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Building and surroundings. Thermal coupling

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Abstract— Energy building performance can be different according to outdoor conditions or urban environment, at the same time that this last assess, buildings are also affected by the building envelope, as obvious consequence of the thermal and Aeraulic coupling existing between the indoor and outdoor conditions in buildings. Thus, in this coupling is fundamental to typify the transmission phenomenon through the building envelope. Doing this, it is possible to estimate transmission heating losses and gains and also the superficial temperatures of the envelope. In order to assess the transient behaviour of the building envelope it is necessary to develop a predictive model, precise enough, to be integrated in a simulating tool. Detailed and multidimensional models, based in numerical methods, like Finite Element Method (FEM), has a high precision, but its complexity imply resources consumption and computational time, too high to be integrated in these kind of tools. On the contrary, simplified methods are good enough because they are simple and fast, with an acceptable precision in almost all the situations. The present work is focused: (a) Firstly, to develop a simplified RC-network model. The aim of the model is to characterize and to implement with precision the behaviour of a wall in a simulating software tool based on urban environment, (b) secondly, to express in form of equivalences, the different indoor and outdoor excitations that can exist in the building envelope, and (c) finally, to calibrate the simplified model through its characteristic parameters. For a homogeneous wall and two types of excitations, it has been obtained the characteristic parameters of the model that represent the better adjustment to the real wall. In a first step, it has been obtained the results of the proposal model and a reference model based on FEM, in terms of wall external surface heat flow. Results of both models have been compared, and the resultant characteristic parameters of the model have been obtained through an optimisation method. Results for the wall and for the excitations under analysis show: (1) Characteristic longitude ec, or capacitive node position, it is determined according to a certain value of Fo equal to 2 for both excitations, this value remains constant in time, (2) useful wall thickness, on the contrary, vary as time function, according to a logarithmic law for both excitations, although this function is different depending on the considered excitation, (3) using a constant excitation, coefficients from the previous logarithmic function depends on the range of the excitation, while these are practically independent of the lineal excitation gradient.

Keywords: Building and surroundings thermal coupling, RC-network model, Urban environment modelling.

1 Introduction

Energy building performance can be different according to outdoor conditions or urban environment, at the same time that this last assess, buildings are also affected by the building envelope, as obvious consequence of the thermal and Aeraulic coupling existing between the indoor and outdoor conditions in buildings [1]-[4]. This Aeraulic coupling can be quantified in the next terms:

- Ventilation and infiltration (from outdoor). It is the main reason for this coupling, and it is function of the temperature and velocities of the outdoor air.
- Transmission heating losses and gains through the building envelope, it also depends on the same variables than the previous therm.
- Winter and Summer solar protections (respectively). The different components of the solar radiation, diffuse, beam and reflected, have an impact in buildings. This radiation leads to gains that counteract losses in Winter and have the opposite effect in Summer, the radiation effect reflects like thermal loads.
- Heating dissipation capacity provoked by the radiant exchange with the sky and the environment. Buildings exchange heating with the urban environment with the

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long-wave radiation, this exchange depends on the superficial temperatures and the can led to losses or gains. On the contrary, the sky exchange, function of the sky temperature and the superficial temperatures of the envelope, always means losses.

Thus, in this coupling is fundamental to typify the transmission phenomenon through the building envelope. Doing this, it is possible to estimate transmission heating losses and gains and also the superficial temperatures of the envelope. In order to assess the transient behaviour of the building envelope it is necessary to develop a predictive model, precise enough, to be integrated in a simulating tool.
Detailed and multidimensional models, based in numerical methods, like Finite Element Method (FEM), has a high precision, but its complexity imply resources consumption and computational time, too high to be integrated in these kind of tools. On the contrary, simplified methods are good enough because they are simple and fast, with an acceptable precision in almost all the situations. Most of the previous models developed, used by tools like EnergyPlus [5], TRNSYS [6], LIDER [7], CALENER [8], ENVI-met [9], are based in very well-known methods, e.g heat transfer function method, which requires Finite Difference Method (FDM), leads to a high computational cost [10, 11]. Other tools, e.g CitySim [12], use simplified methods, like RC-network [11]. Nevertheless, this RC-network method has a high uncertainty level that exists when an specific façade it is characterized through its characteristic parameters.

The present work is focused:

1. To develop a simplified RC-network model. The aim of the model is to characterize and to implement with precision the behaviour of a wall in a simulating software tool based on urban environment.
2. To express in form of equivalences, the different indoor and outdoor solicitations that can exist in the building envelope.
3. To calibrate or to characterise the simplified model through its characteristic parameters.

2 Developing the model

The problem is the conduction heat transfer through the building façades. Assuming transient one-dimensional problem:

$$\frac{1}{\alpha}\frac{\partial T(t)}{\partial t} = \frac{\partial^2 T(t)}{\partial x^2}$$

Where:

$$\alpha = \frac{k}{\rho C_p}$$

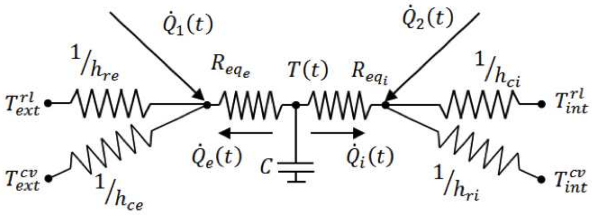

Figure 1: Proposed model RCR

The proposed simplified model is displayed on the figure 1. As shown in the next section, This model is equivalent to a RC-network model of the figure 2.

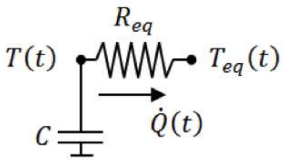

Figure 2: Rd-network model

Thus, this section is focused on the resolution of the RC-network model. Applying energy balance in the capacitive node, we have:

$$C\frac{dT(t)}{dt} = \frac{T_{eq}(t) - T(t)}{R_{eq}}$$

Where $C = \rho C_p e^*$. Then,

$$\frac{dT(t)}{dt} + \frac{1}{\tau}T(t) = \frac{1}{\tau}T_{eq}(t)$$

Where $\tau = C\, R_{eq}$.
Multiplying both side equations by $e^{t/\tau}$ results:

$$\frac{dT(t)}{dt}e^{t/\tau} + \frac{1}{\tau}T(t)e^{t/\tau} = \frac{1}{\tau}T_{eq}(t)e^{t/\tau}$$

From

$$\frac{d}{dt}\Big(T(t)e^{t/\tau}\Big) = \frac{dT(t)}{dt}e^{t/\tau} + \frac{1}{\tau}T(t)e^{t/\tau}$$

results

$$\frac{d}{dt}\Big(T(t)e^{t/\tau}\Big) = \frac{1}{\tau}T_{eq}(t)e^{t/\tau}$$

Integrating

$$\Big[T(t)e^{t/\tau}\Big]_0^t = \int_0^t \frac{1}{\tau}T_{eq}(t)e^{t/\tau}dt$$

So finally

$$T(t) = \frac{1}{\tau}e^{-t/\tau}\int_0^t T_{eq}(t)e^{t/\tau}dt + T(0)e^{-t/\tau} \tag{1}$$

It can be written as follows

$$\int_0^t T_{eq}(t)e^{t/\tau}dt = \int_0^{t-\Delta t} T_{eq}(t)e^{t/\tau}dt + \int_{t-\Delta t}^{t} T_{eq}(t)e^{t/\tau}dt$$

Thus,

$$T(t) = \frac{1}{\tau}e^{-t/\tau}(I_1 + I_2) + T(0)e^{-t/\tau} \tag{2}$$

where

$$I_1 = \int_0^{t-\Delta t} T_{eq}(t)e^{t/\tau}dt \quad \text{and} \quad I_2 = \int_{t-\Delta t}^{t} T_{eq}(t)e^{t/\tau}dt.$$

Solving I_1. On the one hand, it can be written from equation (1),

$$T(t-\Delta t) = \frac{1}{\tau}e^{-\frac{1}{\tau}(t-\Delta t)}I_1 + T(0)e^{-\frac{1}{\tau}(t-\Delta t)}$$

So,

$$I_1 = \Big(T(t-\Delta t) - T(0)e^{-\frac{1}{\tau}(t-\Delta t)}\Big)\tau e^{-\frac{1}{\tau}(t-\Delta t)}$$

Multiplying both sides equations by $\frac{1}{\tau}e^{-t/\tau}$ results:

$$\frac{1}{\tau}e^{-t/\tau}I_1 = T(t-\Delta t)e^{-\frac{\Delta t}{\tau}} - T(0)e^{-\frac{t}{\tau}}$$

Solving I_2. Assuming linear solicitation in each simulation time-step, the integer can be approximated as:

$$\begin{aligned} I_2 &= \int_{t^*=t-\Delta t}^{t^*=t} T_{eq}(t^*)e^{t^*/\tau}dt^* \\ &\sim \int_{t^*=t-\Delta t}^{t^*=t} \big(a + b(t^* - (t-\Delta t))\big)e^{t^*/\tau}dt^* \end{aligned}$$

Where

$$T_{eq}(t^*) = a + b(t^* - (t-\Delta t))$$

with $a = T_{eq}(t-\Delta t)$ and $b = \dfrac{T_{eq}(t) - T_{eq}(t-\Delta t)}{\Delta t}$. Thus

$$I_2 \sim \big(a + b(t^* - (t-\Delta t))\big)I_{21} + bI_{22}$$

where

$$I_{21} = \int_{t^*=t-\Delta t}^{t^*=t} e^{t^*/\tau}dt^* \quad \text{and} \quad I_{22} = \int_{t^*=t-\Delta t}^{t^*=t} t^*e^{t^*/\tau}dt^*.$$

Solving I_{21}:

$$I_{21} = \Big[\tau e^{t^*/\tau}\Big]_{t-\Delta t}^{t} = \tau\Big(e^{t/\tau} - e^{\frac{t-\Delta t}{\tau}}\Big)$$

On the other hand, the I_{22} integer is solved by parts. It yields

$$\begin{aligned} I_{22} &= \Big[t^*\tau e^{t^*/\tau}\Big]_{t-\Delta t}^{t} - \int_{t-\Delta t}^{t} \tau e^{t^*/\tau}dt^* \\ &= \Big[t^*\tau e^{t^*/\tau} - \tau^2 e^{t^*/\tau}\Big]_{t-\Delta t}^{t} \end{aligned} \tag{3}$$

Therefore:

$$\frac{1}{\tau}e^{-t/\tau}I_2 \sim \frac{1}{\tau}e^{-t/\tau}\Big(a - b(t-\Delta t))I_{21} + bI_{22}\Big),$$

$$\frac{1}{\tau}e^{-t/\tau}\Big(a-b(t-\Delta t)\Big)I_{21} = \frac{1}{\tau}e^{-t/\tau}\Big(a-b(t-\Delta t)\Big)\tau\Big(e^{t/\tau}-e^{\frac{t-\Delta t}{\tau}}\Big)$$

and

$$\frac{1}{\tau}e^{-t/\tau}\Big(a - b(t-\Delta t)\Big)I_{21} = \Big(a - b(t-\Delta t)\Big)\Big(1 - e^{\frac{-\Delta t}{\tau}}\Big)$$

By other hand, simplyfing from equation (3),

$$\frac{1}{\tau}e^{-t/\tau}bI_{22} = bt - b\tau - b(t-\Delta t)e^{\frac{-\Delta t}{\tau}} + b\tau e^{\frac{-\Delta t}{\tau}}$$

Then,

$$\frac{1}{\tau}e^{-t/\tau}I_2 \sim \Big(a-b(t-\Delta t)\Big)\Big(1-e^{\frac{-\Delta t}{\tau}}\Big)+b(t-\tau)-b(t-\Delta t-\tau)e^{\frac{-\Delta t}{\tau}}$$

So

$$\frac{1}{\tau}e^{-t/\tau}I_2 \sim (a - b\tau)\Big(1 - e^{\frac{-\Delta t}{\tau}}\Big) + b\Delta t$$

Thus, from (2):

$$T(t) \sim T(t-\Delta t)e^{\frac{-\Delta t}{\tau}} + (a - b\tau)\Big(1 - e^{\frac{-\Delta t}{\tau}}\Big) + b\Delta t$$

And finally,

$$\begin{aligned} &T(t) \sim T_{eq}(t) + \tfrac{\tau}{\Delta t}\Big(T_{eq}(t-\Delta t) - T_{eq}(t)\Big) \\ &+\Big[T(t-\Delta t) - \Big(T_{eq}(t-\Delta t) + \tfrac{\tau}{\Delta t}(T_{eq}(t-\Delta t) - T_{eq}(t))\Big)\Big]e^{\frac{-\Delta t}{\tau}} \end{aligned}$$

2.1 Equivalent indoor and outdoor solicitations in the building envelope

The next condition is imposed

$$\frac{T_{1,1}(t) - T_1(t)}{R_{1,1}} + \frac{T_{1,2}(t) - T_1(t)}{R_{1,2}} + \dot{Q}_1(t) = \frac{T_{eq1}(t) - T_1(t)}{R_{eq1}}$$

$$\frac{T_{1,1}(t)}{R_{1,1}} + \frac{T_{1,2}(t)}{R_{1,2}} + \dot{Q}_1(t) - \left(\frac{R_{1,1} + R_{1,2}}{R_{1,1}R_{1,2}}\right)T_1(t) = \frac{T_{eq,1}(t) - T_1(t)}{R_{eq,1}}$$

and therefore $R_{eq1} = \frac{R_{1,1}R_{1,2}}{R_{1,1}+R_{1,2}}$,

$$T_{eq1}(t) = R_{eq1}\left(\frac{T_{1,1}(t)}{R_{1,1}} + \frac{T_{1,2}(t)}{R_{1,2}} + \dot{Q}_1(t)\right).$$

And for the same reason $R_{eq2} = \frac{R_{2,1}R_{2,2}}{R_{2,1}+R_{2,2}}$,

$$T_{eq2}(t) = R_{eq2}\left(\frac{T_{2,1}(t)}{R_{2,1}} + \frac{T_{2,2}(t)}{R_{2,2}} + \dot{Q}_2(t)\right).$$

This system can be simplified even more:

$$R^*_{eq1} = R_{eq1} + R_1; \quad R^*_{eq2} = R_{eq2} + R_2$$

Finally, under the next condition,

$$\frac{T_{eq1}(t)-T(t)}{R^*_{eq1}}+\frac{T_{eq2}(t)-T(t)}{R^*_{eq2}}+\dot{Q}(t)=\frac{T_{eq}(t)-T(t)}{R_{eq}}$$

results:

$$R_{eq}=\frac{R^*_{eq1}R^*_{eq2}}{R^*_{eq1}+R^*_{eq2}},$$

$$T_{eq}(t)=R_{eq}\left(\frac{T_{eq1}(t)}{R^*_{eq1}}+\frac{T_{eq2}(t)}{R^*_{eq2}}+\dot{Q}(t)\right).$$

The heat fluxes in both surfaces can be obtained from

$$\dot{Q}_{s1}(t)=\frac{T(t)-T_{eq1}(t)}{R^*_{eq1}};\quad \dot{Q}_{s2}(t)=\frac{T(t)-T_{eq2}(t)}{R^*_{eq2}}.$$

And the surfaces temperatures

$$\dot{Q}_{s1}(t)=\frac{T(t)-T_{eq1}(t)}{R^*_{eq1}}=\frac{T(t)-T_{s1}(t)}{R_1};$$

thus,

$$T_{s1}(t)=T(t)-\frac{R_1}{R^*_{eq1}}\Big(T(t)-T_{eq1}(t)\Big),$$

and for the same reason,

$$T_{s2}(t)=T(t)-\frac{R_2}{R^*_{eq2}}\Big(T(t)-T_{eq2}(t)\Big).$$

3 Simplified model characterisation through its characteristic parameters

The proposed parameters that characterise the simplified model are three:

1. Total thermal resistance, which in a general form, for n layers, can be expressed like $R=\sum_{i=1}^{i=n} e_i/k_i$, where e_i and k_i, are thickness and thermal conductivity of each layer, respectively. This value is fixed and, thus, it does not represent a degree of freedom when model is calibrated.

2. Position of condenser or capacitive node through wall thickness in relation to the surface of interest. This value is enclose between 0 and the total wall thickness. It is a free parameter to be determined.

3. Wall capacity value C. This capacity can be expressed, in a general form, for n layers, like $C=\sum_{i=1}^{i=j-1} C_i+\rho_j C_{pj} e^*_j$, where j represents the last useful layer, counting from the surface of interest, referred to the effect of the wall thermal inertia, and where e^*_j represents the truly useful thickness of the last layer j. C value is enclose between 0 and the total storage capacity of the wall, meaning, if every single layer of the wall counts in the wall inertia characterization. Thus, it is a free parameter to determine.

The adjustment or calibration of these parameter to obtain those values that have a better approximation to the wall behaviour depends on different factors as follows: the kind of wall (homogenous or multilayer), wall weight (light, medium or heavy) and the solicitation type .The present work is limited to study what free parameter values are characterized with precision, the wall behaviour under a constant solicitation and standard linear solicitation. For a multilayer wall, it is proposed to do the equivalence to a homogenous wall equivalent [10]. The façade under study has a thickness $e=33$ cm and its thermal conductivity, density and specific heat are respectively: $k=0.3316$ W/mK, $\rho=957.6$ kg/m^3 y $C_p=985.5$ J/kgK. Type solicitations considered are mentioned in table 1.

Type of solicitation	Solicitation (°C) External surface	Solicitation (°C) Internal surface
Constant	$T_{eq1}(t)=20$	$T_{eq2}(t)=20$
Linear	$T_{eq1}(t)=(m/\Delta t)t$	$T_{eq2}(t)=20$

Table 1. Standard solicitation.

For the constant solicitation is imposed a temperature greater than 0 °C in external surface and 0 °C for the internal surface, both of them remain constant during all the simulation time. In the linear solicitation it has been imposed a linear variation of the external temperature through time simulation with a constant gradient for the period, and with an initial value of 0 °C. Reference model which characterizes the real wall behaviour has been developed with ANSYS APDL tool, using FEM. Once both models are compared, in terms of wall external surface heating flow, and using an optimization model, the free parameters of the proposal model have been adjusted for the different solicitation described. The capacitive node position, e_c, has been obtained through the Fourier number, $Fo=(\alpha\Delta t)/e_c^2$, whereas the useful thickness, referred to thermal inertia, e^*, it has been determined from the existent relation between this useful thickness and the capacitive node position, meaning, from the ratio $e^*_c=e^*/e_c$.

4 Results and discusion

Constant Solicitation

Three different solicitation has been established with the purpose of calibrate the characteristic parameters of the suggested model and in function of the amplitude A (mm). Results are shown in the table 2 for the values $a=0.02A+6.72$ and $b=0.17A-53.6$.

Excitation (°C) External surface	Excitation (°C) Internal surface	Fo	e^*_c(m)
$T_{eq1}(t)=10$ $T_{eq1}(t)=20$ $T_{eq1}(t)=30$	$T_{eq2}(t)=0$ $T_{eq2}(t)=0$ $T_{eq2}(t)=0$	2	$a\ln t+b$

Table 2. Characteristic parameters for constant solicitation.

Characteristic length e_c, or capacitive node position, it is determined according to a certain value of Fo equal to 2 for both solicitations, this value remains constant in time. On the contrary, parameter e_c^*, which characterizes the wall thermal inertia vary in a time function according to a logarithmic law and in which it coefficients linearly depends on the excitement gradient.

Linear Solicitation

Three different solicitation has been established with the aim of calibrate the characteristic parameters of the suggested model and in function of the gradient. Results are shown in the table 3.

Excitation (°C) External surface	Excitation (°C) Internal surface	Fo	e_c^*(m)
$T_{eq1}(t) = (1/\Delta t)t$	$T_{eq2}(t) = 0$	2	$0.9 \ln t - 5.2$
$T_{eq1}(t) = (2/\Delta t)t$	$T_{eq2}(t) = 0$		
$T_{eq1}(t) = (3/\Delta t)t$	$T_{eq2}(t) = 0$		

Table 3. Characteristic parameters for linear solicitation.

Characteristic length e_c, or capacitive node position, it is determined according to a certain value of Fo equal to 2 for both solicitations, this value remains constant in time. On the contrary, parameter e_c^*, which characterizes the wall thermal inertia vary in a time function according to a logarithmic law and in which it coefficients, for this solicitation, are practically independent from the solicitation gradient. Thus, it is observed that for the wall and the solicitations under analysis, the capacitive node position remains constant in time, and the useful wall thickness depends on a time logarithmic function for both solicitations. In both cases the capacitive node position is corresponded with a Fo value equal to 2.

5 Conclusions

Energy building performance can be different according to outdoor conditions or urban environment, at the same time that this last assess, buildings are also affected by the building envelope, as obvious consequence of the thermal and Aeraulic coupling existing between the indoor and outdoor conditions in buildings. Thus, in this coupling is fundamental to typify the transmission phenomenon through the building envelope. Doing this, it is possible to estimate transmission heating losses and gains and also the superficial temperatures of the envelope. In order to assess the transient behaviour of the building envelope it is necessary to develop a predictive model, precise enough, to be integrated in a simulating tool. Detailed and multidimensional models, based in numerical methods, like Finite Element Method (FEM), has a high precision, but its complexity imply resources consumption and computational time, too high to be integrated in these kind of tools. On the contrary, simplified methods are good enough because they are simple and fast, with an acceptable precision in almost all the situations.

The present work is focused:

- To develop a simplified RC-network model. The aim of the model is to characterize and to implement with precision the behaviour of a wall in a simulating software tool based on urban environment.
- To express in form of equivalences, the different indoor and outdoor solicitations that can exist in the building envelope.
- To calibrate or to characterize the simplified model through its characteristic parameters.

For a homogeneous wall and two types of solicitations, it has been obtained the characteristic parameters of the model that represent the better adjustment to the real wall. In a first step, it has been obtained the results of the proposal model and a reference model based on FEM, in terms of wall external surface heat flow. Results of both models have been compared, and the resultant characteristic parameters of the model have been obtained through an optimization method.

Results for the wall and for the solicitations under analysis show:

- Characteristic lengthe e_c, or capacitive node position, it is determined according to a certain value of Fo equal to 2 for both solicitations, this value remains constant in time.
- Useful wall thickness e_c^*, on the contrary, vary as time function, according to a logarithmic law for both solicitations, although this function is different depending on the considered solicitation.
- Using a constant solicitation, coefficients from the previous logarithmic function depends on the range of the solicitation, while these are practically independent of the linear solicitation gradient.

It is necessary to mention that the present work suggests a dynamic adjustment for the characteristic parameters of the model and, specifically, it is the parameter that characterizes the thermal inertia or the wall storage capacity which vary with the simulation time, according to the considered solicitation. For future developments, it is proposed to increase the study taking into account different types of walls, multilayer walls, different wall weights and generic solicitations that belong to several climatic severities.

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