Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 28 (2020) 323–329

Structural Integrity Procedia

www.elsevier.com/locate/procedia

1st Virtual European Conference on Fracture

# Calculation of fatigue limits in notches with a micro-mechanical model in a simple way

Victor Chaves[a,*]

[a]Departamento de Ingenieria Mecanica y Fabricacion, Escuela Superior de Ingenieria, Universidad de Sevilla, Camino de los Descubrimientos, 41092, Sevilla, Spain.

**Abstract**

Many methods have been proposed to predict fatigue failure in the presence of notches, among which is the N-R model. The N-R model is based on short-crack fracture mechanics. Specifically, the model analyses the capacity of the crack, which is formed at the notch root by cyclic loading, to overcome successive microstructural barriers such as grain boundaries. This model provides a fairly reasonable explanation of the crack growth from a notch under cyclic loading. The N-R model has been successfully used for many years to predict the fatigue limit in some notched geometries, as shown in several published works, but this model has a certain mathematical complexity. This work shows a simplified version of the N-R model. The elastic problem of a dislocation near a notch is simplified to that of a dislocation in an infinite medium and the study of the equilibrium at the crack line is simplified by using the elastic stress at the midpoint of the crack line. This simplified N-R model has been applied to a circular hole of variable root radius and provides similar fatigue limit predictions to those of the classic N-R model. It has also been compared with results in the literature, where it provides similar predictions to the experimental fatigue limits.



*Keywords:* Fatigue limit; Short-crack; Notch

## 1. Introduction

Fatigue cracks generally grow from geometrical discontinuities, such as holes, grooves and threads, which are called notches. The notch causes an increase in stresses near its root, which favours the formation and propagation of the fatigue crack. Due to its great industrial importance, many researchers have expended much effort to study fatigue in notches and developed several methods to predict fatigue failure in the presence of notches. Most classical methods such as those proposed by Neuber (Neuber (1946)) and Peterson (Peterson (1959)) are based on the calculation of the fatigue notch factor, $K_f$, which is defined as the ratio of

* Corresponding author. Tel.: +34-954-487-311 ; fax: +34-954-487-295.
*E-mail address:* chavesrv@us.es


10.1016/j.prostr.2020.10.038

the fatigue strength of the smooth specimen to the fatigue strength of the notched specimen. More recently, Taylor proposed the Critical Distance Methods, such as the Point Method (PM) and Line Method (LM) (Taylor (1999)). In these methods, the concept of average stress over a certain material characteristic length is used, but the great advance is that this material length is calculated through LEFM from other material properties and not by empirical adjustment.

Focusing on the study of the fatigue limit, it is known that for this case, which is characterized by a low applied stress, the crack spends most of its life during the initiation process and early stages of growth, which is called the short-crack period. A short crack has the size of the material microstructure, which is typically smaller than 1 mm. In the 1980s, theories of short-crack growth were developed, such as the Navarro and de los Rios model (N-R model) (Navarro and de los Rios (1988)). Although the N-R model provides good predictions (Chaves and Navarro (2013)) and has a remarkable physical basis, since it reasonably models the growth of short-cracks, it has more limited use than critical volume methods such as Neuber's, Peterson's or Taylor's models. The reason may be that the critical volume methods are easier to use than the N-R model. This work shows a simplified version of the N-R model. With the proposed simplifications, any notch geometry can be studied by applying few simple equations. The input for this version of the N-R model is the linear elastic stress gradient created by the stress concentrator and simple material properties. Their predictions have been compared to those provided by the classical N-R model without the simplifications to check that they do not greatly differ. Their predictions have also been compared with many experimental results from the literature.

## 2. Brief description of the N-R model for notches

The Navarro and de los Rios model (N-R model) is basically a short-crack growth model. In the N-R model, it is assumed that due to a cyclic applied stress, a crack is initiated in the most favourable grain. The crack and their two plastic zones at the crack tips grow until they reach the first microstructural barrier, such as the grain boundary. If the applied stress is sufficiently high, the crack will exceed this grain boundary and continue to grow through the second grain. The process will be repeated at the second grain boundary and each successive grain boundary thereafter. Mathematically, the problem is analysed using the theory of continuously distributed dislocations. To explain how the model is applied, the simplest crack problem is studied: a crack of length $2a$ is assumed to grow in Mode I in an infinite plate of a polycrystalline body, which is subjected to a cyclic uniform tensile stress ${\sigma_y}^{\infty}$. Suppose that the crack has traversed several grains and their tips have reached two grain boundaries, where the two plastic zones ahead of the crack are practically non-existent, as shown in Fig. 1. The half-crack length $a$ is expressed in terms of the average grain size $D$ and number of grains $i$ that the half-crack has traversed, i.e., $a = (2i-1)D/2$. The two grain boundaries are modelled as barriers of length $r_0 \ll D$. The total length of the crack plus the barriers is $2c$, where $n = a/c$. The crack and barriers are modelled by a continuous distribution of dislocations with Burgers vector $b_y$. The equilibrium of the dislocations at the crack line provides a Cauchy-type integral equation. For this simple case, the problem has an analytical solution, providing relationship between remote applied stress $\sigma_y^{\infty}$ and local stress at the barrier normal to the crack, $\sigma_3$, which is as follows:

$$\sigma_3 = \frac{1}{\arccos{(n)}}\frac{\pi}{2}\sigma_y^{\infty} \tag{1}$$

In the N-R model, the plain fatigue limit is defined by the capability of the first grain boundary to block a microcrack. Thus, by introducing the plain fatigue limit $\sigma_{FL}$ of the material into Eq. (1), the local stress at the first barrier that must be overcome, i.e., the strength of the first material barrier $\sigma_3^{1*}$ is obtained:

$$\sigma_3^{1*} = \frac{1}{\arccos{(n)}}\frac{\pi}{2}\sigma_{FL} \tag{2}$$

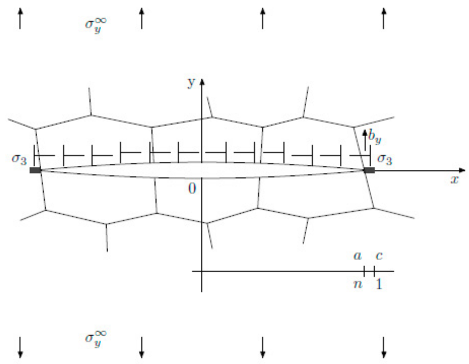


Fig. 1. Crack that has traversed several grains in an infinite polycrystalline body, modelled with distributed dislocations.

The second barrier strength and successive ones, $\sigma_3^{i*}$, are obtained by introducing stresses $\sigma_{Li}$ from Kitagawa-Takahashi diagram of the material into Ec. (1), which is expressed in the form of $\sigma_{Li}$ versus crack length $2a$, for cracks of half-length $a = (2i-1)D/2$, with $i = 2, 3, 4, \ldots$:

$$\sigma_3^{i*} = \frac{1}{\arccos{(n)}} \frac{\pi}{2} \sigma_{Li} \tag{3}$$

Once the barrier strengths $\sigma_3^{i*}$ $(i = 1, 2, 3, \ldots)$ of the material have been calculated with the infinite plate, the model can be applied to calculate the fatigue limit of the notched components. In a notched component subjected to fatigue loading, the crack is assumed to initiate from the notch tip and grow in Mode I. The procedure is repeated: the crack is blocked at the grain boundaries, and it is necessary to calculate the required remote stress for the crack to overcome the successive barriers. For the equilibrium of dislocations, the model requires knowledge of the elastic stresses in the crack line, $\sigma_y(x)$, which is caused by the remote applied stress $\sigma_y^\infty$ from a linear elastic analysis of the solid without crack. The equilibrium of the dislocations in the crack line in the presence of the notch introduces an integral equation that can only be numerically solved. The numerical problem is solved for successive crack lengths ($a = (2i-1)D/2$, with $i = 1, 2, 3, \ldots$). This provides a succession of barrier stresses for the notched component: $\sigma_3^{iN}$ $(i = 1, 2, 3, \ldots)$. The remote stress necessary to overcome each successive barrier in the notched solid $\sigma_{Li}^N$ is calculated, knowing that the barrier strengths $\sigma_3^{i*}$ are identical in the smooth and notched components, since the material is identical in both cases. The maximum value of successive $\sigma_{Li}^N$ is the minimum remote stress required to overcome all barriers. This maximum value is the predicted fatigue limit of the notched component, $\sigma_{FL}^N$.

## 3. A simplified version of the N-R model

The N-R model provides a fairly reasonable explanation of the fatigue failure process and has been successfully used for many years to predict the fatigue limit in some notched geometries. However, the model is not widely used by the scientific community and mechanical engineers, probably due to its mathematical complexity. First, the model requires the knowledge of the elastic field generated by a dislocation near the notch. The second step for the application of the N-R model is the solution of the dislocation equilibrium along the crack line, which introduces a Cauchy-type integral equation. For a crack in an infinite plate subjected to a uniform axial applied stress, the problem has an analytical solution, as previously shown. For a crack in a semi-infinity plate or a crack that grows from a notch, the integral equation must be numerically solved. In summary, the use of the N-R model for fatigue at notches requires studying complicated elastic problems and using numerical integration techniques.

Two main simplifications are made in the present proposal of the N-R model. First, the elastic problem of the forces created by a dislocation near a notch is simplified to that of a dislocation in an infinite medium, which has a known and simple analytical solution. This simplification provides results not far from those of the classic N-R model, as shown in a previous work (Chaves et al. (2017)). This is a great simplification, since the major difficulty of the classic N-R model is the calculation of these forces, i.e., the kernel of the integral equation, for a particular notched geometry. However, the equilibrium still must be solved at the crack line. The presence of the stress gradient due to the notch requires a numerical solution of the equilibrium, which is not simple. In the present proposal, the study of the equilibrium at the crack line is simplified by assuming that the elastic stress gradient created by the notch at the crack line can be described by a uniform stress, which is the stress at the midpoint of the crack line. As shown in the description of the N-R model, there is a known analytical solution for a crack in an infinite plate subjected to uniform loading. With this simplification it is not necessary to numerically solve the problem, since there is a simple analytical solution. So, with the two proposed simplifications, any notch geometry can be studied by applying a few simple equations. The input for the method is the linear elastic stress gradient created by the notch, as in the Critical Distance Methods proposed by Taylor.

Let us explain the proposed technique. Consider a component with a notch subject to cyclic axial loading. A linear elastic analysis of the component, which is analytical or with finite elements, enables the calculation of the point with the maximum principal stress, which is called the hot spot, at the notch contour. The crack line is defined as a straight line normal to the notch contour at the hot spot, i.e., it is assumed to grow in Mode I. Along the crack line, the stress component perpendicular to the crack line is calculated (from the linear elastic model of the component without the crack). A sketch is shown in Fig. 2.

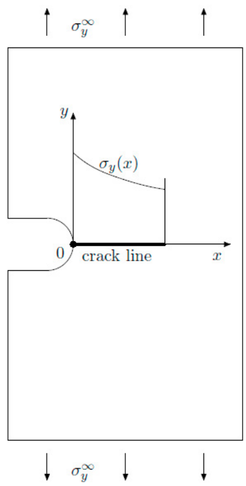


Fig. 2. Solid with a notch subjected to fatigue axial loading.

According to the N-R model, the crack initiates at the hot spot and grows until it reaches the first microstructural barrier (e.g., the grain boundary). It will be stopped unless the local stress at this first barrier, $\sigma_3^{1N}$, is higher than $\sigma_3^1*$. To calculate $\sigma_3^{1N}$, the equilibrium of dislocations at the crack line of length $(D/2)+r_0$ must be solved using the kernel corresponding to the notched geometry and stress $\sigma_y(x)$ along the line. The problem is sketched in Fig. 3, where the grain of the material has been drawn excessively large with respect to the notch size to correctly represent the variables involved. The crack length has been taken as half a grain: $a = D/2$, assuming that the average distance from the notch root to the first grain boundary is $D/2$.

As previously mentioned, the solution of this equilibrium is not easy. Thus, a simplified model to easily calculate $\sigma_3^{1N}$ is proposed, shown in Fig. 4. It is a Mode-I central crack of a grain in length $(2a = D)$ blocked by two barriers at the sides in an infinite plate, which is subjected to uniform stress $\sigma_M^1$. The stress $\sigma_M^1$ is the value of $\sigma_y$ at $x = D/4$ in the original problem. This value is a representative value of the stress gradient

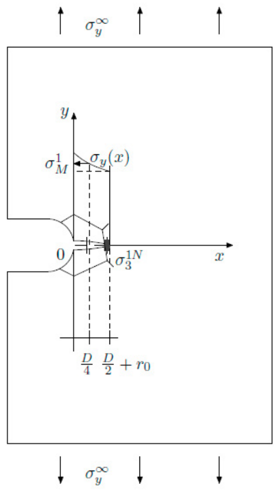


Fig. 3. Calculation of $\sigma_3^{1N}$ in a notched component with the classic N-R model.

in the crack line, since $x = D/4$ is approximately the midpoint of the domain (the midpoint is exactly at $(D/4) + (r_0/2)$, which is very close to the chosen point, since $r_0 << D$). The original edge-crack of length $a = D/2$ has been replaced by a central crack of length $2a = D$.

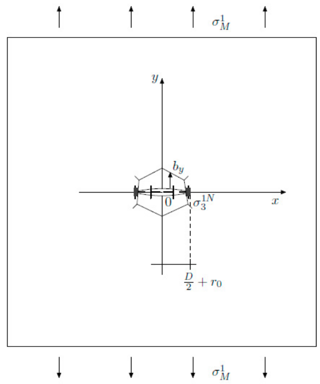


Fig. 4. Calculation of $\sigma_3^{1N}$ in a notched component with the simplified N-R model.

In summary, in the proposed methodology, the original geometry is replaced by an infinite medium, the edge-crack of length $a$ is replaced by a central crack of length $2a$ and the stress gradient is replaced by a uniform stress. This new problem has a simple analytical solution:

$$\sigma_3^{1N} = \frac{1}{\arccos(n)} \frac{\pi}{2} \sigma_M^1 \tag{4}$$

where $n = a/c = (D/2)/((D/2)+r_0)$. If $\sigma_3^{1N}$ is smaller than $\sigma_3^1*$, the crack will stop. If $\sigma_3^{1N}$ is larger than $\sigma_3^1*$, then the crack will overcome the barrier and grow until it reaches the second barrier. The procedure is repeated for the second barrier and successive ones. The stress $\sigma_M^i$ is the value of $\sigma_y$ at $x = (2i-1)D/4$ in the original problem, with $i = 1, 2, 3, \ldots$. The successive values of $\sigma_{Li}$ are deduced from Kitagawa-Takahashi diagram of the material for cracks in plain bodies of length $a^i = (2i-1)D/2$, with $(i = 1, 2, 3, \ldots)$.

The minimum applied stresses necessary to overcome the successive barriers for the notched component is $\sigma_{Li}^{N} = \sigma_{Li}/\sigma_{M}^{i}$. The maximum value of successive $\sigma_{Li}^{N}$ is the fatigue limit of the notched component, $\sigma_{FL}^{N}$.

## 4. Comparison with experimental results

Let us check whether the proposed simplified method provides acceptable predictions for a classic example in fatigue: the circular hole of variable root radius. Fig. 5 shows experimental data of AISI 304L stainless-steel specimens with circular holes of several radii subject to push-pull tests (Chaves et al.-2 (2017)) and the predictions with the two versions of the N-R model: the classic version and the simplified one. The simplified N-R model and classic N-R model provide notably similar predictions for all values of $R$ and below the experimental results. The simplified N-R model converges to $\sigma_{FL}$ when the radius tends to zero and to $\sigma_{FL}/K_t = \sigma_{FL}/3$ when the radius tends to infinity, so it correctly reproduces the size effect in notches.

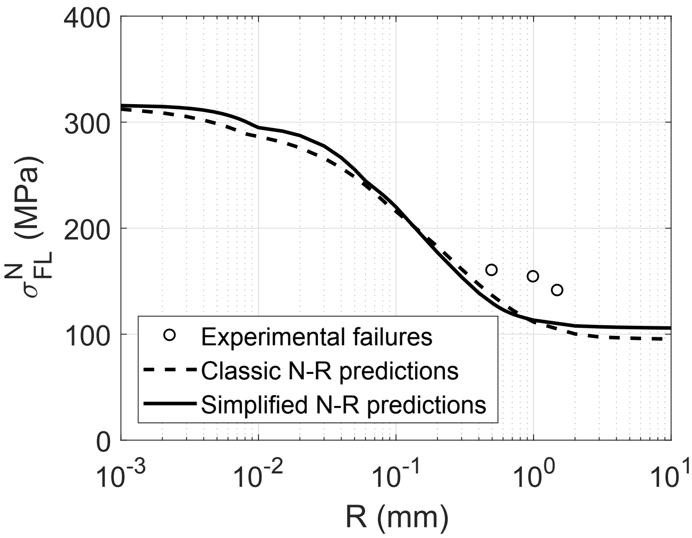


Fig. 5. Experimental results of Chaves et al. Chaves et al.-2 (2017) for stainless steel specimens with circular holes and N-R predictions.

The experimental results in the literature and the predictions of the present version of the N-R model are compared in Fig. 6, with 59 predictions in total. The notch geometry is a circular hole and several types of loading (axial and bending), materials (steel, aluminium alloy and brass), specimen geometries (plate and cylindrical bar) have been studied. The fatigue limit prediction error has been calculated as: Error=(Prediction-Experimental)100/(Experimental) (%). Error bands of ±20% have been included in Fig. 6. Predictions within these bands are considered to have an acceptable error. Average error is 12.5%. As observed, most predictions fall within the 20% error bands, 81.4%. In addition, the results outside of the bands are in the conservative part in all the cases. Thus, the model generally gives acceptable predictions, and when it does not, the results are conservative. These conservative results of the model can favour its use by engineers in industrial applications.

## 5. Conclusions

A simplification of the microstructural model of Navarro and de los Rios (N-R model) to predict the fatigue limit in notched components subjected to axial cyclic loading has been presented. For any notch geometry, the kernel of a set of continuous dislocations in an infinite medium is used. The stress gradient

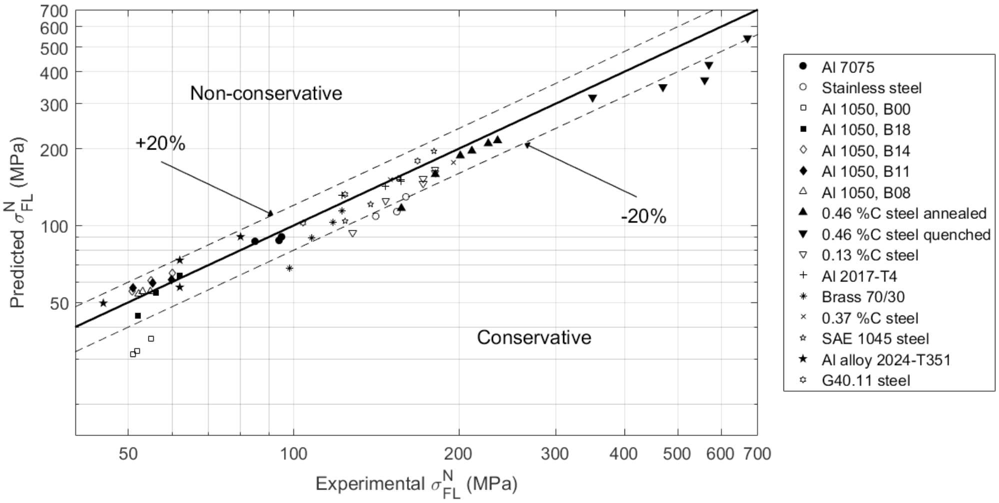


Fig. 6. Fatigue limit predictions with the present version of the N-R model versus experimental fatigue limits for specimens with circular holes.

at the crack line is replaced by an average stress, which is taken from the midpoint of the crack. The basic physics of the N-R model is maintained, which consists of modelling a short crack that grows in the material and overcomes microstructural barriers. The input for this version of the N-R model is the linear elastic stress field ahead of the notch and several common properties of fatigue models. Its use only requires certain knowledge on the conventional finite element analysis. The equations to make the prediction are very simple. The model has been applied to circular holes and many materials from the literature, and it generally provides good predictions or at least conservative ones. This last aspect can be highly appreciated by the engineering community. In conclusion, it can be a useful tool for researchers and engineers in the field of fatigue.

## Acknowledgements

The authors would like to thank the Spanish Ministry of Education for its financial support through grant DPI2017-84788-P.

## References

Neuber, H., 1946. Springer-Verlag; English translation, Theory of notches. Edwards JW. Ann Arbor, MI.

Peterson, R. E., 1959. Notch sensitivity. In: Metal fatigue. Edited by G. Sines and J. L. Waisman, McGraw-Hill, New York.

Taylor, D., 1999. Geometrical effects in fatigue: a unifying theoretical approach. Int J Fatigue21, 413–420.

Navarro, A., de los Rios, E. R., 1988. Short and long fatigue crack growth: a unified model. Philos Mag A 57, 12–36.

Chaves, V., Navarro, A., 2013. Fatigue limits for notches of arbitrary profile. Int J Fatigue 48, 68–79.

Vallellano, C., Navarro, A., Domínguez, J., 2000. Fatigue crack growth threshold conditions at notches. Part I: theory. Fatigue Fract Eng Mater Struct 23, 113–121.

Chaves, V., Madrigal, C., Navarro, A., 2017. Fatigue limit predictions at stress concentrations using FEA and microstructural fracture mechanics. Theor Appl Fract Mec 87, 11–20.

Chaves, V., Beretta, G., Navarro, A., 2017. Biaxial fatigue limits and crack directions for stainless steel specimens with circular holes. Eng Fract Mech 174, 139–154. (2).