egoistic agent, who only cares for her utility, regardless of what the others obtain. Similarly, a pro-social agent cares more for the utility of the others. Note also that if an agent is both pro-self and pro-social, then she is an equanimous agent. The definition of impartial agent means that the agent values the utility of all the others equally.

These types of agents were introduced in Monroy et al. (2017) where the relationship between the social attitude of the agents and the parameters corresponding to the additive representation of preferences was established.

In the present paper, we consider a Rawlsian representation of the preferences, that is, for each $i \in N$ the preference function is

$$\nu^i(u) = \min_{j \in N} \left\{ \frac{u_j}{\gamma^i_j} \right\}$$

with $\gamma^i \in \Delta^n$. Each component of γ^i, γ^i_j, is interpreted as the relative importance that agent i assigns to the individual utility of agent j. In the maximization of the worst individual weighted utility, the situations in which the individual weighted utilities are equal plays a major role, since these levels determine the evaluation of the group. In order to increase this evaluation, the individual weighted utilities must increase from the level in which the equality is attained in the proportions indicated by the weights. Therefore, by attaching a higher weight to one individual utility, its value needs to increase in a higher proportion. For instance, in order to increase the value of the preference function for a two-agent game, if the weight that one agent assigns to her own utility is double than the one she attaches to the utility of the other agent, then the value of her own utility needs to increase twice as much as the other agent's utility.

In the following Proposition the different types of the agents are characterized in terms of these weights.

Proposition 3.2. *If the preference function of agent $i \in N$ is $\nu^i(u) = \min_{j\in N}\left\{\frac{u_j}{\gamma^i_j}\right\}$, with $\gamma^i \in \Delta^n$, then agent i is*

a) equanimous if and only if $\gamma^i_j = \gamma^i_k$ for all $j, k \in N$.

b) impartial if and only if $\gamma^i_j = \gamma^i_k$ for all $j, k \neq i$.

c) egoistic if and only if $\gamma^i_j = 0$ for all $j \neq i$.

d) pro-self if and only if $\gamma^i_i \geq \gamma^i_j$ for all $j \in N$.

e) pro-social if and only if $\gamma^i_i \leq \gamma^i_j$ for all $j \in N$.

The relationship between each type of agent and the set of weights established in this result permits the identification of the subsets of equilibria according with the attitude of the agents towards the other agents.

4 A strategic model with Rawlsian preferences

We start from a symmetric situation with a set of agents $N = \{1, \ldots, n\}$. Since we are dealing with Rawlsian preferences, as stated in Section 3, all the agents consider the same collective vector-valued utility function $u := (u_j)_{j\in N}$. The strategies of the agents are represented by $m = (m^1, ..., m^n)$, with $m^i \geq 0$ for all $i \in N$. Each agent considers a real-valued utility function which, in this framework, represents her own benefit, $u_j(m) = m^j V(M)$, for $j \in N$, where $M = \sum_{i=1}^n m^i$. Function V is twice-continuously differentiable, strictly decreasing, concave and non-negative on some bounded interval $(0, \bar{M})$, and $V(M) = 0$ for $M \geq \bar{M}$ (as in Kreps and Scheinkman, 1983). The benefit functions of the agents, u_j, are strictly concave in their own action since $\frac{\partial^2 u_j}{\partial (m^j)^2}(m) < 0$.

The representation of this situation when each of the agents considers not only her individual benefit but also the benefits of the others is formalized as the vector-valued game $G = \{(A^i, u)_{i\in N}\}$, where $u := (u_j)_{j\in N}$ is the same collective function for all the agents.

Previous analysis of these games have considered a weighted additive function to represent the preferences of the agents (Monroy et al., 2017). In order to accommodate other types of situations we here represent the preferences by adopting a weighted Rawlsian function in which the weights might be different for each agent.

Thus, for agent $i \in N$:

$$w^i_{\gamma^i}(m) = \min_{j\in N}\left\{\frac{u_j(m)}{\gamma^i_j}\right\} = V(M)\min_{j\in N}\left\{\frac{m^j}{\gamma^i_j}\right\},$$

where $\gamma^i \in \Delta^n$ for $i \in N$, $\gamma = (\gamma^i)_{i\in N}$ and the component γ^i_j represents the importance that agent i assigns to the benefit of agent j in the sense explained previously.

This setting includes some standard models widely studied in the literature: on the one hand, oligopoly models in which several firms compete with homogeneous products in a static framework and, on the other hand, the model of the commons, in which when individuals act independently, they spoil a shared-resource through their collective action. In the classic analysis of this problem, the agents must decide their strategy independently and simultaneously, taking into consideration only their own benefit. It is well-known that, under the former assumptions on the utility functions, a unique equilibrium of the scalar game exists, the standard Cournot equilibrium. Let M^* be the positive quantity such that $m^* = (\frac{M^*}{n}, ..., \frac{M^*}{n})$ is the Cournot equilibrium.

Given the game $G = \{(A^i, u)_{i\in N}\}$, and the vector of weights of the agents γ^i, the corresponding Rawlsian game is denoted by $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$. We call the equilibria of the games $E(G^\gamma)$, Rawlsian equilibria.

It follows from Proposition 2.3 that, in general, the set of Rawlsian equilibria is contained in the set of weak equilibria of the vector-valued game G and contains the set of equilibria of G. Moreover, in this model the equilibria of the weighted games G^γ obtained are equilibria of the vector-valued game G by applying Proposition 2.4 since the functions u_j are strictly concave in the action of agent j for all $j \in N$.

In order to analyse the existence of Rawlsian equilibria and to determine the corresponding strategies we rely on the expression of the best response of the agents in relation to the preference function, $w^i_{\gamma^i}$. The explicit expression of the best response function, denoted here as $R^i_\gamma(m^{-i})$, is given in the following Proposition. Let $r^i(m^{-i})$ be the best response of agent i for $u_i(m) = m^i V(M)$.

Proposition 4.1. *The best response of agent i to the actions of the rest of the agents in the Rawlsian game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$ is given by:*

$$R^i_\gamma(m^{-i}) = \min\left\{r^i(m^{-i}), \gamma^i_i \min_{j\neq i}\{\frac{m^j}{\gamma^i_j}\}\right\}.$$

As a consequence of this result the following remarks on the Rawlsian equilibria are straightforward:

- If $\gamma_i^i = 1$ for all $i \in N$, that is, in the case in which all the agents are egoistic, the model coincides with a standard Cournot oligopoly, and the unique equilibrium of the Ralwsian game coincides with the Cournot equilibrium.
- If $\gamma_i^i < 1$ for all $i \in N$, then $m = (0, \ldots, 0)$ is always identified as an equilibrium of the strategic model. That is to say, if the agents are not (completely) egoistic, and their preferences on the benefits of the group are represented by weighted Rawlsian functions, in a situation in which each agent chooses $m^i = 0, i \in N$, no agent increases the value of her preference function by individually choosing a different value.
- If $\gamma_i^i = 0$ for some $i \in N$, then, at any possible equilibrium $m^i = 0$ holds. In other words, at equilibrium, the strategy of a completely pro-social agent is always her lowest value, zero.
- If the set of agents is divided into two groups, N_0 and N_1, such that $N = N_0 \cup N_1$ with $N_o = \{i \in N, \gamma_i^i = 0\}$ and $N_1 = \{j \in N, \gamma_j^j = 1\}$ with $\mid N_1 \mid = p$, then the unique equilibrium is m, such that $m^i = 0$ for $i \in N_0$, and $m^j = \frac{M^*(N_1)}{n-p}$ for all $j \in N_1$, where $M^*(N_1)$ stands for the Cournot quantity in a classic oligopoly game involving only the egoistic agents in N_1.

In this setting, the existence of a non-null equilibrium depends on the relationships between the product of the importances that the agents assign to their own benefits, $\gamma_i^i \cdot \gamma_j^j$, and the product of the importances that they attach to the benefit of the other agent, $\gamma_j^i \cdot \gamma_i^j$, as stablished in the following results.

Lemma 4.2. *Let m be an equilibrium of the Rawlsian game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i \in N}\}$. If for $i, j \in N$, $\gamma_i^i \cdot \gamma_j^j < \gamma_j^i \cdot \gamma_i^j$, then $m^i = 0$ and $m^j = 0$.*

As a consequence, we obtain a sufficient condition for the game having only the trivial (null) equilibrium, $m = \theta$, that is, $m^i = 0$, for all $i \in N$.

Proposition 4.3. *If for all $i \in N$, $\gamma_i^i \cdot \gamma_j^j < \gamma_j^i \cdot \gamma_i^j$ for some $j \in N$, then $m = \theta$ is the unique equilibrium of the game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i \in N}\}$.*

It follows that when all the agents are strictly pro-social, that is, when the agents assign less importance to their own benefit than to the benefit of others, the unique equilibrium has null components. However, other situations in which some of the agents are pro-social and some others are pro-self may also fulfil this sufficient condition.

The following Lemma will help to identify the equilibria in some other relevant cases.

Lemma 4.4. *Let m be an equilibrium of the Rawlsian game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$. If for $i, j \in N$, $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$, with $\gamma^i_i \cdot \gamma^j_j \neq 0$, then $m^i = \frac{\gamma^i_i}{\gamma^i_j} m^j$.*

As a natural consequence, we characterize the set of equilibria when $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$ for all $i, j \in N$.

Proposition 4.5. *If $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$ for all $i, j \in N$, then the set of equilibria of the game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$ is*

$$E(G^\gamma) = \{m \in I\!R^n : \forall\, i \in N,\ 0 \leq m^i \leq r^i(m^{-i}),\ m^i = \frac{\gamma^i_i}{\gamma^i_j} m^j, \forall j \in N\}.$$

The conditions $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$ for all $i, j \in N$, include the cases in which $\gamma^i_i = \gamma^i_j$ for all $i, j \in N$. It follows that when all the agents are equanimous, a multiplicity of equilibria exist. In each of these equilibria, the strategies of all the agents are identical.

The following results refer to the set of equilibria when the inequality $\gamma^i_i \cdot \gamma^j_j > \gamma^i_j \cdot \gamma^j_i$ holds for all $i, j \in N$. The first one establishes the existence of the equilibria when all the agents are pro-self.

Proposition 4.6. *If $\gamma^i_i > \gamma^i_j$ for all $i, j \in N$, then the unique non-null equilibrium of the game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$ is $m \in I\!R^n$ such that $m^i = r^i(m^{-i})$, for all $i \in N$.*

Note that for strictly pro-self agents, two equilibria exist: the null equilibrium and the standard Cournot equilibrium.

The following Lemma establishes conditions in which the strategy of an agent at a non-null equilibrium does not correspond to the individual best response of the agent.

Lemma 4.7. *Let m be a non-null equilibrium of the Rawlsian game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$ with $\gamma^i_i \cdot \gamma^j_j > \gamma^i_j \cdot \gamma^j_i$ for all $i, j \in N$. If $\gamma^i_i < \gamma^i_j$, then $m^i \neq r^i(m^{-i})$.*

4.1 The case of two agents

We now consider the case of a two-person game for which we apply the results established above in order to determine the corresponding equilibria. We simplify notation by setting the parameters $\gamma^i_i = \alpha^i$ and $\gamma^i_j = 1 - \alpha^i$, $i, j = 1, 2, j \neq i$. With this notation α^i represents the importance that agent i assigns to her own benefit and $1 - \alpha^i$ represents the importance that she attaches to the benefits of the other agent, and therefore, agent i is pro-social when $\alpha^i \leq \frac{1}{2}$, equanimous when $\alpha^i = \frac{1}{2}$ and pro-self when $\alpha^i \geq \frac{1}{2}$.

The Rawlsian function is written as

$$w^i_{\alpha^i}(m^1, m^2) = V(m^1 + m^2) \min \left\{ \frac{m^i}{\alpha^i}, \frac{m^j}{1 - \alpha^i} \right\},$$

and the best response of agent i to the actions of agent j is

$$R^i_\alpha(m^j) = \min \left\{ r^i(m^j), \frac{\alpha^i}{1 - \alpha^i} m^j \right\}.$$

It follows that if $\alpha^i = 1$ for $i = 1, 2$, that is, if both agents are egoistic, then the unique equilibrium is the standard Cournot equilibrium. If $\alpha^i < 1$ for $i = 1, 2$ then $m = (0, 0)$ is always an equilibrium and if $\alpha^i = 0$, then at equilibrium $m^i = 0$. From Proposition 2.4, the whole set of equilibria of the Rawlsian game can be generated by changing the parameters that represent the attitude of the agents. And when a profile of strategies is obtained as an equilibrium of some Rawlsian game G^α, then these strategies constitute an equilibrium of the vector-valued game.

The following result summarizes all the cases for two agents.

Proposition 4.8. *Let* $G^\alpha = \{(A^i, w^i_{\alpha^i})_{i=1,2}\}$ *be the Rawlsian game.*

1. *If* $\alpha^1 + \alpha^2 < 1$, *then the unique equilibrium is* $m = (0, 0)$.
2. *If* $\alpha^1 + \alpha^2 = 1$, *and* $\alpha^1 \cdot \alpha^2 \neq 0$, *then multiple equilibria exist*

$$E(G^\alpha) = \{m : m^i = \frac{\alpha^i}{1 - \alpha^i} m^j,\ 0 \leq m^i \leq r^i(m^j),\ for\, i = 1, 2,\ j \neq i\}.$$

3. *If* $\alpha^1 + \alpha^2 > 1$, *a unique non-null equilibrium exists,* m, *and*
   a) *If* $\alpha^i \geq 1/2, \alpha^j > 1/2, i, j = 1, 2$, *then* $m^1 = r^1(m^2)$, *and* $m^2 = r^2(m^1)$.
   b) *If* $\alpha^i < 1/2$ *and* $\alpha^j > 1/2, i, j = 1, 2$, *then* $m^i = \frac{\alpha^i}{1 - \alpha^i} m^j$, *and* $m^j = r^j(m^i)$.

Observe that the condition on the parameters in Case 1 means that either both agents are pro-social, or one of the agents is pro-self and the other is a pro-social agent who attaches a low importance to her own benefit. In Case 2, either both agents are equanimous, or one is pro-social and the other pro-self, and their importances sum up to one. In Case 3a), either both agents are pro-self or one of them is equanimous and the other pro-self. Finally, in Case 3b) one of the agents is pro-social and the other is pro-self and their importances sum up to more than one.

Figure 1 illustrates the reaction functions and the equilibria for two-agent games with Rawlsian preferences fulfilling $\alpha^1 + \alpha^2 < 1$. On the left-hand side, a case of two pro-social agents is shown. On the right-hand side, one of the agents is pro-social and the other one is a pro-self agent. In both cases, the unique equilibrium is $m = (0, 0)$.

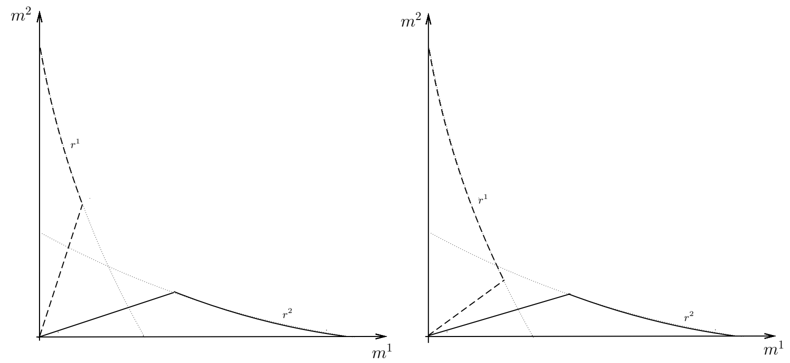

Figure 1: Equilibria for two agents when $\alpha^1 + \alpha^2 < 1$.

When $\alpha^1 + \alpha^2 = 1$, a multiplicity of equilibria exists (Figure 2). On the left-hand side, the case of two equanimous agents is illustrated. On the right-hand side, a case of one pro-social agent and one pro-self agent is shown.

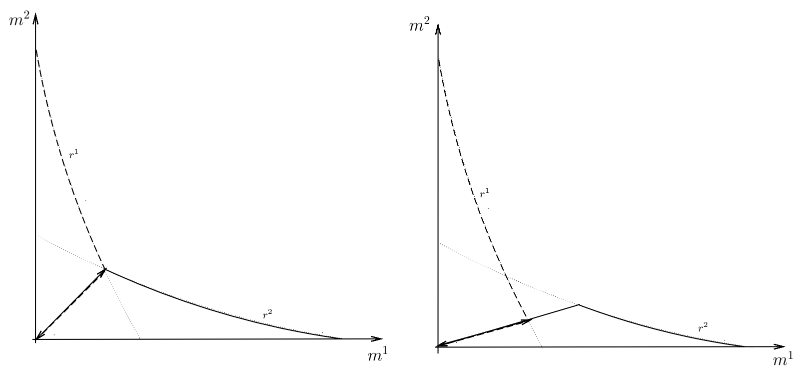

Figure 2: Equilibria for two agents when $\alpha^1 + \alpha^2 = 1$.

Note that when all the agents are pro-self, the unique non-null equilibrium coincides with that of the case without social preferences, that is, the Cournot equilibrium (Figure 3, left-hand side). A non-null equilibrium may exist different from the Cournot equilibrium when $\alpha^1 + \alpha^2 > 1$ and one agent is a pro-self agent and the other is a pro-social agent (Figure 3, right-hand side). This equilibrium is located on the best-response of the pro-self agent.

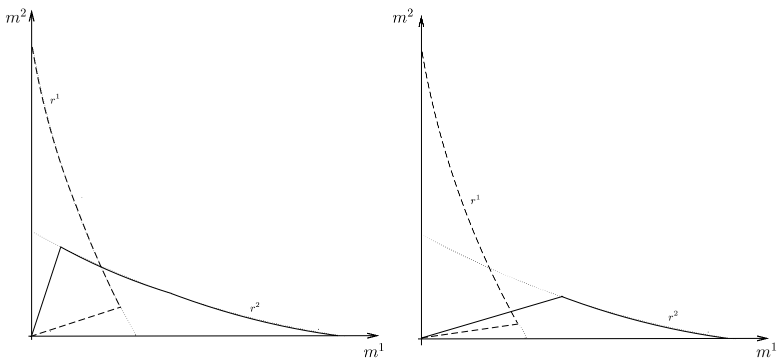

Figure 3: Equilibria for two agents when $\alpha^1 + \alpha^2 > 1$.

5 Concluding remarks

It is well-known that utilitarianism is the most applied theory in the analysis of strategic models. However, we have illustrated the usefulness of the egalitarian framework in contexts where the components of the utility of the agents are not compensable. The adoption of a weighted Rawlsian function as a representation of the agents' preferences presents two interesting features.

On the one hand, the behaviour of the agents with relation to the outcomes of the others is provided by the parameters of such function, which permits the classification of different types of agents. On the other hand, specific conditions on the weights allows the characterization of equilibria for some other-regarding strategic situations. For instance, in the extreme situation in which all the agents are strictly pro-social, the only equilibrium is the null equilibrium. In the opposite situation, if all the agents are pro-self, then the unique non-null equilibrium of the general model coincides with the Cournot equilibrium. In the case in which all the agents are equanimous a multiplicity of equilibria exists. In these equilibria, the strategy of each agent can be expressed proportionally to the strategy of one of the agents. Moreover, in a mixed situation in which agents with different attitudes are included various possibilities of interaction can arise. A particular case is when at least one agent is pro-social and the product of the importances that the agents assign to their own benefits is greater than the product of the importances that they attach to the benefit of the other agent. In this situation, other equilibria different from Cournot equilibrium emerge which are located on the individual reaction curves of the pro-self agents.

The results obtained when considering Rawlsian preferences to identify the equilibria of games with other-regarding preferences show some similarities and differences with respect to those obtained in Monroy et al. (2017) for the additive case. With both preference representations, if only an agent exhibits a pro-social behaviour, regardless of the behaviour of the rest of the agents, the standard Cournot equilibrium is not an equilibrium in the game with other-regarding preferences. And also in both cases, when all the agents are egoistic, the standard Cournot equilibrium is the unique equilibrium. In addition, when agents are equanimous, multiple equilibria exist in both representations. For the Rawlsian representation, all the equilibria are symmetric and the null equilibrium and the standard Cournot equilibrium are included. However, for the additive model, in the case of equanimous agents, the profile formed by the null strategies of the agents and the Cournot equilibrium are not equilibria for the Rawlsian game. There is only one symmetric equilibrium and coincides with one of the equilibria of the Rawlsian representation.

The analysis presented herein may help explain the equilibria that can be achieved in certain strategic situations where other-regarding preferences play a crucial role. Take for instance, the strategic decisions of individuals in a pandemic situation, where they can decide whether to stay at home or to spend some time outside. The utility of an individual depends increasingly on the time outside and also on the probability of not being infected which depends decreasingly on the total time that all the individuals spend outside. It is plausible to assume that individuals are concerned about the well-being of others in a Rawlsian sense, that is, in order to make their decisions they consider the worst-case situations but assigning different importance to different individuals. Our analysis explains why when individuals are pro-social, an equilibrium consists of staying at home. It also explains that depending on the intensity of the social attitudes of the individuals, other equilibria may exist in which some individuals spend some time outside and some others not.

Needless to say that an equilibrium situation does not necessarily entail a social optimal situation. This optimum will clearly be attained when the probability of being infected is null which is only achieved at the null equilibrium.

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6 Appendix

Proof Proposition 3.2: a) If agent i is equanimous, then for all $u \in \mathrm{I\!R}^n$, $\nu^i(u) = \nu^i(u_\pi)$. Suppose on the contrary that $\gamma^i_j > \gamma^i_k$ for some $k, j \in N$, and take $u = (\gamma^i_j)_{j\in N}$. Thus, $\nu^i(u) = 1$. Now consider $\bar{u}$ such that $\bar{u}_r = u_r$ for $r \neq k, j$, $\bar{u}_j = u_k$, and $\bar{u}_k = u_j$. It follows that $\nu^i(\bar{u}) = \frac{\gamma^i_k}{\gamma^i_j} < 1$, which is a contradiction and therefore, $\gamma^i_j = \gamma^i_k$ for all $k, j \in N$.
The reverse is straightforward.
b) The result follows by applying the same reasoning as in a) to the set $N \setminus i$.
c) Suppose on the contrary that $\gamma^i_j > 0$ for some $j \neq i$. Take u such that $u_r = 1$ for all $r \in N$, and $\bar{u}$ such that $\bar{u}_i = 2$ and $\bar{u}_r = 0$ for all $r \neq i$. Now we have that $\nu^i(u) = min_{r\in N}\{\frac{1}{\gamma^j_r}\} > 0$, whereas $\nu^i(\bar{u}) = 0$, and this is a contradiction.
Reciprocally, consider $\gamma^i_j = 0$ for all $j \neq i$, thus $\gamma^i_i = 1$, then $\nu^i(u) = \frac{u_i}{\gamma^i_i}$. For any $\bar{u}$ with $u_i < \bar{u}_i$, $\nu^i(\bar{u}) = \frac{\bar{u}_i}{\gamma^i_i} > \nu^i(u)$.
d) If the agent is pro-self, then for any $u \in \mathrm{I\!R}^n_+$ such that $u_i > u_j$, we have that $\nu^i(u) = \min\left\{\frac{u_i}{\gamma^i_i}, \frac{u_j}{\gamma^i_j}, \min_{k\neq i,j}\left\{\frac{u_k}{\gamma^i_k}\right\}\right\}$, and $\nu^i(\bar{u}) = \min\left\{\frac{u_i}{\gamma^i_j}, \frac{u_j}{\gamma^i_i}, \min_{k\neq i,j}\left\{\frac{u_k}{\gamma^i_k}\right\}\right\}$.
We first prove that, if agent i is pro-self then $\gamma^i_i \neq 0$. Suppose that $\gamma^i_i = 0$, and take u such that $u_i > u_j$, and $u_k = \gamma^i_k(1 + \frac{u_i}{\gamma^i_j})$, for $k \neq i, j$ with $\gamma^i_k \neq 0$. For this u, $\nu^i(u) = \frac{u_j}{\gamma^i_j}$ and $\nu^i(\bar{u}) = \frac{u_i}{\gamma^i_j}$. Therefore, $\nu^i(u) < \nu^i(\bar{u})$, which is a contradiction. Thus, $\gamma^i_i \neq 0$.
Suppose now that $\gamma^i_j > \gamma^i_i$ for some $j \neq i$, and take $u \in \mathrm{I\!R}^n_+$ such that $u_i > u_j$ and $u_k = \gamma^i_k(\frac{u_i}{\gamma^i_j} + \frac{u_j}{\gamma^i_i})$ for all $k \neq i, j$ with $\gamma^i_k \neq 0$. Then $\frac{u_k}{\gamma^i_k} = \frac{u_i}{\gamma^i_j} + \frac{u_j}{\gamma^i_i} > \frac{u_j}{\gamma^i_i} > \frac{u_i}{\gamma^i_j}$. As a consequence, nor the value of $\nu^i(u)$, nor that of $\nu^i(\bar{u})$, is attained at $\min_{k\neq i,j}\left\{\frac{u_k}{\gamma^i_k}\right\}$. Hence, $\nu^i(u) = \min\left\{\frac{u_i}{\gamma^i_i}, \frac{u_j}{\gamma^i_j}\right\} = \frac{u_j}{\gamma^i_j}$ and $\nu^i(\bar{u}) = \min\left\{\frac{u_j}{\gamma^i_i}, \frac{u_i}{\gamma^i_j}\right\}$. Now $\frac{u_j}{\gamma^i_j} < \frac{u_i}{\gamma^i_i}$ and for all $k \neq i, j$, $\frac{u_j}{\gamma^i_j} < \frac{u_k}{\gamma^i_k}$. Therefore, $\nu^i(u) < \nu^i(\bar{u})$. This is a contradiction, and thus, $\gamma^i_i \geq \gamma^i_j$ for all $j \in N$.
Reciprocally, it $\gamma^i_i \geq \gamma^i_j$ for all $j \in N$, let $u \in \mathrm{I\!R}^n_+$ be such that $u_i > u_j$. We are

going to prove that $\nu^i(u) \geq \nu^i(\bar{u})$. Note that $\nu^i(\bar{u}) = \min\left\{\frac{u_j}{\gamma_i^i}, \min_{k\neq i,j}\left\{\frac{u_k}{\gamma_k^i}\right\}\right\}$, since $\frac{u_j}{\gamma_i^i} < \frac{u_i}{\gamma_j^i}$.
On the one hand, if $\nu^i(\bar{u}) = \frac{u_j}{\gamma_i^i}$, then $\frac{u_j}{\gamma_i^i} \leq \frac{u_k}{\gamma_k^i}$ for all $k \neq i, j$. Since $\frac{u_i}{\gamma_i^i} > \frac{u_j}{\gamma_i^i}$, $\frac{u_j}{\gamma_j^i} \geq \frac{u_j}{\gamma_i^i}$ and $\frac{u_k}{\gamma_k^i} \geq \frac{u_j}{\gamma_i^i}$ for all $k \neq i, j$, then $\nu^i(u) \geq \nu^i(\bar{u})$.
On the other hand, if $\nu^i(\bar{u}) = \frac{u_r}{\gamma_r^i}$, for some $r \neq i, j$, then $\frac{u_r}{\gamma_r^i} \leq \frac{u_j}{\gamma_i^i} < \frac{u_i}{\gamma_i^i}$, and $\frac{u_r}{\gamma_r^i} \leq \frac{u_k}{\gamma_k^i}$ for all $k \neq i, j$. Moreover, $\frac{u_j}{\gamma_j^i} \geq \frac{u_r}{\gamma_r^i}$, since, on the contrary, if $\frac{u_r}{\gamma_r^i} > \frac{u_j}{\gamma_j^i} > \frac{u_j}{\gamma_i^i}$, which is a contradiction. Therefore, $\nu^i(u) \geq \nu^i(\bar{u})$.
e) The reasoning is analogous to that in d). $\square$

Proof Proposition 4.1: First, we observe that the preference function of agent i can be written as

$$w_{\gamma^i}^i(m) = V(M)\min\left\{\frac{m^i}{\gamma_i^i}, \min_{j\neq i}\left\{\frac{m^j}{\gamma_j^i}\right\}\right\}.$$

Given $m = (m^1, \ldots, m^n)$, let $s \in N$ be such that $\frac{m^s}{\gamma_s^i} = \min_{j\neq i}\{\frac{m^j}{\gamma_j^i}\}$. For this m,

$$w_{\gamma^i}^i(m) = \begin{cases} \frac{m^i}{\gamma_i^i}V(M) & if \quad \frac{m^i}{\gamma_i^i} \leq \frac{m^s}{\gamma_s^i} \\ \frac{m^s}{\gamma_s^i}V(M) & if \quad \frac{m^s}{\gamma_s^i} \leq \frac{m^i}{\gamma_i^i} \end{cases}$$

The best response of agent i to the strategies of the others, $R_\gamma^i(m^{-i})$, is an action of agent i, $\bar{m}^i$, that has to be selected either in $(\bar{m}^i, m^{-i}) \in A_1 = \{m : \frac{m^i}{\gamma_i^i} \leq \frac{m^s}{\gamma_s^i}\}$, or $(\bar{m}^i, m^{-i}) \in A_2 = \{m : \frac{m^s}{\gamma_s^i} \leq \frac{m^i}{\gamma_i^i}\}$.
Two cases must be considered depending on the value of $r^i(m^{-i})$:
We first analyse the case in which $r^i(m^{-i}) \leq \frac{m^s}{\gamma_s^i}$. If $\bar{m}^i$ is selected such that $(\bar{m}^i, m^{-i}) \in A_1$, then $\bar{m}^i = r^i(m^{-i})$ since $r^i(m^{-i})$ is the best response in A_1. On the other hand, if $\bar{m}^i$ is selected such that $(\bar{m}^i, m^{-i}) \in A_2$, since $w_{\gamma^i}^i(m) = \frac{m^s}{\gamma_s^i}V(M)$ and V is decreasing in $M = m^1 + ... + m^n$, then $\bar{m}^i$ should be set as small as possible, that is, $\bar{m}^i = \gamma_i^i \frac{m^s}{\gamma_s^i}$. If so, $(\bar{m}^i, m^{-i}) \in A_1$ and the best response should be $\bar{m}^i = r^i(m^{-i})$. Therefore, when $r^i(m^{-i}) \leq \frac{m^s}{\gamma_s^i}$, the best response is $R_\gamma^i(m^{-i}) = r^i(m^{-i})$.
Second, we analyse the case in which $r^i(m^{-i}) \geq \frac{m^s}{\gamma_s^i}$. On the one hand, if $\bar{m}^i$ is selected such that $(\bar{m}^i, m^{-i}) \in A_1$, where $r^i(m^{-i})$ is the best response, then the strict concavity of $w_{\gamma^i}^i$ in m^i takes us to choose the highest value in this region, that is, $\bar{m}^i = \gamma_i^i \frac{m^s}{\gamma_s^i}$. It follows that $(\bar{m}^i, m^{-i}) \in A_2$ and in A_2 the best response is $\bar{m}^i = \gamma_i^i \frac{m^s}{\gamma_s^i}$. On the other hand, if $\bar{m}^i$ is selected such that $(\bar{m}^i, m^{-i}) \in A_2$, the best response is straightforward $\bar{m}^i = \gamma_i^i \frac{m^s}{\gamma_s^i}$. Then when $r^i(m^{-i}) \geq \frac{m^s}{\gamma_s^i}$, $R_\gamma^i(m^{-i}) = \gamma_i^i \frac{m^s}{\gamma_s^i}$.
It follows that for any m^{-i} the best response is $R_\gamma^i(m^{-i}) = \min\left\{r^i(m^{-i}), \gamma_i^i \frac{m^s}{\gamma_s^i}\right\}$. $\square$

Proof Lemma 4.2: By definition, the profile of strategies m is an equilibrium of the game $G^\gamma = \{(A^i, w^i_{\gamma^i})_{i\in N}\}$ if and only if $m^i = R^i_\gamma(m^{-i})$ for all $i \in N$. As a consequence, given $i \in N$, $m^i \leq r^i(m^{-i})$ and $m^i \leq \frac{\gamma^i_i}{\gamma^i_j}\, m^j$ for all $j \neq i$.
We prove that if for $i, j \in N$, $\gamma^i_i \cdot \gamma^j_j < \gamma^i_j \cdot \gamma^j_i$, then at equilibrium $m^i = 0$. Suppose on the contrary that for $i, j \in N$ such that $\gamma^i_i \cdot \gamma^j_j < \gamma^i_j \cdot \gamma^j_i$ and $m^i \neq 0$. We have that $m^i \leq \frac{\gamma^i_i}{\gamma^i_j}\, m^j$ for all $j \neq i$, and therefore, $\gamma^i_i \neq 0$ holds. If $\gamma^i_j \cdot \gamma^j_i = 0$, then the inequality $\gamma^i_i \cdot \gamma^j_j \geq \gamma^i_j \cdot \gamma^j_i$ is straightforward, which is a contradiction. If $\gamma^i_j \cdot \gamma^j_i \neq 0$, since m^j is an equilibrium strategy, we also have that $m^j \leq \frac{\gamma^j_j}{\gamma^j_i}\, m^i$, and hence, $m^i \leq \frac{\gamma^i_i}{\gamma^i_j}\, m^j \leq \frac{\gamma^i_i}{\gamma^i_j}\frac{\gamma^j_j}{\gamma^j_i}\, m^i$, where $\gamma^j_j \neq 0$ (since, otherwise $m^i = 0$). Thus $m^i \leq \frac{\gamma^i_i}{\gamma^i_j}\frac{\gamma^j_j}{\gamma^j_i}\, m^i$, and since $m^i \neq 0$, then $\gamma^i_i \cdot \gamma^j_j \geq \gamma^i_j \cdot \gamma^j_i$, which is a contradiction. □

Proof Lemma 4.4: As established in the proof of Lemma 4.2, if the profile of strategies m is an equilibrium of the game G^γ, then $0 \leq m^i \leq r^i(m^{-i})$ and $m^i \leq \frac{\gamma^i_i}{\gamma^i_j}\, m^j$ for all $j \neq i$.
We now prove that if for $i \in N$, $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$ for some $j \in N$, then at equilibrium $m^i = \gamma^i_i \min_{j\neq i}\{\frac{m^j}{\gamma^i_j}\}$. Suppose on the contrary that $m^i = r^i(m^{-i}) < \gamma^i_i \min_{j\neq i}\{\frac{m^j}{\gamma^i_j}\}$, then $m^i = r^i(m^{-i}) < \frac{\gamma^i_i}{\gamma^i_j} m^j$ for each $j \in N$. Since $m^j \leq \frac{\gamma^j_j}{\gamma^j_i} m^i$, we have that $m^i < \frac{\gamma^i_i}{\gamma^i_j} m^j \leq \frac{\gamma^i_i}{\gamma^i_j}\frac{\gamma^j_j}{\gamma^j_i} m^i$. Since $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$, it follows that $m^i < m^i$, which is a contradiction. Therefore, at equilibrium $m^i \neq r^i(m^{-i})$, then $m^i = \gamma^i_i \min_{j\neq i}\{\frac{m^j}{\gamma^i_j}\}$ must hold.
Moreover, if $m^i < \frac{\gamma^i_i}{\gamma^i_j} m^j$ for $j \in N$ such that $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$, then, similarly as before $m^i < \frac{\gamma^i_i}{\gamma^i_j} m^j \leq \frac{\gamma^i_i}{\gamma^i_j}\frac{\gamma^j_j}{\gamma^j_i} m^i = m^i$, which is a contradiction. As a consequence, $m^i = \frac{\gamma^i_i}{\gamma^i_j} m^j$ for $j \in N$ such that $\gamma^i_i \cdot \gamma^j_j = \gamma^i_j \cdot \gamma^j_i$. □

Proof Proposition 4.5: The first inclusion is straightforward by Proposition 4.1 and Lemma 4.4. We now prove that if $m^i = \frac{\gamma^i_i}{\gamma^i_j} m^j$ for all j, and $0 \leq m^i \leq r^i(m^{-i})$, then $m^i = R^i_\gamma(m^{-i})$. If $m^i < R^i_\gamma(m^{-i})$, then $m^i < \frac{\gamma^i_i}{\gamma^i_j} m^j = m^i$, which is a contradiction. If $m^i > R^i_\gamma(m^{-i})$, since $m^i = \frac{\gamma^i_i}{\gamma^i_j} m^j$, for all $j \neq i$, then $R^i_\gamma(m^{-i}) = r^i(m^{-i})$. Hence $m^i > r^i(m^{-i})$, which is also a contradiction. Therefore, $m^i = R^i_\gamma(m^{-i})$. And if $m^i = R^i_\gamma(m^{-i})$ for all $i \in N$, then m is an equilibrium of G^γ. □

Proof Proposition 4.6: For the case of two agents, Kreps and Scheinkman (1983) proved that if $r^j(m^i) < m^i$, then $r^i(r^j(m^i)) < m^i$. This result can straightforwardly be generalized for the case of n agents as follows: If $r^j(m^{-j}) < m^i$, then $r^i(r^j(m^{-j}), m^{-\{i,j\}}) < m^i$ (where $m^{-\{i,j\}}$ denotes the strategy combination of all agents except agents i and j). We will use this fact in the proof of our result.

We first prove that if $m^i = r^i(m^{-i})$ for all $i \in N$, then m is an equilibrium of G^γ. Note that under these conditions, m is the Cournot equilibrium of the strategic model in which each agent values only its own benefit ($u_i(m) = m^i V(M)$) and it follows from the symmetry of this game that $m^i = m^j$ for all $i, j \in N$. As a consequence, since $\gamma_i^i > \gamma_j^i$, $\frac{m^i}{\gamma_i^i} = \frac{m^j}{\gamma_i^i} < \frac{m^j}{\gamma_j^i}$ holds for all $j \neq i$, Therefore, $w^i_{\gamma^i}(m) = V(\sum_{k\in N} m^k)\frac{m^i}{\gamma_i^i} = \frac{u_i(m)}{\gamma_i^i}$.
Consider a deviation from m^i. For $\varepsilon > 0$, $\frac{m^i-\varepsilon}{\gamma_i^i} < \frac{m^i}{\gamma_i^i} = \frac{m^j}{\gamma_i^i} < \frac{m^j}{\gamma_j^i}$ holds. Hence $w^i_{\gamma^i}(m^i - \varepsilon, m^{-i}) = V(\sum_{k\in N} m^k - \varepsilon)\frac{m^i-\varepsilon}{\gamma_i^i} = \frac{u_i(m^i-\varepsilon, m^{-i})}{\gamma_i^i}$ and, since u_i is strictly concave in its own action and the best response is m^i, we have that $w^i_{\gamma^i}(m) > w^i_{\gamma^i}(m^i - \varepsilon, m^{-i})$. Now consider $\varepsilon > 0$ such that $\frac{m^i}{\gamma_i^i} < \frac{m^i+\varepsilon}{\gamma_i^i} < \frac{m^j}{\gamma_j^i}$. For this ε, $w^i_{\gamma^i}(m^i + \varepsilon, m^{-i}) = V(\sum_{k\in N} m^k + \varepsilon)\frac{m^i+\varepsilon}{\gamma_i^i} = \frac{u_i(m^i+\varepsilon, m^{-i})}{\gamma_i^i}$ and, since u_i is strictly concave in its own action and the best response is m^i, $w^i_{\gamma^i}(m) > w^i_{\gamma^i}(m^i + \varepsilon, m^{-i})$. Therefore, m is an equilibrium of G^γ.

Reciprocally, we will prove that if m is an equilibrium of G^γ, then $m^i = r^i(m^{-i})$ for all $i \in N$. Suppose on the contrary that an agent i exists such that $m^i < r^i(m^{-i})$, that is, $m^i = \frac{\gamma_i^i}{\gamma_j^i} m^j$ for some j and $m^i \leq \frac{\gamma_i^i}{\gamma_k^i} m^k$ for all $k \neq i, j$. Since m is an equilibrium of G^γ, for this j one of the following situations happens: a) $m^j = \frac{\gamma_j^j}{\gamma_i^j} m^i$, b) $m^j = r^j(m^{-j})$ and c) $m^j = \frac{\gamma_j^j}{\gamma_k^j} m^k$ for some $k \neq i, j$.

a) If $m^j = \frac{\gamma_j^j}{\gamma_i^j} m^i$, then $m^i = \frac{\gamma_i^i}{\gamma_j^i}\frac{\gamma_j^j}{\gamma_i^j} m^i$ Since $\gamma_i^i > \gamma_j^i$ and $\gamma_j^j > \gamma_i^j$ for all $i, j \in N$, it follows that $m^i > m^i$, which is a contradiction.

b) If $m^j = r^j(m^{-j})$, with $\gamma_i^i > \gamma_j^i$, then $m^j = r^j(m^{-j}) < m^i$. Since when $r^j(m^{-j}) < m^i$, then $r^i(r^j(m^{-j}), m^{-\{i,j\}}) < m^i$, it follows that $r^i(m^{-i}) < m^i$. And this is a contradiction since m is an equilibrium of G^γ and consequently $m^i \leq r^i(m^{-i})$.

c) If $m^j = \frac{\gamma_j^j}{\gamma_k^j} m^k$ for some $k \neq i, j$, an analogous reasoning can be made for m^k, that is, three situations regarding the value of m^k arise. The two first cases yield a contradiction as above. In the third case $m^k = \frac{\gamma_k^k}{\gamma_r^k} m^r$ holds for some $r \neq i, j, k$. Hence, $m^i = \frac{\gamma_i^i}{\gamma_j^i}\frac{\gamma_j^j}{\gamma_k^j}\frac{\gamma_k^k}{\gamma_r^k} m^r$. The recursive application of this reasoning leads to the expression $m^i = \frac{\gamma_i^i}{\gamma_j^i}\frac{\gamma_j^j}{\gamma_k^j}\frac{\gamma_k^k}{\gamma_r^k}\ldots m^i$. Since all the quotients are greater than one, $m^i > m^i$, which is also a contradiction.

Therefore, the result follows. □

Proof Lemma 4.7: Suppose on the contrary that $m^i = r^i(m^{-i})$. Since m is an equilibrium of G^γ and $\gamma_i^i < \gamma_j^i$ for some $j \neq i$, it follows that for this j, $m^i \leq \frac{\gamma_i^i}{\gamma_j^i}m^j < m^j$ holds, that is, $r^i(m^{-i}) < m^j$. Hence, $r^j(m^{-j}) < m^j$ since u_i is strictly concave and V is strictly decreasing, which is a contradiction since m is an equilibrium of G^γ and consequently $m^j \leq r^j(m^{-j})$. □

Proof Proposition 4.8: Note that with the simplified notation, for the case of two agents, the condition $\gamma_1^1 \cdot \gamma_2^2 < \gamma_2^1 \cdot \gamma_1^2$ can be written as $\alpha^1 + \alpha^2 < 1$. The condition $\gamma_1^1 \cdot \gamma_2^2 = \gamma_2^1 \cdot \gamma_1^2$ can be written as $\alpha^1 + \alpha^2 = 1$, and $\gamma_1^1 > \gamma_2^1$ is equivalent to $\alpha^1 > 1/2$. Thus, the results in 1) and 2) follow from the general analysis presented in Propositions 4.3 and 4.5.

In case 3a), when $\alpha^1, \alpha^2 > 1/2$, the result follows from Proposition 4.6. Otherwise, without loss of generality, consider $\alpha^1 = 1/2$ and $\alpha^2 > 1/2$. We are going to prove that if m is an equilibrium of G^γ, then $m^1 = r^1(m^2)$ and $m^2 = r^2(m^1)$.

Since m is an equilibrium of G^γ, then $m^i = r^i(m^j)$ or $m^i = \frac{\alpha^i}{1-\alpha^i}m^j, i, j = 1, 2$. Four different situations can occur: First, if $m^1 = \frac{\alpha^1}{1-\alpha^1}m^2 = m^2$ and $m^2 = \frac{\alpha^2}{1-\alpha^2}m^1$, then $m^1 = \frac{\alpha^2}{1-\alpha^2}m^1$ and since $\frac{\alpha^2}{1-\alpha^2} > 1$, $m^1 > m^1$, which is a contradiction. Second, if $m^1 = r^1(m^2)$ and $m^2 = \frac{\alpha^2}{1-\alpha^2}m^1$, then $m^2 > m^1$ since $\alpha^2 > 1/2$. Moreover, since $m^2 > r^1(m^2) = m^1$, it follows that $m^2 > r^2(m^1)$, which is a contradiction since m is an equilibrium of G^γ, and $m^2 \leq r^2(m^1)$ holds. Third, $m^1 = \frac{\alpha^1}{1-\alpha^1}m^2 = m^2$ and $m^2 = r^2(m^1)$. By definition of the best responses r^1 and r^2, it follows that $V(m^1+m^2)+m^2V'(m^1+m^2) = 0$ and $V(m^1+m^2)+m^1V'(m^1+m^2) > 0$, since u_1 is strictly concave. Hence, $(m^1-m^2)V'(m^1+m^2) > 0$, and consequently $m^1-m^2 < 0$, since V' is negative. And this is a contradiction. Therefore, if m is a non-null equilibrium of G^γ, then $m^1 = r^1(m^2)$ and $m^2 = r^2(m^1)$.

Conversely, we are going to prove that if $m^1 = r^1(m^2)$ and $m^2 = r^2(m^1)$, then m is an equilibrium of G^γ. Since $m^1 = r^1(m^2)$ and $m^2 = r^2(m^1)$, then $m^1 = m^2$. Taking into account that $\alpha^1 = 1/2 = 1-\alpha^1$, the Rawlsian function is $w^1_{\alpha^1}(m^1, m^2) = V(m^1+m^2)\frac{m^1}{1/2} = V(m^1+m^2)\frac{m^2}{1/2} = \frac{u_1(m^1,m^2)}{1/2} = \frac{u_2(m^1,m^2)}{1/2}$. Recall that u_1 is strictly concave, then it follows that $w^1_{\alpha^1}(m^1-\varepsilon, m^2) < w^1_{\alpha^1}(m^1, m^2)$, as we proved in Proposition 4.6. Moreover, $\frac{m^2}{1/2} = \frac{m^1}{1/2} < \frac{m^1+\varepsilon}{1/2}$. Since V is strictly decreasing, $w^1_{\alpha^1}(m^1+\varepsilon, m^2) = V(m^1+m^2+\varepsilon)\frac{m^2}{1/2} < V(m^1+m^2)\frac{m^2}{1/2} = w^1_{\alpha^1}(m^1, m^2)$, and the result follows.

The reasoning for $w^2_{\alpha^2}$ can be made analogously as in the proof of Proposition 4.6.

In order to prove case 3b), consider $\alpha^1 < 1/2$ and $\alpha^2 > 1/2$ with $\alpha^1 + \alpha^2 > 1$. By Lemma 4.7, if m is an equilibrium of G^γ, then $m^1 \neq r^1(m^2)$, and thus $m^1 = \frac{\alpha^1}{1-\alpha^1} m^2$. We now prove that $m^2 = r^2(m^1)$. Suppose on the contrary that $m^2 = \frac{\alpha^2}{1-\alpha^2} m^1$. Hence $m^2 = \frac{\alpha^2}{1-\alpha^2}\frac{\alpha^1}{1-\alpha^1} m^2$. Since $\frac{\alpha^2}{1-\alpha^2}\frac{\alpha^1}{1-\alpha^1} > 1$ is equivalent to $\alpha^1 + \alpha^2 > 1$, it follows that $m^2 > m^2$, which is a contradiction. The reciprocal result follows with an analogous reasoning to that in 3a). □

7 References

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