# Modelling and planning public cultural schedules for efficient use of resources

Francisco A. Ortega [a], Miguel A. Pozo [b,*], Justo Puerto [b]

[a] Department of Applied Math I, University of Seville, Spain
[b] Department of Statistics and Operational Research, University of Seville, Spain

*Keywords:*
Assignment
Scheduling
Mathematical programming
Decision making

A B S T R A C T

This paper addresses a decision making problem concerning the planning of cultural schedules. The model maximizes the overall welfare of the entire system by integrating the different parties involved in the process (artistic agents, sites and administration) in a unified setting. In order to solve the proposed model this paper also derives fast ad hoc heuristics and different Lagrangian relaxations that lead to good lower and upper bounds. These bounds are later used to obtain feasible solutions that improve the currently available lower bounds. All these elements are tested over a testbed of random instances to analyze their computational performance, showing promising results. In addition, we present one particular instance based upon actual data gathered in Andalusia (Spain). The results of this analysis draw interesting conclusions on how to improve the efficient use of public funds devoted to promote cultural activities.

## 1. Introduction

In its Universal Declaration on Cultural Diversity (2001), UNESCO defines cultural goods as commodities that should not be considered as simple consumer goods and, subsequently, it is necessary to pay particular attention to the diversity of the creative as well as the specific nature of cultural services, since they are carriers of identity and immaterial values. Cultural policy can be most usefully considered as the sum of government's activities with respect to the arts, humanities, and heritage that involves governmental strategies and activities that promote the production, dissemination, marketing, and consumption of the arts [26]. A cultural policy can involve support for museums, visual arts, performing arts (music, dance, opera and theater), historic preservation and folklore, such as creative writing and poetry. In many countries there is often an association between culture and civic identity. Consequently, an important political issue leads to subsidizing what is known as cultural industries with the objective of preserving an old cultural heritage or to develop a nascent culture [20].

Typically, private and even public cultural industries are located at cities where there is a high concentration of cultural consumers. With the aim of establishing equal opportunity for all citizens to participate in publicly organized and financed cultural activities, extensive touring programs are designed to bring these activities to those culturally underserved areas. This top-down approach orientated to provide equal opportunities for citizens to be culturally active on their own preferences, is known as democratization of culture [19]. For this purpose several funding measures (ticket-price reduction, 50–50 state-subsidy, etc.) are commonly applied in order to create (and maintain) larger audiences in local theaters and museums. Despite having implemented different compensatory programs, the cultural institutions in many countries do not seem to have reached the desired effect of guaranteeing a successful dissemination of cultural contents.

This paper presents a general setting to handle the planning of public cultural events as a mathematical programming model and a solution framework that address this class of assignment-type optimization problems. It is specifically geared to provide an efficient planning of events at different sites along a time horizon incorporating conflictive points of view coming from several decision makers involved in the process as well as limited resources. In particular, this framework fits better to the planning of public cultural events where sites are theaters and decision-makers are managers of municipal theaters, producers of theater companies and policy makers in the central administration of cultural events.

Assignment problems [24,5] involve matching the elements of two or more sets in such a way that an objective function is optimized. A very interesting variant encountered in the literature is the problem of matching the members of three different sets known as agents,

* Corresponding author. Dpto de Estadística e IO. Facultad de Matemáticas. C/ Tarfia s/n, 41012 Sevilla, Spain.
*E-mail addresses:* riejos@us.es (F.A. Ortega), miguelpozo@us.es (M.A. Pozo), puerto@us.es (J. Puerto).

events (or tasks) and time slots over a time horizon. This kind of three-dimensional assignment problems is also known as multi-period assignment problems [5]. A well-known example is the so-called school or university timetabling problem (see e.g. [10]) that assigns students and teachers to classes and time slots. There is a considerable body of research on multi-period assignment problems taking into account a large variety of features and requirements (the interested reader is referred to [5] for a recent compendium of related references). Exact solution methods for multi-period assignment problems became easily intractable in medium to large size instances [5]. However, as far as we know, there is a lack of references of optimization models specifically geared to the planning and management of the cultural industry although there exists an increasing need of results in this field [9,13].

The Andalusian Institute of Arts and Letters (IAAL in Spanish), concerned about the existence in the Spanish region of Andalucia of municipal theaters without enough cultural activity (more details in Section 6.1), suggested the development of a model that, after an appropriate calibration of parameters, could improve the efficiency of the cultural agenda promoted by public resources in Andalucia. For this purpose, the IAAL provided data on the location of the municipal theaters, the number of performances carried out in 2009–2010 and the theater companies interested. As a first approach, Ortega et al. [23] formulate several models of linear optimization in order to assess decisions to be made independently by the three agents who concur in this context (managers of municipal theaters, producers of theater companies and policy makers in the administration of cultural circuits). Thus, that seminal paper does not integrate all viewpoints into a unified setting and it only illustrates the separate models for the different individual points of view with toy examples. Moreover, no algorithms were developed for any of those models and consequently no computational results for real size instances were reported.

Under a different paradigm, we would find some similarities between the problem of planning cultural events and some variants of generalized assignment problems. For instance, if the performance space is identified with a hospital operating room (the surgical team would be the agent and the operating room would correspond to the site), we may establish a parallelism between the planning of public cultural agendas and the efficient use of operating rooms [18,14]. In both contexts, scheduling is addressed to optimize the use of scarce resources. Nevertheless, one of the main objectives when designing surgery schedules is to minimize the overtime cost of “individual” patients (specially, in the case of urgencies) whereas planning cultural agendas emphasizes a collective point of view related to accessibility and coverage. Another related application consists in elaborating schedules of tournaments for the regional-national sport leagues [11,25], whose objective is the design of feasible routes within competition calendars with a wide range of restrictions. As mentioned in Ortega et al. [12], many criteria have been taken into account in this field like, for instance, transportation savings [3] and time constraints [21]. The approaches for sport league scheduling require, as happens with the cultural management in practice, organizatorial, attractiveness and fairness criteria/constraints. On the other hand, there are intrinsic differences between sports and cultural planning. In sport planning all teams must be allocated in time and space whereas in cultural schedules some agents and/or sites may not be included in a solution. Reviewing the literature one can find some other related models as for instance the scheduling of medical residents and nurses [28,15] scheduling transit system drivers [7], etc. Summarizing, the main difference between planning cultural schedules and the previously mentioned contributions of related literature is that its goal is the maximization of the social welfare, which induces a completely innovative modelling phase to incorporate the attitude of all parties included in the system.

The contribution of this paper is two-fold. First, we describe a complete decision making model for the efficient planning of cultural schedules that maximizes the overall welfare of the system. This new formulation covers the gaps in Ortega et al. [23] and extends that approach since it integrates the different parties involved in the process in a unified setting. In addition, by adapting the constraints it could be applied to handle some other models considered in Ebewo and Sirayi [13] and Kidd [17]. We also analyze this formulation studying its behavior using standard MIP solvers. The second contribution is a solution framework, firstly applied in this kind of problems, that is based on different decompositions of the initial model. This is done by deriving fast ad hoc heuristics that provide good lower bounds. Next, we develop Lagrangian relaxations for the model that lead to good upper bounds that are later used to improve the feasible solutions previously obtained. All these elements are tested over a testbed of random instances to analyze their computational performance, showing promising results. In addition, we present one particular instance based upon actual data gathered in Andalusia (Spain).

The remainder of the paper is organized as follows. Section 2 presents the mathematical programming model that describes the actual problem and introduces the terminology and notation required for the rest of the paper. It will be shown that its exact solution is very difficult to obtain using general purpose solvers, even for medium size instances. Thus, we address next the development of fast heuristics in Section 3. Section 4 deals with the derivation of error bounds between heuristic solutions and the, sometimes unknown, optimal ones. Since optimal solutions are not always available, we provide upper bounds by means of different Lagrangian relaxations. Sections 4.4 and 5 improve previous heuristic solutions (lower bounds) by using, respectively, a repairing and a multi-start greedy algorithm. In Section 6 all previous models and solution methods are tested over a testbed of random instances to analyze its computational performance. In addition, Section 6.1 contains a practical application for the case of Andalusia. There, we have applied our models and solution approaches to the schedule of the cultural agenda of activities of an annual season in the region of Andalusia (Spain) obtaining promising results as compared with the actual ones implemented in previous years. Finally, our concluding remarks and future lines of research are presented in Section 7.

## 2. Problem description

In this section, we present a general setting (a mathematical programming model) for addressing an efficient planning of cultural events, taking into account the different involved parties (municipalities, artistic companies and administration) and specific quantitative requirements (population coverage, length, budget, etc.). Municipalities are interested in providing an attractive cultural offer with respect to a combination of preferences, mainly related to companies choice and calendar schedule. Artistic companies try to maximize their incomes performing as long as possible in the most attractive municipalities. The administration aims to maximize the overall social welfare. Thus, we propose to measure the quality of solutions relating those factors.

In order to present the general model we introduce the following notation that will be linked to the application of cultural events. Let $k \in K$ be the set of agents (companies) and $q \in Q$ the set of modalities (modalities refer to thematic cultural areas like Theater and Dance, Music). Each agent could belong to several modalities, so from now on, we will understand an agent as a pair $(k, q) \in KQ$ joining each agent (artistic group) with its respective modalities. Events (performances) are carried out over the set of sites $h \in H$ (understood as theaters or municipalities) along the set of days $t \in T$ in the planning horizon. The repetition of events at the same site will be described by the index $j \in J$. Thus $|J|$ is the maximum number of repetitions of an agent on a given site. Site $h$ has a budget of $C_{qh}$ for modality $q$ and each agent has a cost of $c_{kq}$. Our goal is to plan a cultural agenda maximizing the overall induced *welfare*. In our setting, welfare must be understood as cultural benefit and therefore there is not a standard agreement on the units to measure it. For this reason, we have adopted ourselves the convention that welfare is induced by the combination of several factors that depend on the attractiveness of agents, sites and days of the calendar. We assume that each entity, namely agent, site or day, has an inherent welfare (attractiveness) factor that is estimated by the data or is given by the planner. These factors are considered parameters in our approach and are denoted by $\omega_{kq}$, $\sigma_h$ and $\theta_t$, respectively, for the agent $(k,q)$, the site $h$, and the day $t$. Additionally, in order to describe the modification of the amount of welfare produced by each repetition of an event carried out in the same site, we define a non-increasing parametric function $\rho(j)$, $0 \le \rho(j) \le 1$, the discount factor (in cost and benefit) for the $j$th repetition of an event in the same site. Finally, we denote by $p^t_{kqh}$ the preference weight associated with the allocation of agent $(k,q)$ to site $h$ on day $t$ (this allows us to consider/reward that, for unknown or personal reasons, some agents may fit better than others at certain sites on specific days). These factors altogether allow us to estimate the overall benefit of a combination of indices $[k, q, h, t, j]$ by means of a multiplicative expression, as

$$b^{tj}_{kqh} = \omega_{kq}\sigma_h\rho(j)\theta_t p^t_{kqh}.$$

The reader may note that this estimation does not compromise the validity of the model since it is valid regardless the estimation method for the welfare coefficients. Additionally, benefits $b^{tj}_{kqh}$ could be computed in many different ways even admitting a flexible different expression for each site and/or agent. In our model, decisions are made on which agents perform at which sites and when. Therefore, we define the binary variable $x^{tj}_{kqh} \in X$ that assumes value 1 if agent $(k,q)$ carries out the event for the $j$th time at site $h$ on day $t$ of the calendar (planning horizon); otherwise, $x^{tj}_{kqh}$ would take a value 0.

### 2.1. Notation

For the sake of readability we include the following short table of symbols summarizing the elements described above. These symbols will be used throughout the paper:

*Sets*

| | |
|---|---|
| $k \in K$ | set of agents |
| $q \in Q$ | set of modalities associated to agents |
| $(k,q) \in KQ$ | set of agents with their associated modalities |
| $h \in H$ | set of sites |
| $t \in T$ | set of days of the planning horizon |
| $j \in J$ | counter of the number of repetitions of events in the same site |

*Parameters*

| | |
|---|---|
| $c_{kq}$ | cost required to choose agent $(k,q)$ |
| $C_{qh}$ | total budget of site $h$ for modality $q$ |
| $\rho(j)$ | discount factor (in cost and benefit) for the $j-$ th event |
| $\omega_{kq}$ | coefficient of attractiveness for agent $(k,q)$ |
| $\sigma_h$ | coefficient of attractiveness for site $h$ |
| $\theta_t$ | coefficient of attractiveness for day $t$ |
| $p^t_{kqh}$ | preference weight for agent $(k,q)$ performing at site $h$ on day $t$ |
| $b^{tj}_{kqh}$ | welfare obtained by the $j$th repetition of an event by the agent $(k,q)$ carried out in $h$ the day $t$ of the calendar. We can assume $b^{tj}_{kqh} = \omega_{kq}\sigma_h\rho(j)\theta_t p^t_{kqh}$ |

*Variables*

| | |
|---|---|
| $x^{tj}_{kqh} \in X$ | binary variable equal to 1 $\Leftrightarrow$ agent $(k,q)$ carries out the event $j$ at site $h$ on day $t$ of the planning horizon. |

### 2.2. Integer formulation

In order to analyze the above-described problem we use the following model of integer programming:

$$P : z = \max \sum_{h \in H} \sum_{(k,q) \in KQ} \sum_{t \in T} \sum_{j \in J} b^{tj}_{kqh} x^{tj}_{kqh} \tag{1}$$

$$\text{s.t.} \quad \sum_{(k,q) \in KQ} \sum_{h \in H} \sum_{j \in J} x^{tj}_{kqh} \le 1 \quad k \in K,\ t \in T \tag{1a}$$

$$\sum_{t \in T} x^{tj}_{kqh} \le 1 \quad (k,q) \in KQ,\ h \in H,\ j \in J \tag{1b}$$

$$x^{tj}_{kqh} \le x^{t-1,j-1}_{kqh} \quad (k,q) \in KQ,\ h \in H,\ t \in T,\ j \in J \setminus \{1\} \tag{1c}$$

$$\sum_{(k,q)\in KQ}\sum_{j\in J}x_{kqh}^{tj}\le 1 \quad h\in H,\ t\in T \tag{1d}$$

$$\sum_{(k,q)\in KQ}\sum_{j\in J}\sum_{t\in T}c_{kq}\rho(j)x_{kqh}^{tj}\le C_{qh} \quad h\in H,\ q\in Q \tag{1e}$$

$$x_{kqh}^{tj}\in\{0,1\} \quad (k,q)\in KQ,\ j\in J,\ h\in H,\ t\in T \tag{1f}$$

The objective function (1) maximizes the overall welfare of a planned schedule of cultural events. Constraints (1a) ensure that an agent (in any modality) does not perform more than once during the same day. Constraints (1b) guarantee that any agent does not perform for the $j$th-time in a site more than once. Constraints (1c) force that the event $j$ of an agent in a site comes the day after the event $(j-1)$ of that agent in that site. Constraints (1d) avoid more than two events on the same day in the same site. Constraints (1e) ensure that no site exceeds the allowed budget of site $h$ for modality $q$ and provides a discount in cost for the $j$th repetition of an agent in a site.

Problem $P$ represents a globalized perspective of the planner that makes decisions at two different levels, allocating agents to cultural sites and time periods, in order to maximize the overall welfare, so fomenting an economy of scale and a reasonable measure of satisfaction for the audiences. We remark that this formulation considers the point of view of the different parties in the problem. Specifically, assuming that constraints (1d) and (1e) are removed, the model represents the point of view of individual agents (when $|K|=1$) that consists in designing a tour among all sites in order to maximize their income. This perspective leads to ignore all information about which agents and days have already been selected by sites and how much budget has already been spent by each site. Analogously, if one removes constraints (1a), the model gives the point of view of individual sites (when $|H|=1$) that consists in designing an optimal schedule for a single site according to the planning horizon and agents preferences. The total cost spent by that site on each modality cannot exceed $C_{qh}$.

The reader may notice that the above formulation does not include conditions on transportation costs or day/s off after events. However, these extensions and many others can be easily included in this formulation by means of additional constraints as described in Section 7.

This type of multi-period assignment problems is known to be hard for general purpose solvers. A similar behavior is also observed in our model as will be shown in Section 6.

## 3. Fast lower bounds: sequential method

The main drawback of applying a general solver as CPLEX to handle the above formulation is memory limitation (for the branch and bound tree) before the time limit is reached (see Section 6). Note that this behavior has already been observed on other related optimization problems dealing with scheduling or multi-period supply chain problems (e.g. [12,7,1,4]). This behavior leads us to develop "ad hoc" solution approaches that, at least, provide good approximate solutions as well as valid bounds to evaluate the quality of our solutions.

A first approach to obtain fast lower bounds for medium to large size instances can be obtained when problem $P$ is separated into smaller subproblems that can be solved independently (see e.g. [12,16,1,14] for similar techniques in different problems). We propose two splittings of our master problem $P$, namely solving $P$ by sites or by agents.

Consider first the splitting by sites. The rationale of this approach is to solve sequentially the subproblems by sites incorporating adequately the information (solutions) of previous subproblems into the current one. Assume that a ranking of the sites is available. First, one allows site 1 to make its own choices among agents and periods and then subsequent sites are restricted to choose among the remaining agents and days.

Let $X_h$ be the set of variables of $P$ when site $h$ has been chosen to be solved. Besides, let $X_h^*$ denote the value of these variables at the solution of subproblem $h$. Finally, let $\tilde{X}_h$ be the set of variables that are fixed when solving subproblem $h$ and that come from the solutions of previous subproblems. Algorithm 1 presents the pseudocode of this first sequential approach.

**Algorithm 1.** (SMS) Sequential method for each $h\in H$.

**input**: Instance of $P$
**Output**: Feasible solution $(\tilde{X},\tilde{z})$ (lower bound)
Sort $H$ in non-increasing order according to $\sigma_h$;
$X^*=\{\}$;
**for** $h=1:|H|$ **do** // Solve $P$ sequentially for every $h\in H$
  **for** $h'=1:h-1$ **do** // $h=0$ does nothing
    **for** $\tilde{x}\in\tilde{X}_{h'}$ // Building $X_h^*$ to be fixed
      $x^*(\tilde{k},\tilde{q},h,\tilde{t},j)=0,\ j\in J$;
  Solve $P$ for $H=\{h\}$ fixing $X^*$. Keep $\tilde{X}_h,\ z_h$;
Return the final solution $\tilde{X}=\{\tilde{X}_1,\ldots\tilde{X}_{|H|}\}$ and the objective value $z=\sum_{h\in H}z_h$;

The same process is analogously described in Algorithm 2, but taking the subdivision by agents. Here, the difference is that it is required an updating process for the remaining budget for each site in each iteration. Additionally, these results can be significantly improved sorting previously sites/agents according to some ranking criterion (e.g. attractiveness factor) in order to let choosing first those sites/agents that are more important.

**Algorithm 2.** (SMA) Sequential method for each $k\in K$.

**input**: Instance of $P$
**output**: Feasible solution $(\tilde{X},\tilde{z})$ (lower bound)

Sort $K$ in non-increasing order according to $w_{kq}$;
$X^* = \{\}, \tilde{X}_0 = \{\}$;
**for** $k = 1 : |K|$ **do** // Solve sequentially for every $k \in K$
  Determine the set of modalities of $k$ : $KQ_k$;
  **for** $k' = 1 : k-1$ **do** // $k = 0$ does nothing
    **for** $\tilde{x} \in \tilde{X}_{k'}$ **do** // Building $X_k^*$ to be fixed
      $x^*(k, q, \tilde{h}, \tilde{t}, j) = 0, \; j \in J, \; q \in (k, q)$;
  Update $C_{qh}$ according to $\tilde{X}_{(k-1)}$ // ($k = 0$ does nothing);
  Solve $P$ for $K = \{k\}$ fixing $X^*$. Keep $\tilde{X}_k, \; z_k$;
Return the final solution $\tilde{X} = \{\tilde{X}_1, \ldots \tilde{X}_{|K|}\}$ and the objective value $z = \sum_{k \in K} z_k$;

We will denote these methods as *Sequential Method by Sites* (SMS) and *Sequential Method by Agents* (SMA). We have observed that, in general, SMS works better than SMA since, at the end of SMS, the portion of budget that cannot be spent in any available agent is generally lower than the one left in SMA. In any case, both methods are heuristic as it is shown by the following counter-example.

**Example 1.** For sizes $|K| = 3, |Q| = 1, |H| = 3, |T| = 3, |J| = 1$, we define $c_k = [5, 3, 2]$, $\sigma_h = [1, 0.9, 0.4]$, $C_h = [10, 10, 10]$ and $\theta^t = [1, 0.7, 0.3]$. Setting benefits as $b_{kqh}^{tj} = c_k \sigma_h \theta_t$, the optimal value of $P$ is equal to $z^{(opt)} = 15.93$ while both sequential methods only provide objective values equal to $z^{(h)} = z^{(k)} = 14.95$ which are only a lower bound of the optimal value. Moreover, redefining $\theta^t = [1, 0.7, 0.4]$, $t = 1, 2, 3$ we obtain $z^{(opt)} = 16.61$, $z^{(h)} = 16.49$ and $z^{(k)} = 15.35$. Note that even when constraints (1e) are removed (or equivalently, when budgets $C_h$ are set to a value large enough) SMS and SMA remain heuristic.

## 4. Improving upper and lower bounds (Lagrangian relaxation)

In order to better evaluate the solutions of our model and given that exact solutions obtained by CPLEX cannot be obtained even for medium size instances (see Section 6), we have developed an alternative approach that provides us with good upper bounds and a constructive heuristic to get feasible solutions of higher quality than the one returned by SMA. Moreover, what is more important, these bounds can be used within our framework to evaluate the quality of our heuristic solution measuring the actual deviation of our approximated solution.

Given an integer program $P : z = \max_x \{cx : Ax \le b, Dx \le e, x \in \mathbb{Z}^+\}$, with $Ax \le b$ being a set of constraints to be relaxed a Lagrangian relaxation of $P$ is defined as $P' : L(\lambda) = \max_x \{cx + \lambda(b - Ax) : Dx \le e, x \in \mathbb{Z}^+\}$. For any value of $\lambda$, $L(\lambda)$ provides an upper bound of $P$. Note that if $\lambda$ is too big, the relaxed constraints $Ax \le b$ will be too well-satisfied and $L(\lambda)$ will not generate good upper bound of $P$. On the other hand, if $\lambda$ is too small, the relaxed constraints could not be satisfied at all. The aim is to solve the Lagrangian Dual problem $L^* = \min_{\lambda \ge 0} L(\lambda)$ which can provide a better bound than the one provided by the linear programming relaxation ($z_{LP}$), that is $z \le L^* \le z_{LP}$.

Our idea is to choose properly the set of constraints to be relaxed in order to split $P$ into smaller programs that can be solved independently.

### 4.1. Lagrangian relaxation decomposing index set $H$

Relaxing constraints (1a) in $P$, we obtain $P'$, the Lagrangian relaxation of $P$ for a given $\lambda$:

$$\begin{aligned} P' : L(\lambda) &= \max_x \left( \sum_{(k,q) \in KQ} \sum_{h \in H} \sum_{t \in T} \sum_{j \in J} b_{kqh}^{tj} x_{kqh}^{tj} + \sum_{k \in K} \sum_{t \in T} \lambda_{kt} \Big(1 - \sum_{(k,q) \in KQ} \sum_{h \in H} \sum_{j \in J} x_{kqh}^{tj}\Big) \right) \\ &= \max_x \sum_{h \in H} \left( \sum_{(k,q) \in KQ} \sum_{t \in T} \sum_{j \in J} b_{kqh}^{tj} x_{kqh}^{tj} - \sum_{k \in K} \sum_{t \in T} \lambda_{kt} \sum_{(k,q) \in KQ} \sum_{j \in J} x_{kqh}^{tj} \right) + \sum_{k \in K} \sum_{t \in T} \lambda_{kt} \\ &\text{s.t. (1b), (1c), (1d), (1e), (1f)} \end{aligned} \tag{2}$$

Note that constraints (1b)–(1f) can now be split by index $h$ giving rise to a reformulation of the relaxed problem $P'$ into $|H|$ subproblems as follows:

$$P'_h : L_h(\lambda) = \max_x \left( \sum_{(k,q) \in KQ} \sum_{t \in T} \sum_{j \in J} (b_{kqh}^{tj} - \lambda_{kt}) x_{kqh}^{tj} \right) \tag{3}$$

$$\text{s.t.} \quad \sum_{t \in T} x_{kqh}^{tj} \le 1 \quad (k, q) \in KQ, j \in J \tag{3a}$$

$$x_{kqh}^{tj} \le x_{kqh}^{t-1, j-1} \quad (k, q) \in KQ, t \in T, j \in J \setminus \{1\} \tag{3b}$$

$$\sum_{(k,q) \in KQ} \sum_{j \in J} x_{kqh}^{tj} \le 1 \quad t \in T \tag{3c}$$

$$\sum_{(k,q) \in KQ} \sum_{j \in J} \sum_{t \in T} c_{kq} \rho(j) x_{kqh}^{tj} \le C_{qh} \quad q \in Q \tag{3d}$$

$$x_{kqh}^{tj} \in \{0, 1\} \quad (k, q) \in KQ, j \in J, t \in T \tag{3e}$$

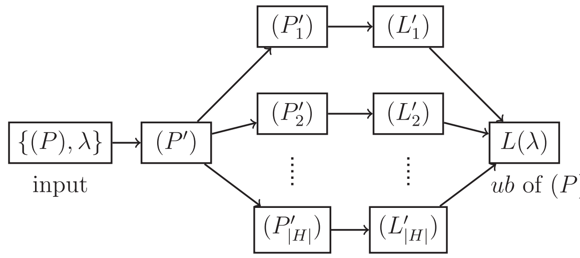


**Fig. 1.** Decomposition into subproblems by using Lagrangian relaxation.

Thus, solving each problem $P'_h$ independently, $L(\lambda)$ can be obtained through

$$L(\lambda) = \sum_{h \in H} L_h(\lambda) + \sum_{k \in K}\sum_{t \in T} \lambda_{kt}$$

This scheme is illustrated in Fig. 1.

### 4.2. Lagrangian relaxation decomposing by index set K

Problem $P$ admits another useful Lagrangian relaxation decomposing by the index set $K$. Relaxing constraints (1d) and (1e), we obtain $P''$ the Lagrangian relaxation of $P$ for a given $(\lambda, \mu)$:

$$\begin{aligned} P'' : L(\lambda,\mu) = \max_x \Bigg( & \sum_{(k,q) \in KQ}\sum_{h \in H}\sum_{t \in T}\sum_{j \in J} b^{tj}_{kqh} x^{tj}_{kqh} + \\ & + \sum_{h \in H}\sum_{t \in T} \lambda_{ht}\Big(1 - \sum_{(k,q) \in KQ}\sum_{j \in J} x^{tj}_{kqh}\Big) + \sum_{q \in Q}\sum_{h \in H} \mu_{qh}\Big(C_{qh} - \sum_{(k,q) \in KQ}\sum_{t \in T}\sum_{j \in J} c_{kq}\rho(j) x^{tj}_{kqh}\Big)\Bigg) \\ = \max_x \sum_{k \in K} \Bigg( & \sum_{(k,q) \in KQ}\sum_{h \in H}\sum_{t \in T}\sum_{j \in J} b^{tj}_{kqh} x^{tj}_{kqh} - \sum_{h \in H}\sum_{t \in T} \lambda_{ht} \sum_{(k,q) \in KQ}\sum_{j \in J} x^{tj}_{kqh} - \\ & - \sum_{h \in H}\sum_{(k,q) \in KQ}\sum_{t \in T}\sum_{j \in J} \mu_{qh} c_{kq}\rho(j) x^{tj}_{kqh}\Bigg) + \sum_{h \in H}\sum_{t \in T} \lambda_{ht} + \sum_{q \in Q}\sum_{h \in H} \mu_{qh} C_{qh} \\ & \text{s.t. } (1a),\ (1b),\ (1c),\ (1f) \end{aligned}$$

Note that constraints (1a)–(1c) can now be split by index $k$ giving rise to a reformulation of the relaxed problem $P''$ into $|K|$ subproblems as follows:

$$P''_k : L_k(\lambda,\mu) = \max_x \Bigg( \sum_{(k,q) \in KQ}\sum_{h \in H}\sum_{t \in T}\sum_{j \in J} (b^{tj}_{kqh} - \lambda_{ht} - \mu_{qh} c^j_k \rho(j)) x^{tj}_{kqh} \Bigg) \tag{4}$$

$$\text{s.t.} \quad \sum_{(k,q) \in KQ}\sum_{h \in H}\sum_{j \in J} x^{tj}_{kqh} \le 1 \quad t \in T \tag{4a}$$

$$\sum_{t \in T} x^{tj}_{kqh} \le 1 \quad h \in H, j \in J \tag{4b}$$

$$x^{tj}_{kqh} \le x^{t-1,j-1}_{kqh} \quad h \in H, t \in T, j \in J \setminus \{1\} \tag{4c}$$

$$x^{tj}_{kqh} \in \{0,1\} \quad (k,q) \in KQ, h \in H, t \in T, j \in J \tag{4d}$$

Analogously, solving each problem $P''_k$ independently, value $L(\lambda,\mu)$ can be obtained through

$$L(\lambda,\mu) = \sum_{k \in K} L_k(\lambda,\mu) + \sum_{h \in H}\sum_{t \in T} \lambda_{ht} + \sum_{q \in Q}\sum_{h \in H} \mu_{qh} C_{qh}$$

From our experience, decomposing problem $P$ by sites ($H$) has always worked better than by agents ($K$). For this reason, in the rest of the paper we will restrict ourselves to report our results based on this approach, namely we will only focus on different decompositions based on sites.

### 4.3. Solving the Lagrangian dual by subgradient optimization

It is well known that $L(\lambda)$ is a continuous concave function over its domain (regardless of the cost structure and constraints of $P$) but non-differentiable due to its piecewise linearity. In this situation, a successful technique for solving the Lagrangian dual problem is subgradient optimization (see [22] for a complete description of this algorithm). This method iteratively adjusts the Lagrangian multipliers in order to produce the best or nearly the best upper bound (in the case of a maximization problem).

The subgradient optimization method can be basically described as follows. Given an initial value of the solution $\tilde{X}^{(0)}$, the associated initial Lagrangian dual $L^{*(0)}$ and initial multipliers ($\lambda^0$), the subgradient algorithm consists basically of an iterated 3-steps approach:

1. Determine the subgradient direction $s^{(n)}$ of $L(\lambda^{(n-1)})$ for a given $\tilde{X}^{(n-1)}$: $s^{(n)} = \|\lambda\|^{-1}\partial L(\lambda^{(n-1)})/\partial\lambda$ (where $\partial a/\partial b$ denotes the differential of expression $a$ with respect to variable $b$).
2. Update Lagrangian multipliers by using $s^{(n)}$: $\lambda^{(n)} = \lambda^{(n-1)} - \alpha^{(n)}s^{(n)}$, where $\alpha^{(n)}$ is the step size.
3. Approximate $L^{*(n)} \simeq L(\lambda^{(n)})$ and check termination conditions.

There are many possible variants to calculate the step size and termination conditions (e.g. [2]). According to the characteristics of our problem, we have observed a good performance in the algorithm when the step size is chosen as follows:

$$\alpha^{(n)} = \max\left\{\overline{\alpha}, \frac{L^{*(0)} - L(\lambda^{(n)})}{\|b - A\tilde{X}^{(n)}\|^2}\vartheta^{(n)}\right\},$$

where $\vartheta^{(n)}$ is determined as

$$\vartheta^{(n)} = \begin{cases} 2 & \text{if } n = 0 \\ \vartheta^{(n-1)} & \text{if } L(\lambda^{(n)}) < L(\lambda^{(n-1)}) \\ \frac{\vartheta^{(n-1)}}{2} & \text{if } L(\lambda^{(n)}) \geq L(\lambda^{(n-1)}) \end{cases}$$

and $\overline{\alpha}$ is an upper bound of $\alpha$ that prevents oscillating jumps. In particular we have selected to stop the subgradient after one hour of CPLEX CPU-time in solving subproblems. Algorithm 3 shows this implementation when the problem is split by sites.

**Algorithm 3.** Sequential subgradient method for each $h \in H$.

**input**: Instance of $P$
**output**: Infeasible solution $(\overline{X}, \overline{z})$ (upper bound)
Determination of initial values $X^{(0)}$ // e.g. using the sequential method
Determination of initial values $\lambda^{(0)}$ // e.g. $\lambda^{(0)}_{kt} = 0, k \in K, t \in T$
**for** $n = 1 : n\_max$ **do** // For a given number $n\_max$ of iterations do...
  Determine the subgradient direction $s^{(n)}$ of $L(\lambda^{(n-1)})$ for a given $\tilde{X}^{(n-1)}$;
  Determine step size $\alpha^{(n)}$;
  Obtain new positive multipliers $\lambda^{(n)} = \max\{0, \lambda^{(n-1)} - \alpha^{(n)}s^{(n)}\}$;
  **for** $h = 1 : |H|$ **do** // Solve $P'$ sequentially for every $h \in H$
    Solve $P'_h$. Keep $\tilde{X}_h$, $L_h(\lambda^{(n)})$;
  Keep $\tilde{X}^{(n)} = \{\tilde{X}_1, \ldots \tilde{X}_{|H|}\}$, $L^{*(n)} = \sum_{h \in H} L_h(\lambda^{(n)}) + \sum_{k \in K}\sum_{t \in T}\lambda_{kt}$;
Return the final (infeasible) solution $\overline{X} = \tilde{X}^{(n)}$ and the objective value $\overline{z} = L^{*(n)}$;

### 4.4. Repairing algorithm

Any solution returned by the subgradient algorithm provides an upper bound of the problem but does not guarantee a feasible solution. According to the constraints relaxed in the Lagrangian relaxation decomposing $H$, a solution $(\overline{X}, \overline{z})$ returned by Algorithm 3, infeasible for $P$, satisfies the following property:

$$\exists k \in K, t \in T / \sum_{(k,q) \in KQ} \sum_{h \in H} \sum_{j \in J} x^{tj}_{kqh} > 1. \quad (5)$$

We use the infeasible solution provided by the subgradient algorithm in a constructive heuristic. Our approach consists in two phases. In a first step, we set to zero all variables that fulfil condition (5), i.e., we transform $(\overline{X}, \overline{z})$ in an integer feasible solution. Then, we take advantage of the application of our SMS algorithm to build a new feasible solution $(\tilde{X}, \tilde{z})$ upon the former one. Algorithm 4 describes the above process. This algorithm repairs and improves all infeasibilities from a solution $(\overline{X}, \overline{z})$ but does not guarantee that the new solution $(\tilde{X}, \tilde{z})$ is optimal in the original problem $P$.

**Algorithm 4.** Repairing algorithm.

**input** Infeasible solution $(\overline{X}, \overline{z})$
**output** Feasible solution $(\tilde{X}, \tilde{z})$
**for** *each* $k \in K, t \in T$ **do**
  Set to zero all $x^{tj}_{kqh} \in \overline{X} / \sum_{(k,q) \in KQ}\sum_{h \in H}\sum_{j \in J} x^{tj}_{kqh} > 1$;
Sort $H$ according to $\sigma_h$;
**for** $h = 1 : |H|$ **do** // Solve sequentially for every $h \in H$
  **for** $h' = 1 : h - 1$ **do** // Fix to 0 each $(\tilde{k}, \tilde{q}, \tilde{t})$ previously selected
    **for** $\tilde{x} \in \tilde{X}_{h'} / \tilde{x} = 1$ **do** $x^*(\tilde{k}, \tilde{q}, h, \tilde{t}, j) = 0,\ j \in J$;
  **for** $x \in \tilde{X}_h / \tilde{x} = 1$ **do** $x^*(\tilde{k}, \tilde{q}, \tilde{h}, \tilde{t}, \tilde{j}) = 1$; // Fix to 1 each $(\tilde{k}, \tilde{q}, \tilde{h}, \tilde{t}, \tilde{j})$ of $\tilde{X}_h$;
  Solve $P$ for $H = \{h\}$ fixing $X^*$. Keep $\tilde{X}_h$, $\tilde{z}_h$;

Returnthe final solution $\tilde{X} = \{\tilde{X}_1, \ldots \tilde{X}_{|H|}\}$ and the objective value $\tilde{z} = \sum_{h \in H} \tilde{z}_h$;

## 5. A multi-start greedy algorithm

In this section we present another family of local search algorithms that could be used to contrast/validate the previous solution methods. Our approach is a Multi-Start Greedy Algorithm (MSGA), along the lines of Caramia et al. [6].

The presented algorithm has three building blocks: a *greedy_scheduler* to find "fast" solutions, an *objective_improver* to improve the objective function in a local search area, and a *improver_trader* to improve the objective function by increasing the search area. We recall that the SMS algorithm decomposes the problem $P$ into $|H|$ subproblems that are solved sequentially incorporating adequately the information obtained in the subproblems already solved. The SMS requires a ranking of sites used to solve the subproblems in such order. We have used in Section 3 the ranking given by the attractiveness factor of the sites in non-increasing order (in the following we will denote this ranking "attractiveness") but we will use another ranking provided by the contribution of each municipality to the objective function (in the following we will denote this ranking as "contribution"). The basis of the MSGA is to find the best order possible in which municipalities have to be sorted in order to achieve the best possible objective function value.

Let $L$ be the list of sites initially ranked by attractiveness. Once a feasible solution is obtained by using the SMS (*greedy_scheduler*), we perform local search (*objective_improver*) trying to improve the objective by re-sorting parts of $L$ (of size $m$) according to the ranking contribution. We repeat this re-sorting until no improvement is achieved or no more consecutive parts of $L$ can be sorted. Then we repeat the process calling again *greedy_scheduler* in order to generate another greedy solution by solving the SMS fixing a percentage $d$ of $x$ variables to zero. Generically, after a call to *greedy_scheduler*, if the objective value cannot be improved by *objective_improver* we call *improver_trader* (if a number of iterations *iter_max* have not been exceeded) in order to provide an oscillation in the search increasing the size of $m$. Algorithm 5 presents the pseudocode of the MSGA.

**Algorithm 5.** (MSGA) Multi-start greedy algorithm.

**input**: Instance of $P$, *greedy_scheduler* diversity $d$, *objective_improver* intensity $m$
**output**: Feasible solution $(\tilde{X}, \tilde{z})$ (lower bound)
$X^* = \{\}$, $l = 0$, $iter = 0$;
**while** $iter < iter\_max$ **do** // `improver_trader`
  **while** $l > 1$ **do** // `greedy_scheduler`
    $improvement = true$, $l = 0$;
    **while** $(improvement = true)$ *and* $(l \leq m)$ **do** // `objective_improver`
      Solve the SMS fixing $X^*$;
      Update $\{\text{best}(\tilde{X}, \tilde{z}),\ improvement,\ iter{:=}iter+1,\ l{:=}l+1\}$; of $L$ by contribution;
      Sort elements $\left\{1 + (l-1)\lceil\frac{|H|}{m}\rceil, \ldots, \min\left\{|H|, l\lceil\frac{|H|}{m}\rceil\right\}\right\}$
    Define $X^*$ as $\tilde{X}$ fixing to zero a percentage $d$ of $x$ variables;
  $m = m + 1$;
Return the final solution $\tilde{X}$ and the objective value $z$;

## 6. Computational results

Formulation $P$ is complex due essentially to the interaction of 5 different factors on the decision process, namely sites, agents, modalities, repetitions and time periods. These elements give rise to an integer programming formulation with 5-index variables.

First of all, we would like to evaluate the behavior of this model. For this purpose, we have tested it in a set of random instances taking values of $|K| = \{30, 60, 80\}$, $|H| = \{25, 45, 65, 85\}$ and $|T| = \{30, 60, 120, 240\}$. For simplicity, we have selected just one modality for each agent ($|Q| = 1$) and a maximum of 2 events in a site by each agent, with discount factors $\rho(1) = 1$, $\rho(2) = 0.8$. Then, we have generated our data according to $\omega_{kq} = \alpha(0.10, 0.30)$, $\sigma_h = \alpha(20, 40)$ and $p^t_{kqh} = \alpha(0.5, 1)$, where $\alpha(a_1, a_2)$ denotes a random number drawn in the interval $[a_1, a_2]$. Additionally, it is widely agreed, observed from actual data, that Fridays and Saturdays are commonly chosen for performances, so the calendar factor has been heterogeneously generated like $\theta_t = \alpha(0.3, 0.5)\theta$, where $\theta$ is equal to 0.7 on Fridays and Saturdays, and equal to 0.2 otherwise. Note that these random values play the same role in other scales since they only appear in the multiplicative expression $b^{tj}_{kqh}$. Finally, we assume that the cost required to choose an agent is estimated as $c_{kq} = 100\omega_{kq}$, and the total budget of a site $h$ for modality $q$ is equal to $C_{qh} = 350$ (this would allow each site to choose a maximum number of agents between 11 and 35).

We have generated 5 instances for each combination of $\{|H|, |T|, |K|\}$, according to the above design. Table 1 reports for each row average relative gaps ($\overline{gap}\%$), maximum relative gaps ($gap^*\%$), average running times ($\overline{sec}$) and maximum running times ($sec^*$), in seconds. All instances have been solved using ILOG CPLEX 11.2.1 on a personal computer with an Intel(R) Core(TM)i7 CPU 2.93 GHz processor and 8 GB RAM, setting all CPLEX parameters to default values (after several preliminary tests the best performance was obtained leaving CPLEX parameters at their default values). Instances have been solved until reaching a relative gap lower than 0.01% or a maximum running time (marked with "*time*") of 3600 s. Those instances where CPLEX runs out of memory loading the problem have been marked with a dash (–). We note that the average running time was computed considering a time limit of 3600 s for those instances that could not be solved to optimality.

From the results of Table 1 we observe that the main drawback of the CPLEX exact method is memory limitation specially in sizes $|K| \geq 60$, $|T| \geq 240$ where no instance could be even loaded in memory. We remark that for the remaining combinations of parameters no

**Table 1**
Results obtained for a time limit (*time*) of 3600 s.

| Instances | | $\lvert K\rvert = 30$ | | | | $\lvert K\rvert = 60$ | | | | $\lvert K\rvert = 80$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lvert H\rvert$ | $\lvert T\rvert$ | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% |
| 25 | 30 | *time* | *time* | 0.44 | 0.55 | *time* | *time* | 0.17 | 0.23 | *time* | *time* | 0.13 | 0.17 |
| 45 | 30 | *time* | *time* | 1.66 | 1.87 | *time* | *time* | 0.75 | 1.11 | *time* | *time* | 0.79 | 0.97 |
| 65 | 30 | *time* | *time* | 0.85 | 0.95 | *time* | *time* | 1.48 | 1.77 | *time* | *time* | 0.99 | 1.38 |
| 85 | 30 | *time* | *time* | 0.89 | 1.09 | *time* | *time* | 3.35 | 4.45 | *time* | *time* | 3.51 | 4.61 |
| 25 | 60 | *time* | *time* | 0.78 | 1.06 | *time* | *time* | 0.77 | 0.92 | *time* | *time* | 0.49 | 0.58 |
| 45 | 60 | *time* | *time* | 2.09 | 2.70 | *time* | *time* | 1.81 | 3.16 | *time* | *time* | 1.74 | 2.89 |
| 65 | 60 | *time* | *time* | 3.76 | 6.09 | *time* | *time* | 4.44 | 5.07 | *time* | *time* | 4.31 | 4.61 |
| 85 | 60 | *time* | *time* | 4.51 | 5.11 | *time* | *time* | 3.18 | 3.68 | *time* | *time* | 4.40 | 4.89 |
| 25 | 120 | *time* | *time* | 1.03 | 1.22 | *time* | *time* | 0.64 | 0.83 | 3520 | *time* | 0.52 | 0.65 |
| 45 | 120 | *time* | *time* | 1.79 | 2.79 | *time* | *time* | 1.43 | 2.32 | 3044 | *time* | 1.37 | 1.65 |
| 65 | 120 | *time* | *time* | 4.01 | 4.50 | – | – | – | – | *time* | *time* | 1.74 | 2.13 |
| 85 | 120 | – | – | – | – | – | – | – | – | – | – | – | – |
| 25 | 240 | *time* | *time* | 1.13 | 1.32 | – | – | – | – | – | – | – | – |
| 45 | 240 | *time* | *time* | 2.71 | 3.29 | – | – | – | – | – | – | – | – |
| 65 | 240 | *time* | *time* | 2.75 | 3.00 | – | – | – | – | – | – | – | – |
| 85 | 240 | – | – | – | – | – | – | – | – | – | – | – | – |

**Table 2**
Results obtained for the sequential method.

| Instances | | $\lvert K\rvert = 30$ | | | | $\lvert K\rvert = 60$ | | | | $\lvert K\rvert = 80$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lvert H\rvert$ | $\lvert T\rvert$ | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% |
| 25 | 30 | **4.8** | **7** | 2.65 | 3.42 | **6.8** | **8** | 1.23 | 1.50 | **23.2** | **80** | 0.66 | 1.19 |
| 45 | 30 | **8.8** | **14** | 5.40 | 5.92 | **11.2** | **14** | 2.17 | 2.72 | **14.0** | **18** | 1.55 | 1.93 |
| 65 | 30 | **6.8** | **10** | 5.44 | 5.69 | **14.0** | **15** | 3.16 | 3.79 | **19.6** | **28** | 2.24 | 2.31 |
| 85 | 30 | **5.4** | **7** | 5.12 | 6.23 | **16.2** | **20** | 4.11 | **4.36** | **22.8** | **28** | **2.93** | **3.09** |
| 25 | 60 | **6.4** | **9** | 2.56 | 2.97 | **9.2** | **10** | **0.42** | **0.51** | **12.4** | **14** | **0.34** | **0.38** |
| 45 | 60 | **24.2** | **66** | 4.23 | 5.15 | **15.6** | **18** | **1.64** | **2.01** | **20.6** | **24** | **0.76** | **0.88** |
| 65 | 60 | **23.2** | **30** | 4.42 | **4.88** | **52.6** | **108** | **2.83** | **3.36** | **28.8** | **33** | **1.75** | **1.98** |
| 85 | 60 | **26.0** | **52** | **4.42** | **4.91** | **64.0** | **104** | 3.46 | 3.93 | **104.2** | **356** | **2.32** | **2.65** |
| 25 | 120 | **13.6** | **19** | **0.52** | **0.58** | **158.2** | **343** | **0.21** | **0.27** | **40.0** | **59** | **0.15** | **0.21** |
| 45 | 120 | **27.2** | **50** | **1.68** | **1.95** | **56.2** | **90** | **0.44** | **0.47** | **190.0** | **399** | **0.33** | **0.50** |
| 65 | 120 | **58.0** | **101** | **1.58** | **1.64** | **82.2** | **143** | – | – | **72.6** | **79** | **0.51** | **0.58** |
| 85 | 120 | **140.4** | **440** | – | – | **108.4** | **217** | – | – | **99.6** | **129** | – | – |
| 25 | 240 | **127.2** | **239** | **0.33** | **0.38** | **851.2** | **3148** | – | – | **491.8** | **815** | – | – |
| 45 | 240 | **245.0** | **427** | **0.57** | **0.66** | **695.8** | **1403** | – | – | **494.4** | **939** | – | – |
| 65 | 240 | **209.0** | **421** | **0.91** | **1.01** | **1063.0** | **1750** | – | – | **926.0** | **1969** | – | – |
| 85 | 240 | **218.4** | **452** | – | – | **829.6** | **1644** | – | – | **1007.4** | **1559** | – | – |

instance could be solved to optimality, with the exception of the groups ($|H| = 25, |T| = 120, |K| = |80|$) and ($|H| = 45, |T| = 120, |K| = |80|$) where some instances were solved which can be observed because we report average running times smaller than 3600 s.

Table 2 shows results obtained for the SMS using the same instances as in Section 6. Note that even solving each subproblem of the SMS to optimality, all instances have been solved for computational times significantly smaller than those reported in Table 1. In addition, we mark in bold all items (times, gaps) that outperform those obtained with the CPLEX exact method. We do not report gaps for those cases where CPLEX was not able to solve the corresponding instance (see Table 1) since we do not have another valid upper bound to make a comparison. From the results in Table 2, we observe that the sequential method is fast and it provides reasonably good feasible solutions with average gaps (w.r.t. the optimal known solutions) around 2%.

Table 3 shows an updated version of Table 2 by using the best upper bounds obtained between the one provided by CPLEX and the one provided by the subgradient algorithm. We mark in italics (also in the following tables) those items that were as good as the best obtained so far with previous methods. In all our experiments, we have stopped the subgradient algorithm after 1 h of CPLEX CPU-time in solving subproblems. In order to calibrate the stepsize we have used the same instances as for the exact and sequential methods. As we can see in Table 3, the average gap is around 2.5%. This means that the quality of the upper bounds obtained by the subgradient algorithm is quite good specially because in all cases we are computing the gaps with respect to the lower bounds provided by the sequential method.

Table 4 shows relative and maximum gaps obtained from the best upper bounds between the subgradient algorithm and the CPLEX solution method, and lower bounds from the repairing algorithm. In addition, we also report running times of the repairing routine. Note that in most instances, duality gaps are smaller than those calculated using the sequential method. Furthermore, we have observed that with longer running times to compute the upper bounds of the subgradient algorithm, the repairing strategy maybe improves the sequential method in all instances. From this computational experience we conclude that in general, the repairing algorithm outperforms the sequential one, but it would need longer running times than 1 h of CPLEX-cpu time for the subgradient routine, in order to get the improvement in some difficult or big size instances. Therefore it is a very competitive method to solve problem $P$ since (1) it allows solving instances that general solvers such as CPLEX cannot even load in memory and (2) the gaps reported in our computational experiments validate its usage as a good approximation of the optimal solution.

**Table 3**
Computational results for the subgradient optimization.

| Instances | | $\|K\|=30$ | | | | $\|K\|=60$ | | | | $\|K\|=80$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|H\|$ | $\|T\|$ | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% |
| 25 | 30 | 4.8 | 7 | 2.65 | 3.42 | 6.8 | 8 | 1.23 | 1.50 | 23.2 | 80 | 0.66 | 1.19 |
| 45 | 30 | 8.8 | 14 | 5.40 | 5.92 | 11.2 | 14 | 2.17 | 2.72 | 14.0 | 18 | 1.55 | 1.93 |
| 65 | 30 | 6.8 | 10 | 5.44 | 5.69 | 14.0 | 15 | 3.16 | 3.79 | 19.6 | 28 | 2.24 | 2.31 |
| 85 | 30 | 5.4 | 7 | 5.12 | 6.23 | 16.2 | 20 | 4.11 | 4.36 | 22.8 | 28 | 2.93 | 3.09 |
| 25 | 60 | 6.4 | 9 | 2.56 | 2.97 | 9.2 | 10 | **0.33** | **0.40** | 12.4 | 14.0 | **0.27** | **0.33** |
| 45 | 60 | 24.2 | 66 | 4.23 | 5.15 | 15.6 | 18 | 1.64 | 2.01 | 20.6 | 24 | **0.73** | 0.88 |
| 65 | 60 | 23.2 | 30 | 4.42 | 4.88 | 52.6 | 108 | 2.83 | 3.36 | 28.8 | 33 | 1.75 | 1.98 |
| 85 | 60 | 26.0 | 52 | 4.42 | 4.91 | 64.0 | 104 | 3.46 | 3.93 | 104.2 | 356 | 2.32 | 2.65 |
| 25 | 120 | 13.6 | 19 | **0.43** | **0.53** | 158.2 | 343 | **0.12** | **0.16** | 40.0 | 59 | **0.09** | **0.10** |
| 45 | 120 | 27.2 | 50 | 1.68 | 1.95 | 56.2 | 90 | 0.44 | 0.47 | 190.0 | 399 | 0.33 | 0.50 |
| 65 | 120 | 58.0 | 101 | 1.58 | 1.64 | 82.2 | 143 | **1.58** | **2.18** | 72.6 | 79 | 0.51 | 0.58 |
| 85 | 120 | 140.4 | 440 | **7.30** | **8.15** | 108.4 | 217 | **3.13** | **3.73** | 99.6 | 129 | **1.73** | **1.99** |
| 25 | 240 | 127.2 | 239 | 0.33 | 0.38 | 851.2 | 3148 | **0.11** | **0.17** | 491.8 | 815 | **0.05** | **0.06** |
| 45 | 240 | 245.0 | 427 | 0.57 | 0.66 | 695.8 | 1403 | **0.57** | **0.72** | 494.4 | 939 | **0.23** | **0.30** |
| 65 | 240 | 209.0 | 421 | 0.91 | 1.01 | 1063.0 | 1750 | **1.49** | **1.78** | 926.0 | 1969 | **0.87** | **1.06** |
| 85 | 240 | 218.4 | 452 | **5.61** | **7.20** | 829.6 | 1644 | **2.85** | **3.62** | 1007.4 | 1559 | **1.42** | **1.66** |

**Table 4**
Computational results for the repairing algorithm.

| Instances | | $\|K\|=30$ | | | | $\|K\|=60$ | | | | $\|K\|=80$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|H\|$ | $\|T\|$ | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% |
| 25 | 30 | **2.2** | **3** | 2.20 | 3.08 | **3.0** | **3** | 1.27 | 1.53 | **3.4** | **4** | 0.96 | 1.21 |
| 45 | 30 | **3.2** | **4** | 2.34 | 3.15 | **5.0** | **5** | 1.58 | 2.01 | **6.0** | **7** | 1.42 | 1.59 |
| 65 | 30 | **2.6** | **3** | 0.91 | 1.49 | **8.4** | **9** | 1.65 | 1.88 | **9.6** | **14** | 1.64 | 1.98 |
| 85 | 30 | **3.0** | **4** | 1.29 | 1.75 | **11.4** | **12** | **2.43** | **2.76** | **14.2** | **17** | **1.63** | **1.84** |
| 25 | 60 | **3.0** | **3** | 2.52 | 2.70 | **5.4** | **6** | 0.68 | 0.84 | **6.6** | **7** | 0.59 | 0.67 |
| 45 | 60 | **8.4** | **14** | 2.42 | 3.08 | **11.0** | **13** | **1.47** | **1.73** | **13.4** | **15** | 0.90 | 1.06 |
| 65 | 60 | **16.4** | **19** | **2.25** | **3.18** | **31.0** | **82** | **1.31** | **1.98** | **22.0** | **24** | **1.34** | **1.52** |
| 85 | 60 | **16.6** | **22** | **2.73** | **3.29** | **32.0** | **39** | **2.57** | **2.98** | **37.2** | **45** | **1.29** | **1.65** |
| 25 | 120 | **6.2** | **7** | 0.87 | 1.09 | **13.0** | **16** | 0.36 | 0.45 | **15.8** | **18** | 0.34 | 0.40 |
| 45 | 120 | **15.2** | **16** | 1.71 | **1.93** | **27.6** | **29** | 0.60 | 0.72 | **40.6** | **60** | 0.53 | 0.72 |
| 65 | 120 | **30.6** | **40** | **1.23** | 1.69 | **54.4** | **88** | 1.64 | 2.23 | **51.4** | **55** | 0.64 | 0.72 |
| 85 | 120 | **55.0** | **125** | **6.99** | **7.95** | **69.2** | **91** | 3.16 | 3.82 | **84.4** | **116** | 1.74 | 2.02 |
| 25 | 240 | **21.4** | **35** | 0.53 | 0.74 | **79.0** | **149** | 0.21 | 0.34 | **39.6** | **48** | 0.15 | 0.24 |
| 45 | 240 | **123.4** | **345** | 0.68 | 0.76 | **110.6** | **219** | 0.66 | 0.82 | **120.4** | **244** | 0.32 | 0.38 |
| 65 | 240 | **73.2** | **92** | 0.96 | 1.05 | **347.0** | **607** | 1.55 | 1.83 | **323.0** | **572** | 0.96 | 1.15 |
| 85 | 240 | **200.0** | **365** | 5.67 | 7.30 | **638.0** | **1599** | 2.90 | 3.64 | **492.2** | **784** | 1.46 | 1.71 |

Table 5 shows relative and maximum gaps obtained from the best upper bounds between the subgradient algorithm and the CPLEX solution method and lower bounds from the repairing algorithm. We have calibrated the MSGA parameters according to a set of preliminary tests over the previous instances. In our study, we consider $m=3$, $d=15$ and $iter\_max=6$. Since now we have to call the SMS several times, we have established a time limit of 10 s to solve each subproblem of the sequential method. In most cases the optimal solution of the subproblem is still found even though the associate gap cannot always be closed. Note that in most instances gaps are at least as good as those obtained by the SMS and in several cases improved. However, only few items (in bold) outperform the previous results. This shows that the MSGA is an effective approach to improve the SMS at the price of increasing the computational burden.

### 6.1. *Application to Andalusia*

#### 6.1.1. *Andalusian framework*

In addition to the general framework for the planning process of cultural events and the computational tests performed, we present one particular instance based upon actual data gathered in Andalusia (Spain) on the season 2009–2010. Moreover, we would like to assess the solution that was actually implemented on that season to evaluate the overall welfare obtained by that solution as compared with ours.

In Andalusia, the region south of mainland Spain with an overall population of more than 8 million inhabitants, there are about 100 publicly owned theaters that provide service in the municipalities where they are located (scenic rooms for carrying out educational activities, music concerts, and performances of theater and dance).

Since its start in 2006, in Andalusia there exist four thematic cultural circuits (Theater and Dance, Music, Cinema and ABC of educational content), also called modalities, which represent an instrument for the democratization of Andalusian culture with the primary objective that culture reaches all parts of the region, as established by the Spanish Constitution and the Andalusian laws. The municipalities are free to decide if their theaters must join these circuits in order to guarantee a series of advisory services and cost sharing of expenses for holding events. Fig. 2 illustrates the distribution map of theaters which form the basis of the so-called Cultural Circuits.

**Table 5**
Computational results for the multi-start greedy algorithm.

| Instances | | $\|K\| = 30$ | | | | $\|K\| = 60$ | | | | $\|K\| = 80$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|H\|$ | $\|T\|$ | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% | $\overline{sec}$ | $sec^*$ | $\overline{gap}$% | $gap^*$% |
| 25 | 30 | 100.6 | 229 | 2.50 | 3.01 | 72.7 | 76 | 1.23 | 1.50 | 122.4 | 220 | 0.64 | 1.14 |
| 45 | 30 | 111.0 | 180 | 5.26 | 5.92 | 121.5 | 167 | 2.12 | 2.45 | 155.3 | 210 | 1.51 | 1.89 |
| 65 | 30 | 143.0 | 173 | 4.92 | 5.19 | 138.0 | 181 | 3.15 | 3.79 | 192.7 | 234 | 2.22 | 2.29 |
| 85 | 30 | 120.3 | 162 | 4.80 | 5.55 | 155.5 | 191 | 3.94 | 4.24 | 206.6 | 209 | 2.93 | 3.09 |
| 25 | 60 | 80.4 | 87 | 2.42 | 2.85 | 110.6 | 129 | **0.31** | **0.37** | 164.7 | 274 | **0.26** | **0.31** |
| 45 | 60 | 105.7 | 108 | 4.11 | 5.15 | 156.2 | 168 | 1.61 | 1.91 | 242.9 | 352 | *0.73* | *0.88* |
| 65 | 60 | 131.1 | 145 | 4.20 | 4.55 | 201.5 | 213 | 2.79 | 3.29 | 292.5 | 302 | 1.75 | 1.98 |
| 85 | 60 | 148.3 | 165 | 4.26 | 4.91 | 244.0 | 256 | 3.42 | 3.93 | 339.9 | 351 | 2.29 | 2.65 |
| 25 | 120 | 281.9 | 386 | *0.43* | *0.53* | 441.5 | 645 | *0.12* | *0.16* | 381.3 | 503 | *0.09* | *0.10* |
| 45 | 120 | 425.1 | 605 | **1.65** | 1.95 | 641.1 | 757 | **0.43** | *0.47* | 677.0 | 918 | *0.33* | *0.50* |
| 65 | 120 | 677.5 | 929 | 1.57 | *1.64* | 762.7 | 971 | *1.58* | *2.18* | 772.7 | 873 | *0.51* | *0.58* |
| 85 | 120 | 623.1 | 789 | 7.30 | 8.15 | 771.0 | 997 | *3.13* | *3.73* | 848.2 | 927 | *1.73* | *1.99* |
| 25 | 240 | 746.0 | 1354 | **0.31** | **0.33** | 826.0 | 1208 | *0.11* | *0.17* | 810.3 | 981 | *0.05* | *0.06* |
| 45 | 240 | 1096.4 | 1300 | *0.57* | *0.66* | 1133.3 | 1199 | *0.57* | *0.72* | 1402.3 | 1655 | *0.23* | *0.30* |
| 65 | 240 | 1010.4 | 1211 | *0.91* | *1.01* | 1725.5 | 2324 | *1.49* | *1.78* | 1841.4 | 2123 | *0.87* | *1.06* |
| 85 | 240 | 1273.5 | 1778 | *5.61* | *7.20* | 1808.4 | 2061 | *2.85* | *3.62* | 2306.1 | 2521 | *1.42* | *1.66* |


**Fig. 2.** Regional networks of Andalusian cultural circuits.

In practice, the activity of the municipal theaters attached to the above-mentioned cultural circuits shows a high dispersion coming into conflict with the initial political objective of achieving equity in the cultural offerings. Fig. 3 shows the number of performance days on each municipality along the Andalusian theatrical season 2009–2010 (note that only some bars have been labeled).

Focusing on the routes that companies carry out annually in Andalusia, we observe the existence of underserved geographic regions due to socio-demographic characteristics of the zones (low population, low income) where the theaters are located. Analyzing why some municipalities do not reach the goal of attracting theater companies to perform, we conclude that the reasons are mainly conflicts between the schedules of artistic groups and municipalities, as well as the resource allocation policy implemented by the municipal managers.

### 6.1.2. Data generation

Data provided by the *Dirección General de Innovación e Industrias Culturales de la Consejería de Cultura* show a complete schedule of that season (1044 performances) that is, groups that made a performance in each municipality on each day of the calendar. On the season 2009–2010 there were 129 groups in the panel and some of them participated in more than one of the three existing modalities: theater and dance, music and ABC. We note that cinema circuit is excluded from this study since it did not require artistic companies and the films of cultural interest were directly provided by the IAAL (Andalusian Institute of Arts and Letters), not by a private company (therefore $Q = \{1, 2, 3\}$). There were 91 municipalities some of them included in several cultural circuits and in principle there were no restrictions on the category of performances that could have been chosen for any of the 365 days of the planning horizon. We also observed that no more

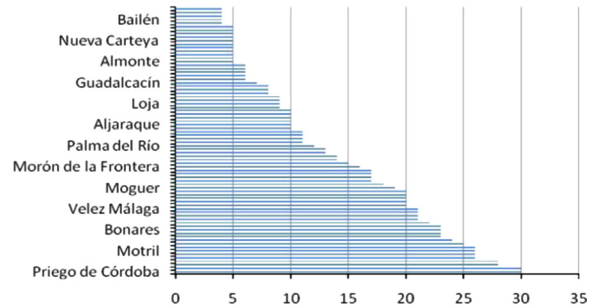


**Fig. 3.** Number of performance days along the Andalusian theatrical season 2009–2010.

**Table 6**
Data associated to solution $\tilde{X}$ of 2009–2010 (1044 performances)[a].

| # | $k$ | $q$ | $h$ | $t$ | $j$ |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 45 | 228 | 1 |
| 2 | 1 | 2 | 59 | 188 | 1 |
| 3 | 2 | 3 | 1 | 96 | 1 |
| 4 | 2 | 3 | 2 | 55 | 1 |
| 5 | 2 | 3 | 12 | 10 | 1 |
| 6 | 2 | 3 | 23 | 54 | 1 |
| 7 | 2 | 3 | 37 | 95 | 1 |
| … | … | … | … | … | … |
| 1039 | 129 | 3 | 56 | 250 | 1 |
| 1040 | 129 | 3 | 59 | 256 | 1 |
| 1041 | 129 | 3 | 64 | 286 | 1 |
| 1042 | 129 | 3 | 69 | 265 | 1 |
| 1043 | 129 | 1 | 77 | 26 | 1 |
| 1044 | 129 | 3 | 87 | 270 | 1 |

[a] *Data source*: Dirección General de Innovación e Industrias Culturales de la Consejería de Cultura of the Regional Government of Andalusia.

than 2 performances ($|J|=2$) on a row of the same group in the same theater were reported in that season. Table 6 shows a piece of the actual cultural agenda (the actual solution $\tilde{X}=\{\tilde{x}_{kqh}^{tj}:(k,q,h,t,j)\text{ is given}\}$) that was active on the season 2009–2010.

The above solution gives rise to the number of performances that were carried out in each municipality for each modality. We know that, during the season 2009–2010, each municipality paid according to the number (absolute) of performances carried out for each modality. Thus, it is reasonable to assume that $C_{qh}=\sum_{(k,q)\in KQ}\sum_{t\in T}\sum_{j\in J}\tilde{x}_{kqh}^{tj}$ and $c_{kq}=1$. That is, in this evaluation we will consider the number of performances carried out on each municipality bounded by those carried out on season 2009–2010. In this way, we will optimize over the quality of choices among days and groups, losing (by lack of data) another source of improvement consisting in increasing the number of performances with a better usage of the available credits. Subsequently, in the application to Andalusia $\rho(j)$ will be assumed constant (equal to 1) for consecutive performances ($j>1$). Moreover, the number of groups that performed more than once was very small (37 out of 1044 performances) and none performed more than twice.

The calendar factor on day $t$ has been obtained from the relative frequency of performances:

$$\theta_t=\frac{\sum_{(k,q)\in KQ}\sum_{h\in H}\sum_{j\in J}\tilde{x}_{kqh}^{tj}}{\sum_{(k,q)\in KQ}\sum_{h\in H}\sum_{j\in J}\sum_{t'\in T}\tilde{x}_{kqh}^{t'j}} \quad (6)$$

This factor has special interest since there is an heterogeneous preference for performances along days of the week and months of the year. As an example, data from season 2009–2010 show that Fridays and Saturdays were the most preferred days, with a percentage of performances of 33.33% and 19.25%, respectively. The month of the year with the most performances was November (22.89%) followed by May (14.08%). However, there is no data about particular preferences among specific groups performing on specific dates in certain municipalities. For this reason, we have assumed equal preferences among groups, municipalities and days, namely $p_{kqh}^{t}=1$.

Finally, we need to evaluate the attractiveness of municipalities and groups. After our experience, it is reasonable to assume $\sigma_h$ equal to the capacity of the theater located in municipality $h$. With respect to groups, we have assigned the attractiveness according to the number of performances that they carried out on the season 2009–2010 weighted with the calendar factor and the municipality attractiveness:

$$\omega_{kq}=\sum_{h\in H}\sum_{t\in T}\sum_{j\in J}\sigma_h\theta_t\tilde{x}_{kqh}^{tj} \quad (7)$$

### 6.1.3. *Alternative solution*

The table included in Fig. 4 reports the improvement in the quality of the solutions regarding data from season 2009–2010. Since absolute values of the solutions do not have a specific meaning (recall that we use *welfare units*) we have compared solutions by means of their relative gaps with respect to the upper bound given by the subgradient algorithm. First, we remark that the actual solution implemented in 2009–2010 has a 55.54% of gap, thus having a lot of room for improvement. Second, any of our methodologies, namely SMS, repairing algorithm or MSGA improve that solution more than 50%, providing in this case, all of them the same cultural agenda in Andalusia in that season. We recall that MSGA always provides a solution at least as good as SMS whereas nothing can be said of the repaired solution compared to MSGA (this can also be seen in Tables 3–5).

Fig. 4 also depicts the results obtained for the eight provinces of Andalusia. Considering the upper bound obtained for the overall welfare, we have calculated which percentage of this amount is received by each one of the eight provinces. As it is shown, the solution provided in 2009 (dark left bars) is significantly lower than the one obtained with MSGA (light right bars). As an example, we have applied a zoom in province of Málaga in order to show the improvement by municipality. For the sake of visibility, bars inside this province have been amplified ten times (percentages remain the same).

Note that the improvement over the solution implemented in 2009–2010 is due to a better allocation among municipalities, companies and days of the calendar. This implies that municipalities now receive companies with a greater pull factor and, simultaneously, the companies operate in municipalities with more demanding population. As a result of the use of the above model, repetition of performances by the same company in the same site will occur only when a suitable calendar, a population mass in the municipality and a degree of attractiveness of the artistic company coincide. In any case, we observe from our results that even without preferences and credit structure the new plan is superior to the one that was actually implemented. Thus, providing a much better access of the population to the cultural activities.

From the above comparisons based on the solution of 2009 and the approach presented in the paper, it can be concluded that optimization techniques allow increasing the efficiency of public resources in terms of welfare. Increasing welfare implies improving production, dissemination, accessibility, marketing and consumption of the arts.

#### 6.1.4. Sensitivity analysis

The solution quality of the proposed method may seem substantially better than the actual implementation since municipalities may have made their decisions based on different goals. However, it is out of the scope of the paper identifying the true objective of each municipality involved in this application. Such effort could be worth in a future step in case of an actual implementation of this methodology. In order to be able to accommodate any goal in our methodology, we have adopted a multiplicative expression of the attractiveness factors which will be valid regardless of the estimation method for the welfare coefficients.

To show how the structure of attractiveness factors affect the quality of solutions, we now provide a sensitivity analysis for the application to Andalusia. We have generated several instances varying, in different ways, the terms involved in the objective function. First, we consider the municipality attractiveness factor $\sigma_h$ according to (1) the capacity of the theater located in municipality $h$ and (2) the number of inhabitants of municipality $h$. Second, with respect to groups, we have assigned the attractiveness $\omega_{kq}$ according to (1) the number of performances that they carried out on the season 2009–2010 weighted with the calendar factor and the municipality attractiveness and (2) the number of performances that they carried out on the season 2009–2010 weighted with the calendar factor. Third, we have considered the attractiveness

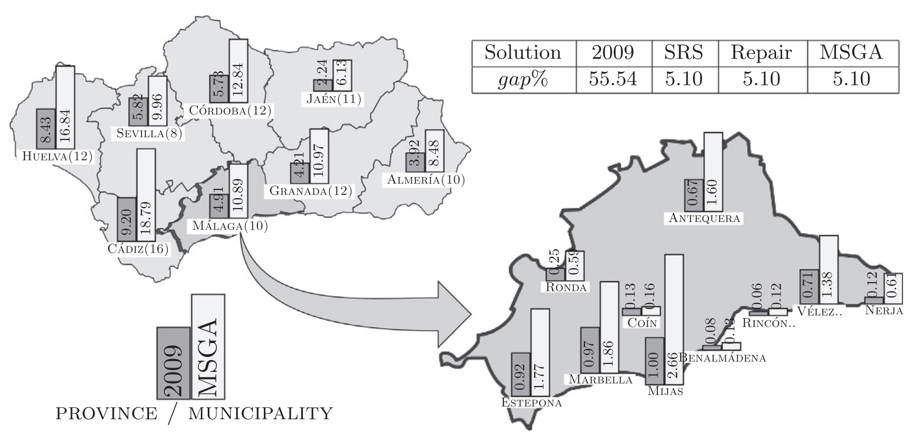


**Fig. 4.** Percentage obtained from the total welfare by each province/municipality.

**Table 7**
Computational results for the sensitivity analysis.

| $\theta_t$ | $\omega_{kq}$ | $\sigma_h$ | Improv. % | | |
|---|---|---|---|---|---|
| | | | SMS | Repair | MSGA |
| $\theta_t^{(1)}$ | $\omega_{kq}^{(1)}$ | $\sigma_h^{(1)}$ | 53.46 | 53.34 | 54.02 |
| $\theta_t^{(1)}$ | $\omega_{kq}^{(1)}$ | $\sigma_h^{(2)}$ | 52.13 | 52.05 | 52.13 |
| $\theta_t^{(1)}$ | $\omega_{kq}^{(2)}$ | $\sigma_h^{(1)}$ | 51.82 | 51.77 | 51.82 |
| $\theta_t^{(1)}$ | $\omega_{kq}^{(2)}$ | $\sigma_h^{(2)}$ | 52.14 | 51.84 | 52.25 |
| $\theta_t^{(2)}$ | $\omega_{kq}^{(1)}$ | $\sigma_h^{(1)}$ | 36.48 | 35.63 | 36.48 |
| $\theta_t^{(2)}$ | $\omega_{kq}^{(1)}$ | $\sigma_h^{(2)}$ | 35.77 | 34.97 | 35.77 |
| $\theta_t^{(2)}$ | $\omega_{kq}^{(2)}$ | $\sigma_h^{(1)}$ | 35.76 | 35.13 | 35.76 |
| $\theta_t^{(2)}$ | $\omega_{kq}^{(2)}$ | $\sigma_h^{(2)}$ | 36.09 | 35.80 | 36.09 |

calendar factor $\theta_t$ defined as in expression (5) and also considering $\theta_t = 1$ for all $t \in T$. All these considerations give rise to 8 different parameters' combinations that we denote (according to the above description) by using $\theta_t^{(1)}, \theta_t^{(2)}, \sigma_h^{(1)}, \sigma_h^{(2)}, \omega_{kq}^{(1)}, \omega_{kq}^{(2)}$ and triplets $\{\theta_t^{(1)}, \omega_{kq}^{(1)}, \sigma_h^{(1)}\}$, $\{\theta_t^{(1)}, \omega_{kq}^{(1)}, \sigma_h^{(2)}\}$, … as it is shown on Table 7. In order to better show how sensitive the solution is to changes in the objective function coefficients, we have redefined the two attractiveness factors including a random value (or noise) as $\sigma_h := \sigma_h + \alpha(\min_{h \in H} \sigma_h, \max_{h \in H} \sigma_h)$, $\omega_{kq} := \omega_{kq} + \alpha(\min_{(k,q) \in KQ} \omega_{kq}, \max_{(k,q) \in KQ} \omega_{kq})$, giving rise to 5 instances per each triplet.

Table 7 shows, for each triplet, the improvement of the SMS, Repair algorithm and MSGA over the solution implemented in season 2009–2010. The reader may note that the percentage of improvement increases when the calendar factor is introduced, as it can be seen in the first four rows of the table. Apart from this, the percentages of improvement obtained are quite similar which lead us to conclude that the estimation method of the benefits does not compromise the validity of the model.

## 7. Conclusions and further research

In this paper we provide a formulation and a solution approach for a decision making problem concerning the planning of cultural schedules. The model maximizes the overall welfare of the entire system by integrating the different parties involved in the process (agents, sites and administration) in a unified setting. Different solution methods have been developed and tested over a testbed of random instances to analyze their computational performance, showing promising results. In addition, we present one particular instance based upon actual data gathered in Andalusia (Spain).

Computational results over a testbed of random instances have shown an efficient performance of the proposed methodology. It is important to note that optimization techniques are a powerful tool for increasing the efficiency of public resources, something of special interest when budgetary sources are very limited. A measure of global welfare has been used in order to size the improved production, dissemination, accessibility, marketing and consumption of the arts.

The model developed admits several extensions in order to include different and specific features for the region in study and the cultural schedules in consideration. Two interesting possibilities of further analysis are penalizing transportation costs and/or day/s off after events, and constraining cultural diversity; as described in the following.

Routing costs are not included in the provided model since distances are not significant compared with the benefit of an event. However, we can easily penalize traveling costs denoting by $h' \in \overline{H}_h \subseteq H \setminus \{h\}$, $h \in H$ the set of sites “far” from site $h$ and imposing that an agent $(k,q)$ is forced to rest at least one day between events at sites “far” from each other:

$$\sum_{(k,q) \in KQ} \sum_{j \in J} x_{kqh}^{tj} + \sum_{(k,q) \in KQ} \sum_{h' \in \overline{H}_h} \sum_{j \in J} x_{kqh'}^{(t+1)j} \le 1 \quad k \in K, h \in H, t \in T \setminus \{|T|\} \tag{8}$$

By sorting sites according to some sort of attractiveness criterion (theater capacity, population, etc.), we can select a set of clusters $w \in W$ in order to split set $H$ into a family of subsets $H_w$ of sites with similar attractiveness. An interesting constraint consists in imposing that agents have to perform at sites of different attractiveness, that is, the number of events in a site $h \in H_w$ cannot exceed an upper bound $M_w$:

$$\sum_{h \in H_w} \sum_{t \in T} \sum_{j \in J} x_{kqh}^{tj} \le M_w \quad (k,q) \in KQ, w \in W \tag{9}$$

Clearly, apart from the above-mentioned extensions, this type of models based on mathematical programming formulations allows the incorporation of any further requirements that can be cast into the form of constraints in the model. In addition, it has been clearly identified in the literature the necessity of a better management of cultural policy [8,27,13,17] which makes this model rather useful to be applied in other contexts and countries. A recent directive of the Andalusian Regional Government published in 2013 has launched the program “Enredados”, which will be a new opportunity to apply this methodology in order to improve the management of the cultural policy. These can be topics for a follow up paper.


## Acknowledgments

This work was partially supported by Ministerio de Ciencia e Innovación under Grants MTM2010-19576-C02-01, MTM2013-46962-C2-01 and MTM2012-37048, and by FEDER-Junta de Andalucía under Grant FQM-5849. We gratefully acknowledge the helpful comments of the anonymous referees.


## References


[1] Albareda-Sambola M, Fernández E, Hinojosa Y, Puerto J. The multi-period incremental service facility location problem. Comput Oper Res 2009;36(5):1356–75.
[2] Bazaraa MS, Sherali HD. On the choice of step size in subgradient optimization. Eur J Oper Res 1981;7(4):380–8.
[3] Bean JC, Birge JR. Reducing travelling costs and player fatigue in the national basketball association. Interfaces 1980;10(3):98–102.
[4] Blanco V, Carpente L, Hinojosa Y, Puerto J. Planning for agricultural forage harvesters and trucks: model, heuristics, and case study. Netw Spat Econ 2010;10:321–43.
[5] Burkard R, Dell'Amico M, Martello S. Assignment Problems. Philadelphia, PA, USA: Society for Industrial and Applied Mathematics; 2009.
[6] Caramia M, Dell'Olmo P, Italiano GF. Novel local-search-based approaches to university examination timetabling. INFORMS J Comput 2008;20(1):86–99.
[7] Carraresi P, Gallo G. A multi-level bottleneck assignment approach to the bus drivers' rostering problem. Eur J Oper Res 1984;16(2):163–73.
[8] Casacuberta D, Mestres A. Redes culturales: una introducción. Technical report, Boletín Gestión Cultural N$^0$ 14: Redes culturales (ISSN:1697-073X), julio 2006. Portal Iberoamericano de Gestión Cultural; 2006 ⟨www.gestioncultural.org⟩.
[9] Castells R. Es viable la gestión privada de la cultura?. Technical report, Boletín Gestión Cultural N$^0$ 18: La Gestión Cultural desde el ámbito Empresarial Privado (ISSN:1697-073X), julio 2009. Portal Iberoamericano de Gestión Cultural; 2009 ⟨www.gestioncultural.org⟩.
[10] Daskalaki S, Birbas T, Housos E. An integer programming formulation for a case study in university timetabling. Eur J Oper Res 2004;153(1):117–35.
[11] de Werra D, Ekim T, Raess C. Construction of sports schedules with multiple venues. Discr Appl Math 2006;154(1):47–58.
[12] Drexl A, Knust S. Sports league scheduling: graph- and resource-based models. Omega 2007;35(5):465–71.
[13] Ebewo P, Sirayi M. The concept of arts/cultural management: a critical reflection. J Arts Manag Law Soc 2009;38(4):281–95.
[14] Fei H, Meskens N, Chu C. A planning and scheduling problem for an operating theatre using an open scheduling strategy. Comput Ind Eng 2010;58:221–30.

[15] Franz LS, Miller JL. Scheduling medical residents to rotations: solving the large-scale multiperiod staff assignment problem. Oper Res 1993;41(2):269–79.
[16] Hinojosa Y, Kalcsics J, Nickel S, Puerto J, Velten S. Dynamic supply chain design with inventory. Comput Oper Res 2008;35(2):373–91.
[17] Kidd D. Public culture in America: a review of cultural policy debates. J Arts Manag Law Soc 2012;42(1):11–21.
[18] Lovejoy WS, Li Y. Hospital operating room capacity expansion. Manag Sci 2002;48(11):1369–87.
[19] Mannheim K. The democratization of culture. In: Essays on the sociology of culture. London: Routledge & Kegan Paul Ltd; 1956. p. 171–246.
[20] Mulcahy K. Cultural policy: definitions and theoretical approaches. J Arts Manag Law Soc 2006;35(4):319–30.
[21] Nemhauser G, Trick M. Scheduling a major college basketball conference. Oper Res 1998;46:1–8.
[22] Nemhauser GL, Wolsey LA. Integer and combinatorial optimization. New York, NY, USA: Wiley-Interscience; 1988.
[23] Ortega FA, Pozo MA, Torres E. Planificación de la difusión cultural en el territorio mediante el uso de modelos de optimización lineal. Rev Int Investig Oper 2010;1:47–56.
[24] Pentico DW. Assignment problems: a golden anniversary survey. Eur J Oper Res 2007;176(2):774–93.
[25] Russell RA, Urban TL. A constraint programming approach to the multiple-venue, sport-scheduling problem. Comput Oper Res 2006;33:1895–906.
[26] Schuster JM. Mapping state cultural policy: the state of Washington. Cultural Policy Center, Chicago, USA: University of Chicago; 2003.
[27] Sempere AM. Circuitos. In: Revista de Espacios Escénicos, vol. 11; 2009.
[28] Warner DM, Prawda J. A mathematical programming model for scheduling nursing personnel in a hospital. Manag Sci 1972;19(4):411–22.