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Procedia Structural Integrity 28 (2020) 1167–1175

Structural Integrity Procedia

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1st Virtual European Conference on Fracture

An iterative technique to assess the fatigue strength of notched components

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Abstract

The present work provides an efficient formulation to assess the growth of short fatigue cracks in metallic components. The proposed technique consists on the iterative combination of a micromechanical short-crack growth model and the Finite Elements Method. The interaction of the crack with the microstructure of the material is evaluated through the dislocations distribution technique. The finite elements analysis of the problem is needed to obtain the stress gradient ahead of the notch. The division of the main problem into simpler scenarios makes the resolution of the method easier since cases with known solutions are required exclusively. The iterative method formulation is properly described and application examples are given in order to show its usefulness.

Keywords: Crack; Notch; Fatigue; Dislocations; Finite Elements;

Nomenclature

a	crack length
b_y	Burgers vector
c	crack length including the microstructural barrier
D	average grain size
h	notched component's height
n	dimensionless crack length
r_0	microstructural barrier length
σ	remote applied stress

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10.1016/j.prostr.2020.11.098

σ_3^i	stress at the i-th barrier
$\sigma_3^{i,*}$	stress required to overcome the i-th barrier
$\sigma_3^{i,N}$	stress at the i-th barrier in a notched component
σ_{Li}	remote stress required to overcome the i-th barrier
σ_{Li}^N	remote stress required to overcome the i-th barrier in a notched component
σ_{FL}	plain fatigue limit
σ_{FL}^N	notched fatigue limit

1. Introduction

In mechanical or structural components subjected to cyclic loading, notches play a very important role in terms of fatigue endurance. For this reason, the presence of stress concentrators must be taken into account to carry out an adequate fatigue design. There has been many authors who have developed different methods to deal with this fact such as the well-known critical volume or distance methods (Neuber (1937); Peterson (1959); Taylor (1999)), for instance.

A notch also represents one of the most common locations where short fatigue cracks could appear. Therefore, a fatigue assessment technique which considers the presence of a short crack in a component under cyclic loading seems to be a good alternative to perform fatigue estimations. Microstructure-sensitive models could be appropriate and represent an acceptable resource to study the behaviour of short fatigue cracks. For this reason, it has been decided to apply the Navarro and de los Rios microstructural model (Navarro and de los Rios (1988)), NR model hereto, in this work.

The combination of the NR model and the finite elements technique has been performed to evaluate the fatigue strength of notched components and represents a continuation of the line of work started by Larrosa (Larrosa et al. (2015)). The objective in this work is the description of the iterative superposition process necessary to analyse components with any notch shape. Furthermore, the application of the iterative method to compare with experimental results available in the literature and other classic methods is shown.

2. Short crack growth model

The NR model uses the assumption that the crack, due to the application of a cyclic load, originates in the most favorable grain and, from that moment, propagates to the first microstructural barrier. The most common example of microstructural barrier is the grain boundary of a material. The crack does not overcome the barrier until the local stress at that location exceeds the barrier strength. When it happens, the crack quickly spreads through the adjacent grain until it reaches the next barrier where it will be stopped again. This process is repeated successively in every microstructural barrier producing an oscillating crack propagation rate.

The problem of the crack is studied through the dislocation's theory (Bilby et al. (1963)). The microstructural model is extensively described in previous publications, so here there is a brief example to understand dislocation modeling. Consider the case of a crack of length $2a$ growing through the microstructure of a material of infinite dimensions and subjected to a uniform stress σ (Mode I), modeled with dislocations with Burgers vector b_y, as represented in Fig. 1. The NR model assumes that all the grains of the material are of the same diameter D and are equally oriented. The microstructural barriers have a length $r_0 \ll D$. The total length of the crack plus the barrier is $c = a + r_0$, with $n = a/c$.

The NR model sets a relationship between the remote applied stress to the component σ and the local stress at the i-th barrier σ_3^i:

$$\sigma_3^i = \frac{1}{\cos^{-1} n} \frac{\pi}{2} \sigma \tag{1}$$

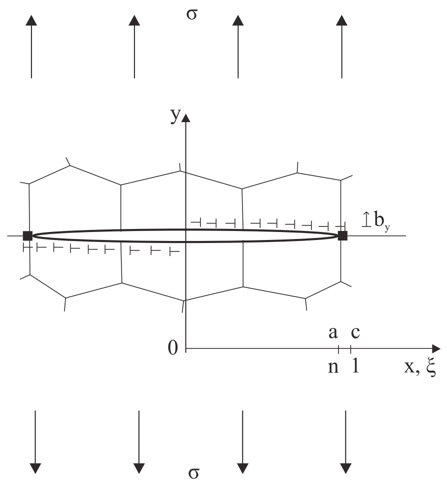

Fig. 1: Crack and barrier in an infinite medium modeled by means of distributed dislocations.

The authors of the model established that the plain fatigue limit of a material σ_{FL} is defined by the resistance it offers so that the crack is not able to overcome the first microstructural barrier. Thus, the strength of the first microstructural barrier, $\sigma_3^{1,*}$, is defined:

$$\sigma_3^{1,*} = \frac{1}{\cos^{-1} n} \frac{\pi}{2} \sigma_{FL} \tag{2}$$

The calculation of the barrier strength for the second and the successive barriers is carried out by replacing the plain fatigue limit σ_{FL} with the stress σ_{Li} in Eq. 2:

$$\sigma_3^{i,*} = \frac{1}{\cos^{-1} n} \frac{\pi}{2} \sigma_{Li} \tag{3}$$

The stress σ_{Li} is obtained from the Kitagawa-Takahashi (KT) diagram of the material and represents the fatigue limit as a function of the crack length. If this diagram is not available, it can be approximated through any of the methods found in the literature (El Haddad et al. (1979); Chapetti (2003); Vallellano et al. (2000)), among others.

2.1. Applying the NR model to noched components

The NR model can also be applied to evaluate the growth of short cracks in components that have a stress concentrator, as is the present case. Thus, it is necessary to take into account several considerations. First, the crack is assumed to originate at the notch root and to grow in Mode I, stopping at each microstructural barrier. Secondly, the presence of the notch produces a stress gradient, $\sigma_y(x)$, over the crack line due to the application of the remote stress σ.

Once the stress gradient produced by the notch is known, the equilibrium of dislocations can be performed on the crack line, whose numerical resolution is detailed in (Chaves and Navarro (2009)), to obtain the local stress in each barrier $\sigma_3^{i,N}$ for the notched case. The problem is solved for different crack lengths $a = iD/2$, where i represents the number of half grains spanned by the crack ($i = 1, 3, 5 \ldots$). Furthermore, in this case where the crack arises from a notch, the resistance of each microstructural barrier $\sigma_3^{i,*}$ is exactly the same for a notched component as for a plain component, since this parameter only depends on the material. Thus, it is possible to calculate the remote applied stress in a notched body, σ_{Li}^N, the crack needs to overcome the i-th barrier:

$$\sigma_{Li}^N = \sigma \frac{\sigma_3^{i,*}}{\sigma_3^{i,N}} \tag{4}$$

Finally, the fatigue limit of a notched component, σ_{FL}^{N}, is the remotely applied stress level from which the crack is able to overcome all the microstructural barriers. This means that the notched fatigue limit corresponds to the maximum value of the stresses σ_{Li}^{N} obtained for the different crack lengths:

$$\sigma_{FL}^{N} = max\left\{\sigma_{Li}^{N}\right\} \tag{5}$$

3. Iterative superposition method

This section describes a method that is able to provide fatigue limit estimations of notched components through the combination of the NR model and the Finite Elements Method. The proposal detailed here responds to the problem of a crack modeled by dislocations that intersects with a close contour, that is, the case of a crack that arises from the notch root. The background is that there are few cases in which there is an analytical solution for specific notched geometries modeled by means of distributed dislocations (Dundurs and Mura (1964)). However, the present method can evaluate a great variety of notched geometries since the known solution of a crack in an infinite medium is uniquely required. The technique presented here is based on the work of Hartranft and Sih where an example of applying the alternating method to the common space between two half planes with specific boundary conditions is detailed (Hartranft and Sih (1973)). Based on this, the iterative technique can be applied to components in which the geometry and dimensions of the notch can be varied, without having to calculate the analytical solution for each type of notch.

The formulation is explained below through an illustrative example. The original problem to be solved is that of a notched component in which a crack arises from the notch tip and is subjected to cyclic loading. Firstly, the division of the problem into several simpler scenarios, as shown in Fig. 2, is needed. In this way we have, on the one hand, scenario 1 that represents the uncracked notched component and, on the other hand, scenario 2 which defines the problem of a crack in an infinite medium growing across the microstructure of the material. The basis of the formulation described here consists in a convergent process that carries out the iterative superposition of the solution of both scenarios 1 and 2, in order to obtain the solution to the original problem. The previous division into simpler scenarios makes the solutions easier than the original problem. Iterative superposition means that several iterations are necessary, where the solution to one scenario represents the boundary conditions, which must be changed in sign, in order to solve the next scenario and the sum of all of them is equivalent to the solution of the original problem. This process is repeated until the convergence is achieved after several iterations.

In each iteration scenario 1 is solved first. This step is performed using any commercial finite elements software and the objective is to obtain the elastic stress gradient over the fictitious crack line. Then, scenario 2 is solved following

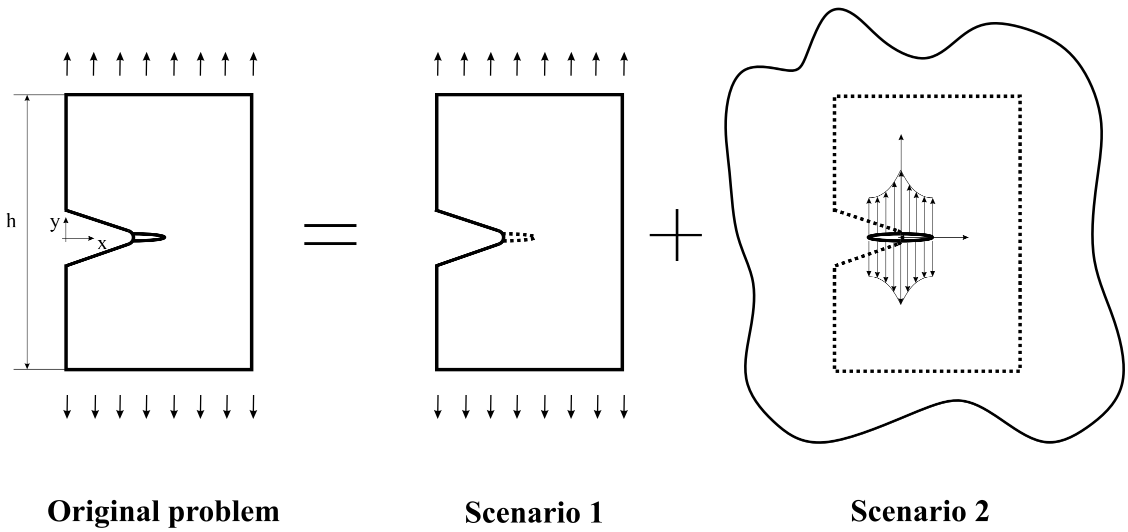

Fig. 2: Division of the original problem into simpler scenarios.

the NR model technique where the crack is subjected to the stress gradient calculated in the previous step but negated. The numerical resolution of the NR model provides, among other values, the dislocations function in the crack that is needed to obtain the value of the stresses in the fictitious notched contour (Hills et al. (1996)) in scenario 2, caused by the presence of the crack. At this point the current first iteration ends. The next iteration starts again solving scenario 1 but bearing in mind that the contour stresses calculated in the last resolution of scenario 2 in previous iteration represent the initial boundary conditions for scenario 1 in this current second iteration and, once more, negated. The rest of the second and successive iterations is exactly the same.

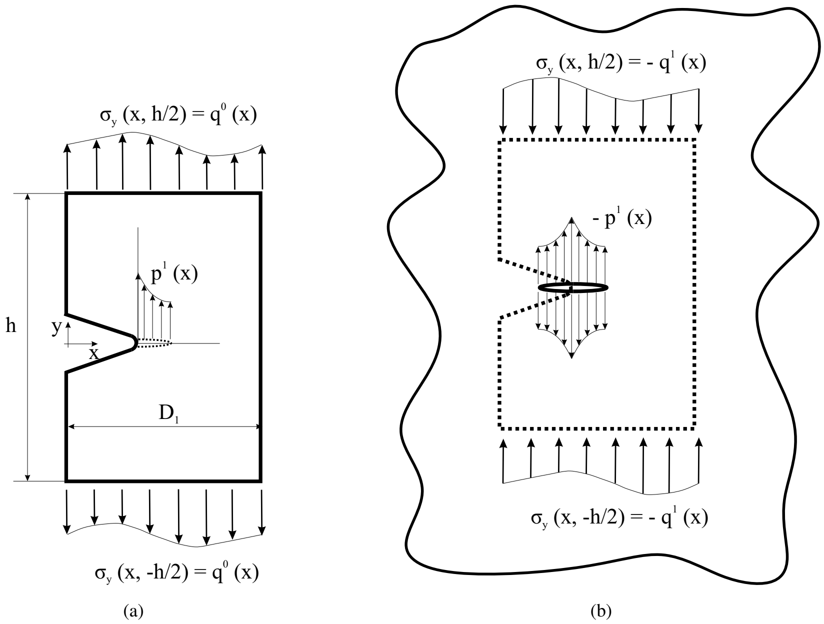

Fig. 3: Detail of first iteration for a) Scenario 1 and b) Scenario 2

This iterative process can be materialized in the following example. Suppose that the original problem in Fig. 2 is subjected to a stress $\sigma_y(x, h/2) = \sigma_y(x, -h/2) = q^0(x)$ at the top and bottom edges. In the first iteration, scenario 1 (see Fig. 3a) is solved by applying the stress $q^0(x)$, which produces the stress gradient $\sigma_y(x, 0) = p^1(x)$ on the fictitious crack line. Then, scenario 2 (see Fig. 3b) is solved, according to the NR model, applying the previously calculated stress gradient over the crack line but negated, that is, $-p^1(x)$. This causes stresses $-q^1(x)$ at the fictitious contour of the notched component. The second iteration begins and scenario 1 is solved by applying the stress $q^1(x)$ to the component (top and bottom edges). This produces a stress gradient $p^2(x)$ over the fictitious crack line. Now scenario 2 is solved by applying $-p^2(x)$ on the crack line and this leads to stress $-q^2(x)$ on the contour of the notched component. This iterative process is repeated for each crack length $a = iD/2$ until convergence is obtained, which is evaluated on the local stress at the barrier $\sigma_3^{i,N}$ obtained after each resolution of scenario 2. Finally, the iterative superposition of all the solved scenarios, after k iterations, implies:

$$\sigma_y(x, h/2) = \sigma_y(x, -h/2) = q^0(x) - q^1(x) + q^1(x) - q^2(x) + q^2(x) + ... \tag{6}$$

$$\sigma_y(x, 0) = p^1(x) - p^1(x) + p^2(x) - p^2(x) + ... \tag{7}$$

A cancellation of the terms is achieved and, after k iterations, the boundary conditions are finally:

$$\sigma_y(x, h/2) = \sigma_y(x, -h/2) = q^0(x) - q^k(x) \tag{8}$$

$$\sigma_y(x, 0) = 0 \tag{9}$$

By performing a considerable number of iterations and superimposing the independently solved scenarios, the stress residual in the notched component contour, $q^k(x)$, is almost negligible compared to the original applied stress $q^0(x)$. Therefore, it is expected:

$$\lim_{k\to\infty} q^k(x) = 0 \tag{10}$$

Finite elements models have been solved using the commercial software *Abaqus* and the entire iterative process has been implemented in Python language since, among other reasons, Abaqus models can be created using this programming language.

4. Application examples and discussion

In this section, the iterative superposition method has been applied to make predictions and compare with the experimental results of fatigue tests in plate specimens with circular hole subjected to cyclic tension-compression loading (R = -1) (DuQuesnay et al. (1986)). The dimensions of the specimens were 44.45 × 116.8 × 2.54 mm and four different hole radius were analysed, 0.12, 0.25, 0.5 and 1.5 mm. The material was 2024-T351 aluminum alloy with the mechanical properties: σ_{UTS} = 466.1 MPa, $\Delta\sigma_{FL}$ = 248 MPa, ΔK_{th} = 7.04 MPa $\sqrt{m}$. The average grain size was not reported by the authors, so it has been estimated as D = 53 μm using the microstructural equation for the KT diagram (Navarro et al. (1997)).

Once the iterative method is applied, the stresses $\Delta\sigma^N_{Li}$ are calculated for every crack length. Please note that the stress data obtained through the iterative method has been transformed into range of stresses in order to make the comparison with the reported experimental fatigue data. Fig. 4 shows the evolution of stress $\Delta\sigma^N_{Li}$ as a function of crack length for different notch radii. It can be seen the big difference between the point where the maximum of this stress $\Delta\sigma^N_{Li}$ is located. For larger radii (blunt notches) the maximum value is located at the smallest crack length. However, in sharp notches, that is small hole radii, the maximum value of $\Delta\sigma^N_{Li}$ is obtained for cracks larger than 0.15 mm.

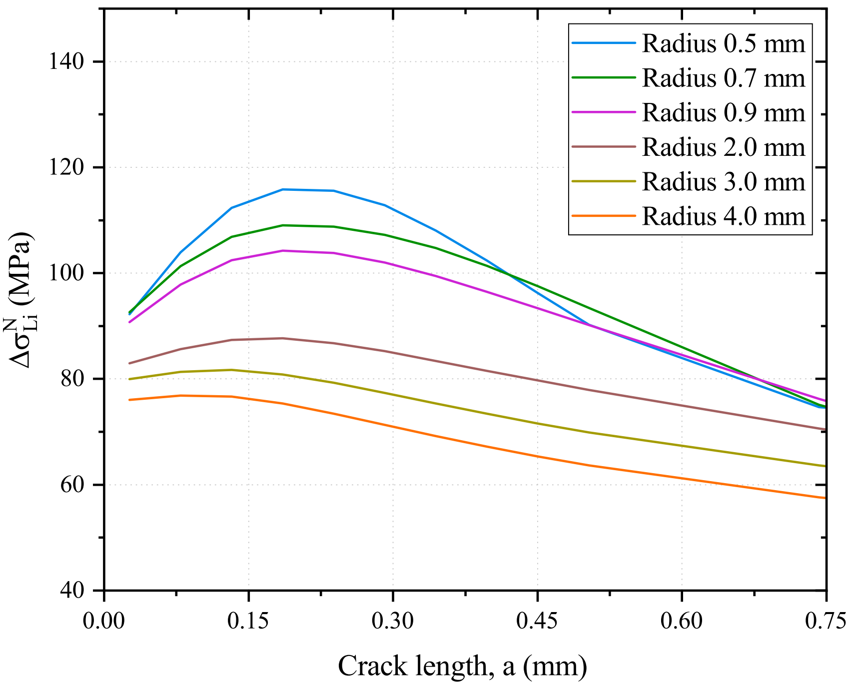

Fig. 4: Stress $\Delta\sigma^N_{Li}$ as a function of crack length for different hole radii.

Calculating the notched fatigue limit for each case is the next step. As described before in Eq. 5, it corresponds to the maximum value of stress $\Delta\sigma^N_{Li}$. Fig. 5 shows the fatigue limit estimations obtained with the iterative method compared to the experimental data reported in (DuQuesnay et al. (1986)). Predictions made with critical distance methods of Peterson (a_1 = 0.51 mm) (Peterson (1959)) and Taylor's point method (L = 0.26 mm) (Taylor (1999)) have also been included. It can be seen that the estimations obtained with the present iterative method fit well the experimental data and also show the same trend as the critical distance methods.

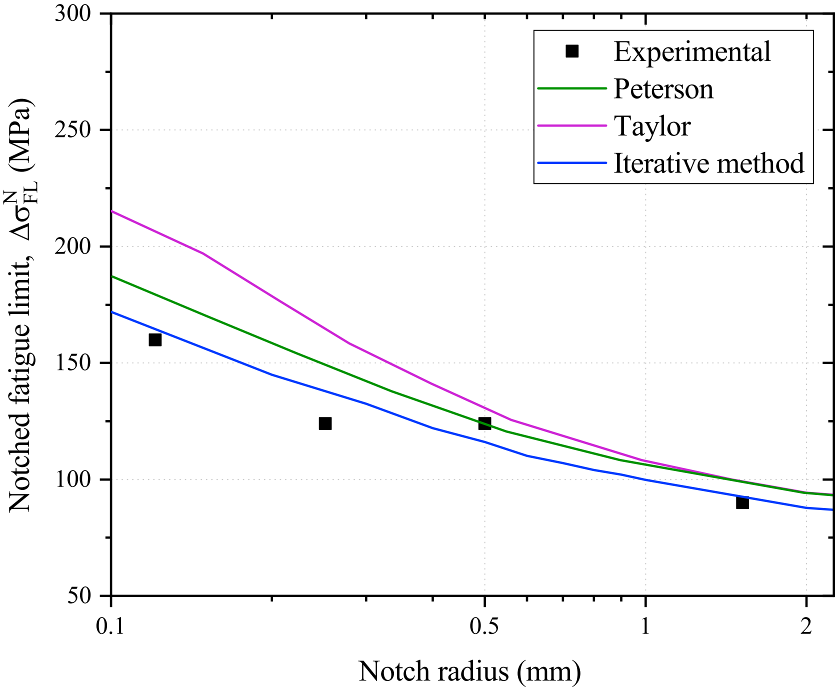

Fig. 5: Tests done by Duquesnay. Estimations performed with different methods.

The fatigue limit predictions of the iterative superposition method are slightly closer to experimental data than the other estimation methods presented in Fig. 5. In order to make a more accurate analysis, Table 1 shows the % relative error between the experimental fatigue limits and the estimations of the three different methods. A negative relative error value represents a conservative estimation and a positive value means just the opposite. As can be seen, the iterative method presents marginally closer fatigue limit estimations in almost all notch radii assessed.

Table 1: Relative error (%) of notched fatigue limit estimations for different methods.

Notch radius (mm)	Peterson	Taylor	Iterative method
0.12	-15.00	-21.87	-7.50
0.25	-23.38	-29.83	-11.29
0.50	0.81	-4.03	6.45
1.50	-11.11	-12.22	3.33

As explained above, the iterative superposition process between scenarios 1 and 2 is repeated in successive iterations until the solution converges. For this reason, a brief analysis of the number of iterations, necessary to obtain convergence during the fatigue endurance estimation process of the plate with circular hole, has been carried out. Fig. 6 shows a bar plot where the number of conducted iterations is indicated for five different i microstructural barriers. Three notch radius have been evaluated, namely, 0.4, 0.7 and 2.0 mm. It is shown that the iterative superposition method does not perform the same number of iterations at each barrier and notch radius. In the current example, it

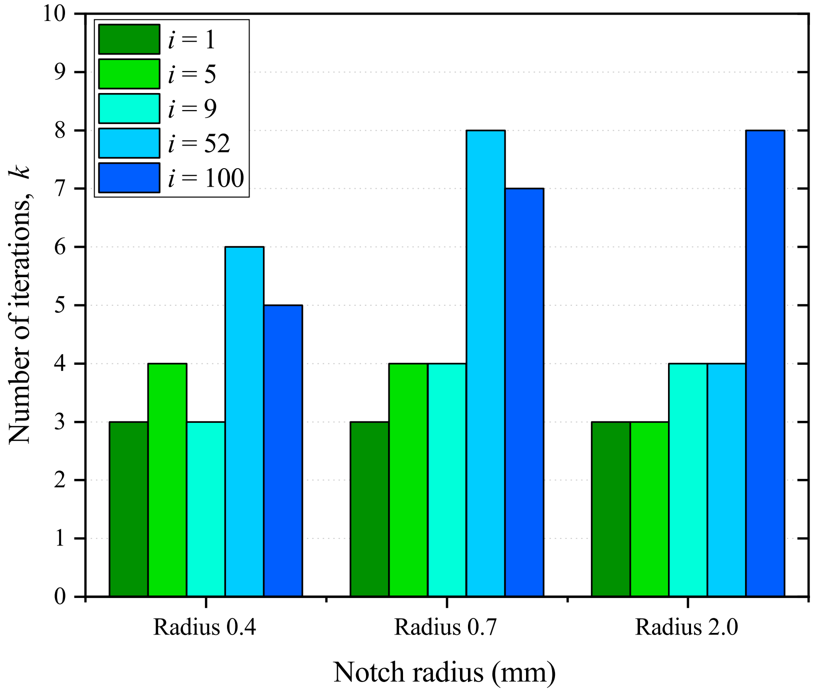

Fig. 6: Number of iterations until convergence is reached.

seems that the number of iterations performed is greater at longer crack lengths (i=52 and i=100), while convergence is achieved earlier at the first microstructural barriers spanned by the crack (i=1, i=5 and i=9). It should be noted that the number of iterations greatly depends on several factors such as the imposed convergence level, the numerical precision of the method's resolution process and the designed mesh for the finite elements model, among others.

5. Conclusions

The use of a short-crack growth microstructural model and the Finite Elements Method have been combined to predict the fatigue strength of notched components. The microstructural model is indicated to evaluate crack propagation, that is, the interactions of the crack with the microstructural barriers, and the Finite Elements Method captures the stress gradient that the notched geometry produces over the crack line.

An iterative formulation has been applied to find the solution to the real notched problem that consists in the division of the original problem into simpler scenarios and iteratively superimposing them. The simplicity of the different scenarios in which the real problem is separated is what adds value to this methodology, since their resolutions are easier and it allows to analyse any notch geometry. Furthermore, the effect of the back surfaces could also be taken into account since it is included in the stress gradient.

Acknowledgements

The authors would like to thank the Spanish Ministry of Education and the Junta de Andalucía for their financial support through grants DPI2014-56904-P, DPI2017-84788-P and P18-FR-4306.

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