

Repairing Conceptual Relations in Ontologies by means of an Interactive Visual Reasoning: Cognitive and Design Principles

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Abstract—The technologies of visual representation are a great help for understanding items of information and the relations among them, especially, for non-expert users on Knowledge Based Systems, e.g. ontologies, which represent a world that can evolve and, therefore, has to be eventually refined. To ensure a cognitive communication soundness, we must integrate into solutions both, usability for these non-expert users and a logical accuracy on all involved elements. In this paper, the design principles for a tool for the visual repair of anomalies in consistent ontologies is presented. It helps users to obtain an ontological agreement between his mental model of the concepts into discourse domain and the intended model of the ontology (that is supposed to be consistent).

I. INTRODUCTION

The design of Artificial Intelligence (AI) tools to represent and process knowledge is a persistent goal which must deal with challenging problems such as the fluidity of knowledge's semantics itself. Having in mind this idea, in general, users do not understand Knowledge Engineering Techniques and Tools and, in order to popularize the use of such skills, researchers have to bridge the gap between tools and end-users. This is usually done by designing visual interfaces [7], [22].

A particularly interesting case occurs when dealing with problems of semantic interoperability between agents/users. That is, when agents need to share knowledge and they have to understand and accept it, in order to work with. In this case, ontologies play a key role being used for tagging metadata, in an effective way, with consensus and properly understood by all. However, this requirement becomes a challenge when agents are illiteracies in Semantic Web technologies.

The cognitive gap, which arises between the model that agents think it is and the model which it really is, must be avoided if we want that communication and interaction are cognitively sound. In fact, this problem must be solved with cognitive interfaces that allow users both to understand and to adjust involved ontologies. Visual metaphors are a powerful source of methodologies for providing solutions.

In this paper we address the problem of visualizing the representation and the effect of the automated reasoning on ontologies (their Description Logics representation [14]). This

issue may include practical management of ontologies, such as extension, refinement and versioning techniques. The design principles for the tool aims to solve the obstacles above described by using visual reasoning, and specializing the repair on little pieces of ontologies that witnesses the anomaly (an anomalous argument).

The idea is based on a previous theoretical analysis of mereotopological representation of relationships among the ontology's concepts [6], as well as the foundational principles for visual ontology cleaning introduced in [5]. In this way, the information about concepts appearing in the arguments is nicely represented in a clear way. Often, an argument is a very small portion of ontology, therefore, it is easily represented and, despite its moderate size, it provides a more useful information about the anomaly than the full ontology (a similar argumentative case for debugging is presented in [16]). Lastly, the tool has to allow (and strongly induces to) user interactions on visual representation (the information space) with no limits for classical user analysis interaction, querying and navigation/browsing.

The design principles described in this paper have been considered in a tool implemented as a plug-in of Protégé¹. The tool allows the arrangement of concepts relationships because it implements a soundly logical translation to ontology code. This feature is specific; other end-user tools which are mostly based on facilitating the understanding of the ontology (see e.g. [9], [19]) but displaying a very limited graphical transformation of the ontology source. Another essential feature is that the reparation process has to be model-based, that is, it has to exploit the fact the user keep in mind the intended model that the ontology should represent, and this model induces the changes (user only aims to fit spatial model into real one). Therefore, arguments repairing is a relatively easy task, because it is more feasible to be represented than the full ontology. Formally, the problem that the paper addresses is described in [20] as *Ontology Repair* in the following terms:

¹<http://protege.stanford.edu/>

For an ontology Σ , for any statement ϕ that is derivable from Σ but is discovered to be incorrect according to the world, a map π exists such that either $\pi(\phi)$ is correct according to the world and derivable from $\pi(\Sigma)$ or, if this cannot be satisfied, then $\pi(\phi)$ is not derivable from $\pi(\Sigma)$;

$$\Sigma \vdash \phi \wedge \not\models \phi \implies \exists \pi (\pi(\Sigma) \vdash \pi(\phi) \wedge \models \pi(\phi) \vee \pi(\Sigma) \not\vdash \pi(\phi))$$

In such terms, the tool has to provide to the user the definition of a map π on an argument (the piece of the ontology used in the proof of ϕ).

In order to implement the system, several decisions have to be made.

A. Requirements

- The tool conforms the cognitive principles for visual ontology cleaning described in [5] (see section below).
- The meta-ontological interpretation of the conceptual structure of the ontology [5] is used in both directions, namely from ontology source to spatial representations, and also when visual arrangements are interpreted as logical revisions of ontology code.
- The tool uses two well-known systems, as black-box modules: Protégé (by using the tool as a new plug-in) and Racer² (by computing mereotopological relationships among concepts).
- The main goal is not the visualization of full ontologies. It is designed to visualize relations among concepts implied in an deficient representation of the world.

B. Structure of the paper

The structure of the paper is as follows. Next section is devoted to summarize the main features of the mereotopological interpretation of concept relationships in an ontology. It is an abridged version of [5]. Sections 3 illustrate step by step the workflow by means of an example. Finally, we conclude with some conclusions of the analysis and a comparison with similar tools.

II. MEREOTOPOLOGY AND VISUAL COGNITIVE PRINCIPLES

The spatial metaphors are a powerful tool in human information processing, although there exists three main obstacles. Firstly, it can be hard to facilitate the code of the ontology that has been repaired from visual arrangements. It is the case of a tool not supported by a formal semantics that, at the same time, supports such changes. Secondly, the repairing can be, from the point of view of computational logic, complex to be represented. Finally, visual representation of huge ontologies is an intricate task because it is hard both to comprehend and to be shown in a suitable way. It makes its use appropriate on light-weight ontologies [10]. The problem is also exacerbated because logical formalisms (the basis of ontology web languages) are a major barrier for end users' understanding of ontologies, among other challenges [2]. This

barrier suggests to address the problem by transforming and weakening the complexity of the knowledge representation and reasoning (KRR) applied to the system, in order to facilitate the ontology repair task to non expert users.

In this section we briefly introduce the mereotopological and metaontological principles on which the development and design of this tool are based. We Work on the principle that if we aim to use spatial reasoning techniques for cleaning ontologies, then we need to provide with a theory on spatial entities which allows us the translation of the spatial arrangements to changes on the ontology source. This is the idea captured by the Main Cognitive Principle (MCP) (see [5]) and, in order to meet this principle, we have selected a Qualitative Spatial Reasoning (QSR) theory. In this way, we can also satisfy the First Cognitive Principle (CP1) [5] by representing topologically the concepts of the conceptualization associated with an ontology by means of non empty regular regions. Note that regular regions comprise polygons and circles.

Due to we are interested in working with models of the ontology which universe is a two-dimensional or three-dimensional space and having in main those models interpret concept symbols as regions, it is clear that the depicted knowledge depends on the topological relations among regions. The QSR theory we have chosen is Region Connection Calculus (RCC).

A. The mereotopological theory RCC

RCC theory [8], a mereotopological approach to QSR, describes topological features of the spatial relations. It has been useful in several fields of Artificial Intelligence such as Geographic Information Systems (GIS) and Spatial Databases [24] [12]. With the help of RCC, CP1 can be satisfied. It allows us both to reason on spatial regions and interchange knowledge between ontologies and their spatial models. We consider a ground relation, the connection between two regions, which enjoys the reflexive and symmetrical properties. The meaning of connection is: the topological closures of two connected regions intersect. The set of axioms expressing the properties and definitions of the remaining relations (Fig. 1 (left) conforms the set of axioms of RCC (see [8]).

On one hand, the set of the eight binary relations depicted in Fig. 1 is denoted by RCC8. These relations are jointly exhaustive and pairwise disjoint (JEPD) and RCC8 is regarded a calculus for Constraints Satisfaction Problems (CSP) [26]. On the other hand, there is another interesting calculus, $RCC5 = \{DR, PO, PP, PPI, EQ\}$. The difference between them is that while the former allows us to enrich the representation of knowledge by using frontiers of the regions, the latter do not. This fact will be discussed above. Although it has been empirically established [17] that RCC8 is more suitable than RCC5 for the representation of topological relations discriminated by humans, both of them are used here: RCC5 is appropriate for solving CSPs associate to a mereotopological representation and RCC8 is useful to design a rich translation of a spatial representation to the ontology code.

²<http://www.racer-systems.com/>

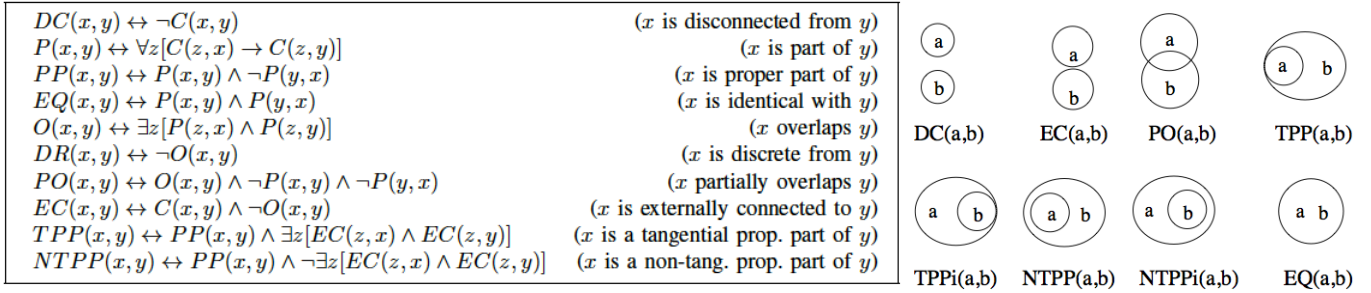


Fig. 1. Axioms of RCC (right) and RCC8 spatial relations (left)

B. RCC interpreted as a meta-ontology

RCC was the QSR theory selected as formal support for an ontology cleaning cycle presented at [1], and now, the use of RCC to represent visually the concepts, turns to RCC8 into an ontology on conceptual relations. The most straightforward approach consists of interpreting concepts such as regions in some model of the theory. Thus, the *strong interpretation* is defined with the intended meaning of *two concepts C_1, C_2 are connected, denoted by $C_\Sigma(C_1, C_2)$, if there exists a common element in some model of the ontology Σ , i.e. $\Sigma \models C_1 \sqcap C_2 \equiv \perp$.*

The knowledge we can represent by means of the strong interpretation of RCC, is limited by the following feature.

Theorem II.1 [5] *The strong interpretation does not discriminate the elements of RCC8 like ontological relations between concepts.*

In fact, if C_1, C_2 are concepts and $R \in \{EC, TPP, NTPP, TPPi, NTPPi\}$, then $\neg R_\Sigma(C_1, C_2)$. It means that we can not directly consider to RCC8 as a metaontology for analysing concept relationships in the strong interpretation, because such an interpretation can not represent *frontier-sensitive* knowledge (it is not able to distinguish among TPP and PP, for example). This limitation is solved by using other interpretation provided by the Second Cognitive Principle (CP2): The frontier of a spatial interpretation of a concept C represents the individuals with possible reclassification C .

The strong interpretation of the remaining RCC relations is obtained from their corresponding definitions (Fig. 1). Note that the strong interpretation does not work on a concrete but abstract spatial interpretation of concepts. By a *spatial model* of an ontology Σ is an interpretation I of Σ whose universe is \mathbb{R}^2 (or \mathbb{R}^3) which interprets concepts as regular regions and such that $C_\Sigma(C_1, C_2)$ if and only if $I \models C(C_1, C_2)$. The existence of an spatial model is made sure by the theorem:

Theorem II.2 *Every consistent ontology has a spatial model*

Proof: (Sketch) If an ontology Σ is consistent, then the constraint satisfaction problem (CSP) defined by the weak interpretation of RCC-relationships [5] among its concepts is consistent. Thus, it holds that the CSP is spatially consistent

[25], that is, there exists a solution interpreting concepts as regular regions ■

It should be noted that Renz's result on which is based above theorem only ensures that polygonal (no necessarily connected) regions can be selected to interpret concepts. We have selected only uses rectangular regions. Thus there exist spatial configurations that can not be properly represented. Two reasons justify this decision. On the one hand, it aims to show user friendly representations on relations among concepts, and on the other hand, concepts to represent are those involved in arguments, which usually deal with a relatively small set of concepts.

Once an argument is spatially represented, in next section we analyse how the topological borders of regions are endowed of cognitive features, by means of RCC8. This tool interprets concept's border as the set of individuals which might be candidates for reclassification.

III. PRAGMATICS IN SPATIAL INTERPRETATION AND REARRANGEMENT

Based on foundational decisions made in above section in this section it describes all the steps which the tool is based on.

In order to exploit RCC8 features by solving the limitation above commented, it interprets $EC_\Sigma(C_1, C_2)$ as follows:

$$\Sigma \models C_1 \sqcap C_2 \equiv \perp \text{ and } \neg \exists C_3 \text{ s.t. } \Sigma \models C_3 \sqsubseteq (C_1 \sqcap C_2)$$

Thus, the relations $\{EC, TPP, NTPP, TPPi, NTPPi\}$ can be re-interpreted and it allows us to use RCC8. The rationale design is based on three layers (see Fig. 2). They uses different representation and reasoning systems to connect automated reasoning on ontologies with spatial reasoning. This connection must be showed to the end user by means of a graphical interface where spatial representation of mereotopological relations is displayed.

The (Fig. 2) workflow starts with a spatial representation of the RCC relationships among concepts of a set. It is done by using an automated reasoning system to compute these relations. In fact, this key step consists in a translation from logical information on conceptualization to a CSP on spatial relations. By solving this CSP, a spatial encoding of conceptualizations is obtained and graphically represented for user feedback. The solution is visualized. If the user considers that there exist

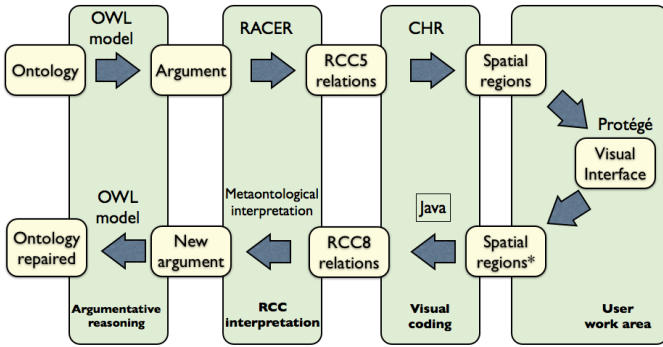


Fig. 2. Repairing Workflow

an anomaly (it represents an anomalous representation of the world, it is usually a subset of concepts for which ontology states a relationship which is not true in the world), then it provides an interface to spatially rearrange the representation. Next, it translates the new spatial configuration to ontology source code.

IV. AN EXAMPLE

To illustrate the full editing process we have chosen a well-known ontology, the *Instructional Objects*³ ontology, which can be found at SchemaWeb⁴

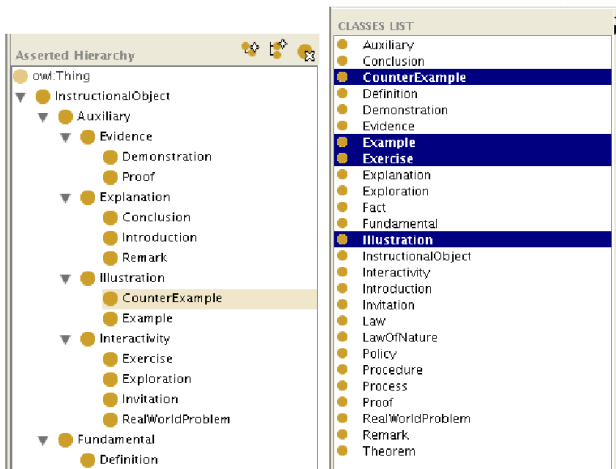


Fig. 3. Classes hierarchy view in Protégé and classes selection

A. Arguments

The first step of the process to repair the ontology, is to select the arguments from the ontology to study up on. Particularly, this ontology has not individuals, it just has classes, hence we only need the *Classes List* to select the arguments.

This example looks interesting when we study the “Example” and “CounterExample” classes. They bear an overlapping

relationship between themselves, which means that it could exist some individuals or classes which belong to the intersection of them. Suppose, for example, that our user thinks that it is not possible in his mental state (he aims to apply the ontology in a concrete kind of problems) and, therefore, he determines that the relation among these classes has to be the disjoint one (DC).

Fig. 3 shows how the user selects needed arguments from the classes list. He selects “Example”, “CounterExample” and some other ones related to them in order to improve the reasoning process.

B. Graphical representation

As it is explained above, all the relations among arguments, in RCC5, are computed and optimized to be sent to the CSP. From this task, we obtain the coordinates belonging to the regular regions and this action let us to build the graphical representation of all the relations.

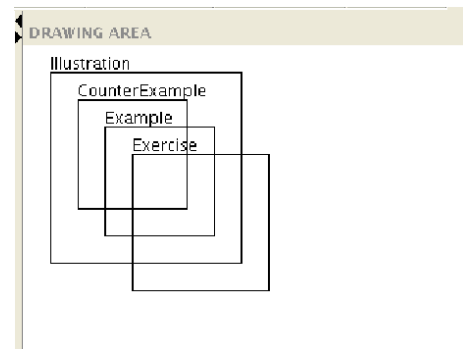


Fig. 4. Initial graphical representation

C. Movements

Once represented the selected concepts (see Fig. 4) note that there are some relations which are not correct, in spite of being consistent. In our example, “Example” and “CounterExample” classes have to be disjoint, which it means that we have to resize “Illustration” class and move “Example” class up to make it disconnected to “CounterExample”. We realize all the possible movements in plug-in, and we turn an overlapping into a disjoint relation (see Fig. 5).

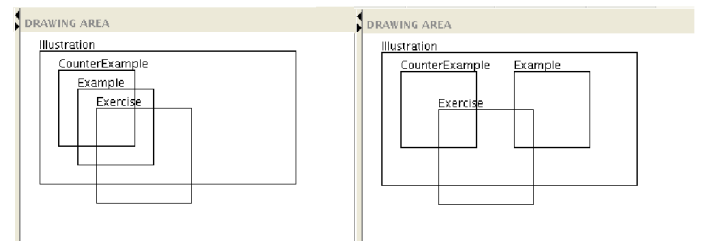


Fig. 5. Resizing (left) and translation (right)

These movements could insert anomalies into the ontology, essentially on classes and individuals which are not graphically represented. This kind of anomalies is solved by assuming

³<http://www.activemath.org/~cullrich/oio/InstructionalObjects.owl>

⁴<http://www.schemaweb.info/>

some criteria as default and, if the repairing is not feasible, a warning message is displayed and all classes and individuals which are involved in the anomaly are included.

D. Saving and final result

When all the movements are performed and the mental user states he is according to the ontology, we can proceed to save permanently all the changes in our knowledge database or ontology.

In addition, at this stage is when, if the user has inserted some inconsistency into the ontology, it has to be detected and decisions to avoid it have to be made. The tool has to apply some set of criteria as default which helps the user to make these decisions, if he has not a clear answer. These criteria are always based on the intention to modify knowledge base as less as possible.

In our example, it could exist a class or individual which belongs to “Example” and “CounterExample” classes simultaneously, and therefore, when they turns into disjoint, the user is inserting an inconsistency. It could fix this situation in two ways: On one hand, if the classes have been represented, when they are moved the user has specified, explicitly, the new relation between the classes which introduce the inconsistency, and hence it disappears. On the other hand, if there exists a class affected and it is not represented, it considers that the affected class belongs to the class which was not moved.

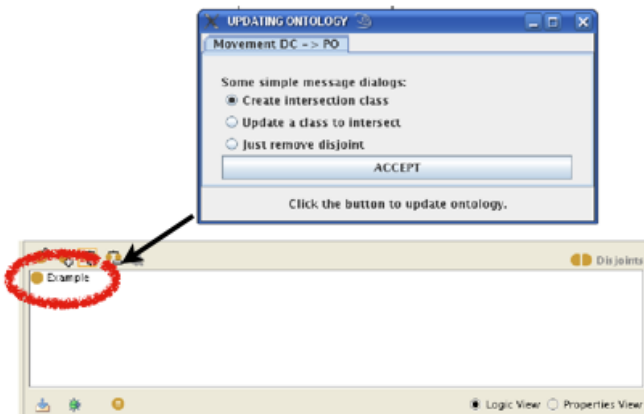


Fig. 6. Updating ontology source

As the last movement involved in our example, we have made the change $PO(Example, CounterExample) \Rightarrow DC(Example, CounterExample)$ in the ontology, and obtained the changes in axioms (see Fig. 6 down, from Protégé interface).

Along this process, the user has obtained a modification of his ontology without needing expert knowledge on ontological engineering.

V. CLOSING REMARKS AND RELATED WORK

In this paper an intelligent tool for ontology repair designed for non-experts on ontology engineering is presented. The rationale behind the tool is that logical relationships among

concepts can be spatially represented, as well as the changes in the representation can be translated to ontology code. RCC8-based spatial encoding provides us with a formal semantics where spatial arrangements mean ontology revision. The encoding establishes a correspondence between the conceptualization implicit in the Ontology and a realm well known to the user. We described several spatial encodings based on different mereotopological interpretations of ontologies.

Furthermore, we exploit logical features of RCC to analyze the impact of revision on the ontology itself. It is worth to note that the tool can be reused for other related problems as for example ontology alignment, and the analysis of the relations among concepts in different versions of an ontology. Particularly interesting is the case of analyzing the difference between two DL ontologies, where the spatial representation of concepts (and anonymous classes) of both ontologies can facilitate the work to ontology engineers [11].

The idea of combining spatial calculi with Description Logics is a research line both in the Semantic Web and also in the Description Logics community (see e.g. [13], [23]). The approach presented does not attempt to combine both theories, just use a translation of DL to QSR to facilitate user interaction by visual reasoning. This approach was successful in previous case studies [4].

A great number of tools for ontologies that supports visual representation exists, even also supporting a variety of tasks such as data analysis and queries [9], [10]. For example, Antarti.ca’s Visual Net⁵ or AquaBrowser display ontology sources as maps or graphs, and queries can be made but they cannot be updated. They also have problems to display individuals and intersections. However these works are mostly focused on visual representation and they lack both inference mechanisms and formal semantics representations outlined here operates beyond just primarily mapping the ontology information/conceptualization structure. A large number of tools are based on GraphViz⁶ and they have a nice visual representation without providing tools for changing the ontology. Other example is *Jambalaya*, a plug-in created for Protégé which uses Shrimp to visualize the knowledge bases that the user has created⁷. As above tools, *Jambalaya* does not provide spatial rearrangements to repair the ontology. Other tool which addresses the problem of user’s comprehension of the structure of the ontology is KC-Viz [21] which provides a relevant visualization of the ontology, facilitating the navigation and browsing.

It is appropriate to indicate that the anomalies could be originated from other different reasons, they not only come from the conceptualization. Roles (object properties) are also an important source of anomalies. We will be focused on mereotopological encodings of them, and develop a tool that can assist to users to repair the anomalies coming from roles.

Finally, note that we use small ontologies pieces because our reparation method is argumentative and it does not need

⁵<http://www.antarcti.ca>

⁶<http://www.graphviz.org>

⁷<http://www.thechiselgroup.org/jambalaya>

the whole ontology. For medium and large size ontologies, visual representation could be unmanageable, however it could be interesting to adapt the spatial semantics to work with other visual encodings as the hyperbolic plane [18]. From the point of view of Multiagent Systems, Ontology Repair System (ORS)[20] was developed to repair ontology mismatches between agents with similar ontologies in a multiagent system (MAS) environment see also [15]. Currently the design principles provide assistance for solving the mismatches to humans. However, we have developed automated semantic negotiation methods [3] which can enrich the method in the future.

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