MULTIVARIABLE ROBUST LTR-i CONTROLLER FOR A SHIP

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Abstract: This paper describes the application of a multivariable robust controller based on LTR-i methodology for roll damping and course steering of a ship by rudder and fins. The controller uses a non observer based control structure, and a partial recovery procedure over the band of interest frequencies. Robustness characteristics of the controller in the face of uncertainties are analyzed and the benefits are proved by simulation with a non linear model.

Keywords: Multivariable control, uncertainty, interconnection system, robust control.

1 INTRODUCTION

Classical design methodologies proceeded by "shaping" the open loop transfer function in the frequency domain to modify the feedback properties of the closed loop single-input/single-output system (SISO). This strategy is succesful because the relations between open loop and closed loop system porperties are well understood. Hence, specifications imposed upon closed loop transfer functions may be mapped into equivalent specifications upon the open loop transfer function. In the last few years, this methodology has been formalized and generalized to multi-input/multi-output (MIMO) design problems. The whole design procedure of altering the shapes of magnitude plots (or sigma-plots) is refered as *"loop shaping"* (Freudenberg and Looze, 1988).

When an observer based control structure is used, and the model of the plant is completely known, the separation principle enables a designer to separate a design into two different tasks: 1) state feedback control design and 2) observer design to reconstruct the state knowing only the output. However, in the presence of uncertainties in the model, the controller design following the separation principle does not necessarily yield the same performance achievable by a state feedback design, or in the worst case, the closed-loop system could even be unstable. Thus, there is a definite need to develop observer design schemes and non observer-based control structures which recover the properties achievable by state feedback design while assuring the closed loop stability. This is the aim of the LTR design methodologies.

A ship is a system subject to a considerable degree of uncertainty in its modelling, fundamentally due to different factors such as: 1) load conditions, 2) cruising speed and 3) environmental disturbancies. Thus robustness properties of any automatic control system must be taken into account if satisfactory performance is desired under realistic conditions. In this work we develop a multivariable design methodology based on LTR procedure for a ship, with two different control structures: 1) an observer based controller and 2) a non observer based controller. We analyze the robustness properties of the controllers, we compare their characteristics and we develop adequate multivariable controllers which are recommended in each case. The design of the controllers is carried out with a linearized model of a non-linear mathematical model of a ship (Kallstrom and Ottosson, 1982), and the control algorithms developed are proven with the non-linear model for different conditions.

2 SYSTEM MODEL

A ship, which is considered as a rigid body, has six degrees of freedom: the longitudinal motions (heave, pitch and surge) and the transverse motions (roll, sway and yaw). The transverse motions are strongly coupled to one another, so that the steering and roll regulation control systems have multivariable properties. Generally, these systems are interactive in that rudder angle and stabiliser fin variations, respectively, induce both

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yaw and roll changes. Cargo, passenger and naval vessels usually employ steering and stabilisation systems in order to provide improved manoeuvering characteristics and motion control. Roll is certainly the most severe angular motion experienced by a ship. Large roll angles can make working on the ship difficult and can lead to motion sickness. The reasons for introducing active roll stabilisation systems in ships are basically: 1) security conditions, 2) transport costs reduction, 3) passenger comfort, 4) personnel efficiency; and additionally in naval vessels: 5) stable weapon platform maintenance and 6) stable platform for helicopter landing on the ship.

If only the motions of roll, sway, yaw and surge are considered, the system is reduced to a problem of four degrees of freedom. The ship model described by Kallstrom and Ottosson (1982) has been used in the simulations carried out in this work. This model has demonstrated to be of great utility for evaluating the control algorithms by simulation, as a previous phase to sea trials (Kallstrom and Ottosson, 1982; Messer and Grimble, 1992). The ship model is a non-linear multivariable model, and the motion equations are (Kallstrom and Ottosson, 1982):

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ \ddot{\phi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} X_{tot} \\ Y_{tot} \\ K_{tot} \\ N_{tot} \end{bmatrix}$$

Where *"tot"* indicates the total forces and torques acting on the ship, due to the following effects: hydrodynamics, wind, waves and current. The state variables are respectively: $x_1 = V_y$ (transversal speed), $x_2 = \dot{\phi}$, $x_3 = \dot{\psi}$, $x_4 = \phi$ and $x_5 = \psi$.

Actuators dynamic are modelled as:

$$\dot{\delta} = (\delta_c - \delta)/\tau_R \,, \qquad |\dot{\delta}| \leq \dot{\delta}_{max} \,, \; |\delta| \leq \delta_{max}$$

$$\dot{\alpha} = (\alpha_c - \alpha)/\tau_F \,, \qquad |\dot{\alpha}| \leq \dot{\alpha}_{max} \,, \; |\alpha| \leq \alpha_{max}$$

The control magnitudes are $\alpha(t)$ and $\delta(t)$, the angles of the fins and rudder respectively, and the magnitudes to be controlled are $\phi(t)$ and $\psi(t)$, the angles of roll and heading. To design the controller, a linearized model has been chosen for nominal conditions of cruising speed $V = 10.8 m/s$. Forces and torques expressions, hydrodynamic derivatives and coefficients are taken from Kallstrom and Ottosson (1982).

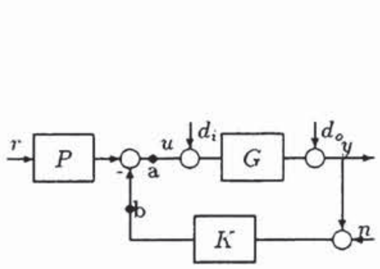

Fig. 1. Feedback control configuration

3 CONTROL ALGORITHM

Consider the control system of fig. 1. It consists of the plant (G), controller (K), pre-compensator P, reference signal (r), measurement noise (n), and disturbancies (d_i, d_o). All signals are multivariable, and nominal mathematical models for G, K, P are LTI. The control objetives can be expressed at different levels of demanding: 1) Nominal stability (NS): bounded outputs for all bounded disturbancies, and bounded reference inputs. 2) Nominal performance (NP): small errors in the presence of disturbancies d_i, d_o and reference inputs r. 3) Robust stability (RS): consider the feedback system in fig. 1. Suppose that the plant is not precisely known, and is modelled as belonging to a class of possible transfer matrices $\mathcal{G}$. A controller K satisfies the robust stability condition if K stabilizes all $G' \in \mathcal{G}$. 4) Robust performance (RP): this requirement is said to be met if the performance specifications are satisfied for all possible plants $G' \in \mathcal{G}$.

The LTR (*Loop Transfer Recovery*) design methodology seeks to define the MIMO compensator $K(s)$ so that the stability robustness and performance specifications are met to the possible greatest extent. This involves two basic steps: 1) We generate a MIMO target loop transfer function (TLTF). 2) A special compensator $K(s)$ is used, so that performance of the feedback system in fig. 1 approximates the performance of the TLTF established in step one. The degree of approximation (or *recovery*) depends on characteristics of the plant. If the plant is minimum phase, then the degree of recovery of the TLTF can be arbitrarily good (Stein and Athans, 1987). If the plant is nonminimum phase and the frequencies of the unstable zeros are beyond the bandwidth of the TLTF, the recovery will take place in low frequencies, and for all practical purposes the presence of far-away nonminimum phase zeros does not degrade the low frequency characteristics of the design.

Different approaches have been suggested in the control literature. obtain the TLTF. One of these is based on *Kalman filter techniques* (which generates the LTR-o procedure (Athans, 1986). An-

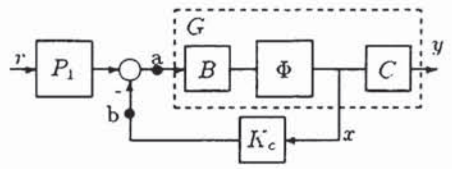

Fig. 2. TLTF synthesis

other one is based on the *linear quadratic regulator* (LQR, or also known as LQSF: linear quadratic state feedback) theory, and it generates the LTR-i procedure (Zhang and Freudenberg, 1990; Maciejowski, 1989). In this work we have employed the latter one: LTR-i.

Target Loop Transfer Function Synthesis

Consider the plant model (which includes the scaling of the variables and augmentation dynamics that the designer has appended to meet specifications):

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y &= Cx(t) \end{aligned}$$

The transfer function matrix of the plant is: $G(s) = C\Phi(s)B$, where $\Phi(s) = (sI - A)^{-1}$, and we assume that $[A, B]$ is stabilizable and that $[A, C]$ is detectable. The structure of the TLTF is shown in fig. 2. It is simply defined by the parameters B and $\Phi(s)$ of the plant model, and by a constant matrix K_c (optimal state feedback matrix). If we break the loop at the input of the plant we obtain the TLTF:

$$H_c(s) = K_c\Phi(s)B$$

For stability robustness to hold, in the face of multiplicative uncertainties at the input of the plant ($G' = (I+E)G, \overline{\sigma}(E) < e(w)$, the interconnection system (Morari and Zafiriou, 1989) is in this case $M(s) = T_c(s)$), the following inequality must be true for all ω (*small gain theorem*):

$$\overline{\sigma}[T_c(j\omega)] < 1/e(\omega)$$

or $\mu[T_c(j\omega)] < 1/e(\omega)$ in the case of structured uncertainties (diagonal structure); where $\overline{\sigma}$ is the maximum singular value and μ represents the structured singular value [MoZa89].

Control demand, command-following and disturbance-rejection can be evaluated from fig. 2 for the matrix K_c obtained. Frequency-domain analysis is made and the temporal responses of the system are obtained by simulating the TLTF in fig. 2, in order to prove if design specifications are satisfied.

To obtain matrix K_c we solve the LQR problem, which consists of meeting the control signal which will minimize the cost:

$$J = \int_0^\infty (x^T M^T Q M x + u^T R_c u)dt$$

with: $Q = Q^T \geq 0, R_c = R_c^T > 0, Q_c = M^T Q M$. The solution is $u = -K_c x$, and K_c is given by:

$$K_c = R_c^{-1} B^T P_c$$

where $P_c = P_c^T \geq 0$ satisfies the algebraic Riccati equation:

$$A^T P_c + P_c A - P_c B R_c^{-1} B^T P_c + Q_c = 0$$

Some remarkable characteristics of the TLTF obtained in this way are: 1) optimal control law, 2) $\overline{\sigma}(T_c) \leq 2$, 3) $\overline{\sigma}(S_c) \leq 1$, 4) at least 60° of phase margin in each input channel, and infinite gain margin; if the loop is conditionally stable it has a margin of at least $6dB$ against gain reductions (Stein and Athans, 1987; Maciejowski, 1989).

LTR procedures

Once the TLTF has been obtained, we can ask ourselves if would be possible to construct a compensator $K(s)$ in fig. 1 with the property that the feedback system of fig. 1 approximates the behaviour of the TLTF in fig. 2. This would happen if the following equality were true (where $K(s)G(s)$ is the loop transfer function LTF): $K(s)G(s) = H_c(s)$. However, for the purposes of design it is not necessary for us to have exact equality. Indeed, if we are interested in finding $K(s)$ so that the approximate relation

$$K(j\omega)G(j\omega) \approx H_c(j\omega)$$

over the band of interest frequencies is satisfied. This is the point of view of the LTR-i method presented in this work.

We now examine two procedures to obtain the LTR controller $K(s)$, one observer based, and the other non observer based. The respective structures are shown in fig. 3 and fig. 4. As we can see fig. 3 shows the conventional LQG observer based controllers structure (OBC), and fig. 4 illustrates the compensator structure developed by

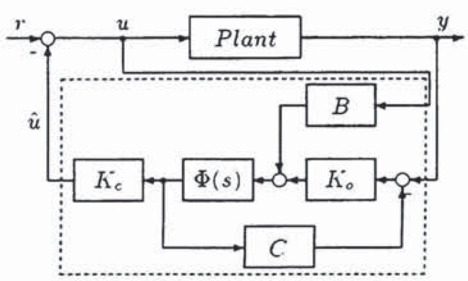

Fig. 3. LTR-i (OBC) structure

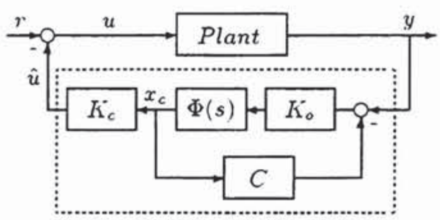

Fig. 4. LTR-i (NOBC) structure

Chen et al. (1991) (NOBC). The difference between them is that the NOBC removes the link from the control signal u to the observer via the control distribution matrix B, which is outside the realm of observer theory and hence the separation principle is no longer valid. In this case to guarantee the closed-loop stability K_o must be such that $A - K_oC$ has all its eigenvalues in the left complex half plan. The respective controllers are:

$$\text{OBC: } K(s) = K_c(sI - A + BK_c + K_oC)^{-1}K_o$$

$$\text{NOBC: } K(s) = K_c(sI - A + K_oC)^{-1}K_o$$

The procedure to obtain the matrix K_o is the same in both cases. One way is that proposed by Doyle and Stein (1981), and is based on the Kalman filter problem (KBF). For this the following algebraic Riccati equation is solved:

$$P_oA^T + AP_o - P_oC^TR_o^{-1}CP_o + Q_o = 0$$

where:

$$P_o = P_o^T \geq 0, R_o > 0, Q_o = \Gamma W \Gamma^T, W \geq 0$$

and the Kalman filter gain matrix is obtained from:

$$K_o = P_oC^TR_o^{-1}$$

If we obtain $K_o(q)$ by choosing the covariance matrix Q_o as:

$$Q_o = Q_o^0 + qZ, \quad Z = Z^T \geq 0, \quad Q_o^0 = \Gamma W \Gamma^T$$

it can be proved [DoSt81] for the minimum phase plant that

$$\lim_{q \to \infty} K(s)G(s) = H_c(s)$$

Therefore: $\quad \text{LTF} \overset{q \to \infty}{\longrightarrow} \text{TLTF}$

The NOBC characteristics for $q \geq q_0$ (the value of q_0 must be calculated in each case) are that (Chen et al., 1991, 1992): 1) The compensator is open-loop stable, 2) closed-loop stability is guaranteed and above all c) much smaller values of *gain recovery gain* q are required than the conventional OBC for the same degree of recovery. This fact implies that the compensator band-width is much smaller than that of the conventional controller and thus we have the advantage of avoiding, in some circumstancies, the saturation of the actuators as well as an improvement in the insensitivity to noise or other high-frequency disturbances.

The approach followed in this work is based on the following points: 1) We are only interested in a partial recovery in the interest frequency range (low and medium frequencies). 2) At high frequencies the singular values of $H_c(j\omega)$ roll-off at -20 dB/dec, while those of $K(j\omega)G(j\omega)$ roll-off at -40 dB/dec. Thus, LTR loops offer some additional robustness to high frequency unmodelled dynamics as compared to the TLTF. 3) The command-following and disturbance rejection performance in the low frequency region between the TLTF and the LTF with LTR will be essentially the same.

4 SIMULATION STUDIES

First we design a LTR-i controller to achieve adequate responses to changes in the reference signal. For this we use the linearized nominal model of the ship for $V = 7.72\ m/s$ and we employ the following design parameters:

$$R_c = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Q_c = C^TQC$$

$$R_o = I_2, \quad Q_o = BB^T$$

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}, \quad q = 10^6$$

In figure 5 the temporary responses and control signals for different values of the cruising speed of the ship ($V_1 = 5.14$ m/s, $V_2 = 10.8$ m/s, and $V = 7.72$ m/s) are shown. As can be seen the

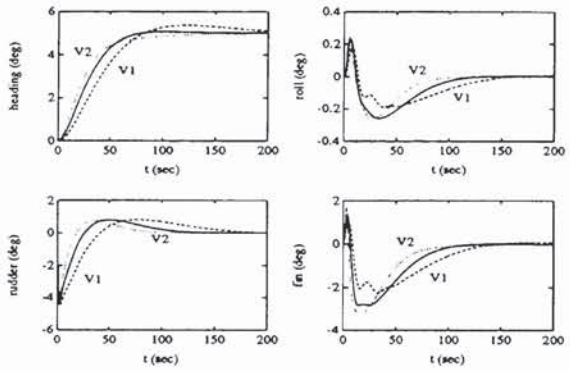

Fig. 5. Time responses for different conditions

controller is characterized for a good properties of robustness, because its behaviour is adequate for extreme working conditions in the plant. The improvement of this LTR-i MIMO controller compared to a good tuning SISO controller is the reduction of coupling roll angle. The responses of the system for both controllers, for a change in the reference vector $r = [0° \ 180°]^T$, are shown in figs. 6 and 7. We can see that the roll angle due to coupling is reduced to more than 50%. The robustness characteristics of the controller to the non-linear behaviour of the plant is tested in fig. 6, where the control signals take high values during relatively long periods of time and produce a considerable loss of the speed and therefore important changes in the plant dynamics.

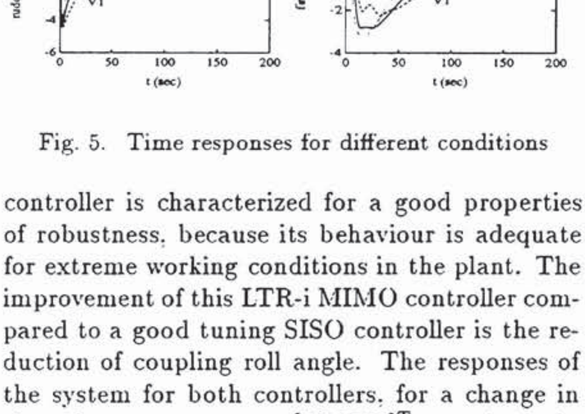

Fig. 6. SISO controller time responses

In fig. 8 the tolerance to uncertainties (iMi: input multiplicative uncertainty, iMo: output multiplicative uncertainty) is shown. This figure represents $1/\overline{\sigma}(M)$, where $M(s)$ is the interconnection system for each particular type of uncertainty, and $\overline{\sigma}(M)$ is its maximum singular value. In the face of unstructured uncertainties, fig. 8 shows that iMi $E_i(s)$ (where $G' = G(I + E_i)$)

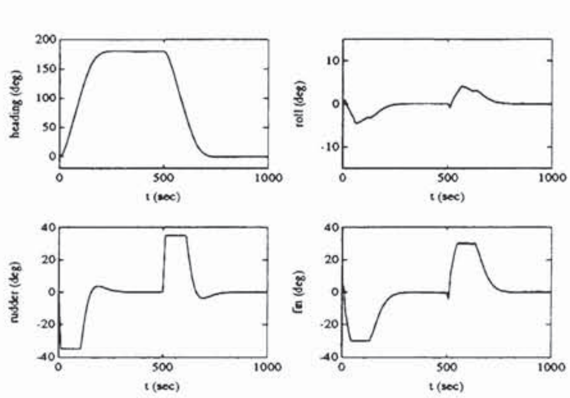

Fig. 7. MIMO controller time responses

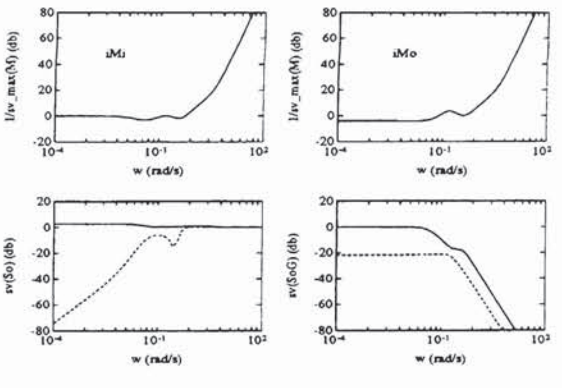

Fig. 8. Tolerances to uncertainties and disturbancy rejection characteristics

do not cause inestability, provided $\|E_i\|_\infty < 0.69$ (or $I_{11i} = 69\%$ relative uncertainty allowed at the input); and that the system remains stable, provided any output multiplicative uncertainties $E_o(s)$ (such that $G' = (I + E_o)G$) satisfy the condition $\|E_i\|_\infty < 0.61$ (or $I_{11o} = 61\%$ relative uncertainty allowed at the output of the plant). In the case of structured uncertainties (diagonal structure), $I_{12i} = 79\%$ and $I_{12o} = 72\%$ are obtained respectively, which supposes a less conservative estimation of the tolerance to uncertainties. In case of multiple input-output multiplicative uncertainties ($G' = (I+E_o)G(I+E_i)$) a relative uncertainty allowed of $I_{1s} = 44\%$ is obtained (this involves the system remains stable provided that: $\max\{\|E_i\|_\infty, \|E_o\|_\infty\} < 0.44$). The respective robustness indicators I_{11i}, I_{11o} are obtained by:

$$\min_\omega 1/\overline{\sigma}[M(j\omega)] \times 100\%$$

and I_{12i}, I_{12o}, I_{1s} are obtained by:

$$\min_\omega 1/\mu[M(j\omega)] \times 100\%$$

where $M(s)$ is calculated in each case.

Control system behaviour in presence of disturbances (at the output d_o and input d_i of the plant)

OBC	NOBC		$\|K_o\|_F$
INP (%)	INP (%)	q	db
5.4	99.83	10^4	36
45.5	99.98	10^5	46
89.5	99.99	10^6	56
99.0	100	10^{10}	96

TABLE 1 INP for OCB and NOBC for different q.

in steering condition are given by:

$$e = -S_o d_o - S_o G d_i$$

As we can see in fig. 8 vectorial perturbations acting in some directions can exist which the controller can not reject adequately, since in the low frequency range $\overline{\sigma}(S_o)$ and $\overline{\sigma}(S_o G)$ are not small enough for all directions. In order to have adequate disturbances rejection we propose incorporate integral action in the controller. We use the linearized nominal model of the plant with speed of $10.8 m/s$ and the following design parameters:

$$R_c = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad Q_c = M_a^T Q M_a$$

$$Q = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}; \quad M_a = \begin{bmatrix} C & \begin{matrix} 0.1 & 0 \\ 0 & 0.1 \end{matrix} \end{bmatrix}$$

$$R_o = I_2; \quad Q_o = \Gamma\Gamma^T$$

$$\Gamma = \begin{bmatrix} B \\ O_{2\times 2} \end{bmatrix}; \quad q = 10^6$$

In this case we have chosen the LTR-i NOBC structure. If we define the indicator of nominal performance (INP) as:

$$INP = \min\left\{ \frac{\overline{\sigma}[L(j\omega_o)]}{\overline{\sigma}[L_t(j\omega_o)]}, \frac{\underline{\sigma}[L(j\omega_o)]}{\underline{\sigma}[L_t(j\omega_o)]} \right\}$$

for $\omega_o = 10^{-5}$ rad/seg we obtain the following values given in the table 1, for OBC and NOBC. As can be seen, it is necessary to increase the gain recovery "q" from 10^6 to 10^{10} to achieve a similar INP with both controllers. However this produces an increase of the size in the K_o matrix ($\|K_o\|_F = trace\sqrt{K_o K_o^T}$), and therefore an unnecessary increment of the controller band-width as well as sensitivity to noise or other high-frequency disturbances. In fig. 9 the extreme singular values of $S_o(j\omega)$ and $S_o(j\omega)G(j\omega)$ are shown. As we can see $\overline{\sigma}[S_o(j\omega)G(j\omega)] < 1$ $(-4.96db)$ for all ω, and $\overline{\sigma}[S_o(j\omega)G(j\omega)]$ and $\overline{\sigma}[S_o(j\omega)]$ are both sufficient small in the low frequency range, as is desirable. With this controller we obtained the following robustness indicators: $I_{11i} = 62\%, I_{12i} = 70\%, I_{11o} = 61\%, I_{12o} = 63\%, I_{1s} = 38\%$.

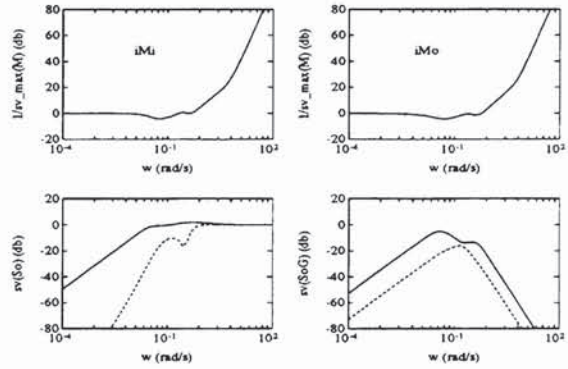

Fig. 9. Tolerances to uncertainties and disturbancies rejection characteristics for MIMO controller with integral action

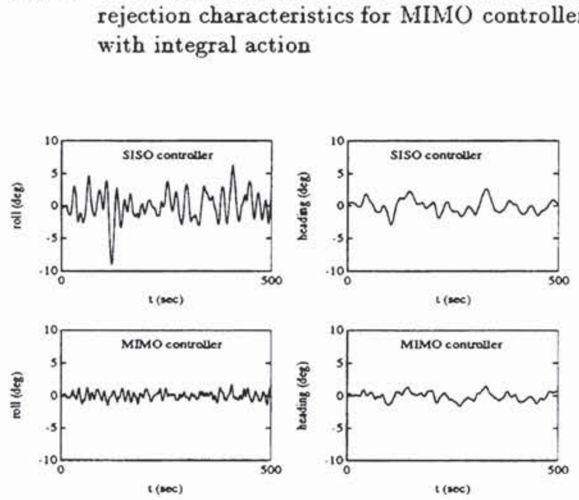

Fig. 10. Behaviour in ship steering condition

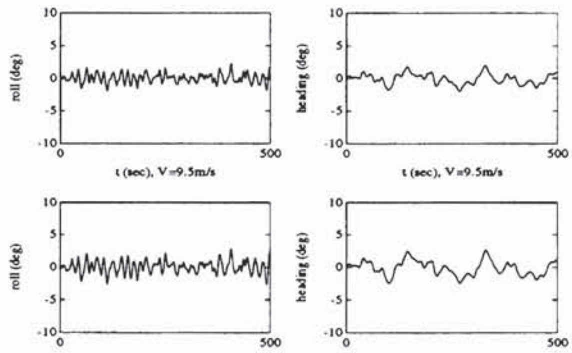

Fig. 11. Behaviour for non-nominal conditions with MIMO controller

Figure 10 shows the temporary responses of the system with the LTR-i (NOBC) controller design, and for a SISO controller that does not take into account the multivariable nature of the plant. A significant wave height of $4m$ with $45°$ relative to ship reference course is chosen in the simulations. We can see that there is a remarkable improvement in roll damping with the LTR-i MIMO controller. Figure 11 shows heading and roll for non-nominal speed conditions ($9.5m/s$ and $8m/s$); we can see that the behaviour is adequate, which is another proof of the controller robustness. In order to improve performance characteristics a gain scheduling controller can be used with the speed of the ship as the auxiliary variable. Due to plant and regulators dynamics, we can implement the controller directly in a digital computer with a sample time of 0.1 seconds, without explicitly taking into account the sample-data character of the system. All the algorithm implementations used in the simulations with the non-linear model of the ship are realized in this way.

5 CONCLUDING REMARKS

Multivariable controllers based on LTR-i (Loop Transfer Recovery at the input of the plant) have been developed: a) for course changing, with considerable decrease in the coupling roll angle and b) for ship steering and roll regulation, with a considerable decrease in roll angle due to waves. The controller uses a non observer based control structure, and a partial recovery procedure over the band of interest frequencies. Robustness characteristics of the controller in the face of uncertainties are analyzed, and the benefits of the controller are proved by simulation with a multivariable non-linear model of a ship.

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REFERENCES

Athans M. (1986). *A Tutorial on the LQG/LTR Method*, ACC., pp 1289-1296.

Chen B.M., A. Saberi and P. Sannuti (1991). *A New Stable Compensator Design form Exact and Approximate Loop Transfer Recovery* Automatica, Vol. 27, No. 2, pp 257-280.

Chen B.M., A. Saberi and P. Sannuti (1992). *Necessary and Sufficient Conditions for a Non-minimum Phase Plant to have a Recoverable Target Loop- A Stable Compensator Design form LTR*, Automatica, Vol.28, No.3, pp 493-507.

Doyle J. C., and G. Stein (1981). *Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis.* IEEE Trans. on Automatic Control, Vol. AC-26, No. 1, pp 4-16.

Doyle J.C., K. Glover, P.P. Khargonekar, and B.A. Francis (1989). *State-Space Solutions to Standard H_2 and H_∞ Control Problems*, Transactions on Automatic Control, Vol. 34, No. 8, pp 831-846.

Freudenberg J.S., D.P. Looze (1988). *Frequency Domain Properties of Scalar and Multivariable Feedback Systems.* Springer-Verlag.

Kallstrom C.G. and P. Ottosson (1982). *The Generation and Control of Roll Motion of Ships in Close Turns*, Fourth International Symposium on Ship Operation Automation, Genova (Italy).

Maciejowski J.M. (1989). *Multivariable Feedback Design.* Adison Wesley.

Messer A.C. and M.J. Grimble (1992). *Robust Track Keeping Control*, CAMS'92 ,IFAC, Genova (Italy).

Morari M. and E. Zafiriou (1989). *Robust Process Control*, Prentice Hall.

Stein G. and M. Athans (1987). *The LQR/LTR Procedure for Multivariable Feedback Control Design* IEEE Transactions on Automatic Control, Vol. AC-32, No.2, pp 105-113.

Zhang Z. and J.S. Freudenberg (1990). Loop Transfer Recovery for Nonminimum Phase Plants IEEE Trans. Automat. Contr., Vol. 35, No. 5, pp 547-553.