Les Cahiers du GERAD ISSN: 0711–2440

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G–2002–09

February 2002

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February, 2002

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G–2002–09

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Abstract

This paper investigates several questions related to the location of facilities in multi-storey buildings in the presence of lifts. Where should facilities be located? What will be the catchment area of each facility? What will be the relative use of each lift? Simple rules and geometrical interpretations are provided.

Résumé

Cet article examine diverses questions reliées à la localisation d'installations dans des édifices à plusieurs étages en présence d'ascenseurs. Où devrait-on localiser les installations? Quelle sera l'aire d'attraction de chaque installation? Quel sera l'usage relatif de chaque ascenseur? On fournit des règles simples et des interprétations géométriques.

1 Introduction

Throughout the twentieth century high-rises have proliferated in city centers, mostly in response to soaring land prices. Corporate headquarters often occupy several floors of the same building, thus allowing easier communication between departments or functional areas. As a rule, the geometry of high-rises follows a relatively simple pattern, in order to help reduce construction costs and facilitate the analysis of horizontal and vertical loads (Cobreros, 1998). Of particular importance is the location of lifts which come to bear on structural loads as they can help lower the center of gravity of the building. Lifts also provide vertical bridges facilitating displacements in the building. While structural problems are central to the design and engineering of high-rise buildings and have received a great deal of attention, issues related to traffic flows, accessibility and facility location are also important, but have been less investigated (Escrig, 1998).

We are interested in locational problems in multi-storey buildings in the presence of lifts. Our unit of analysis may be an entire building, or several (usually contiguous) floors occupied by a company in a high-rise building. We make no specific assumption about the location of lifts. We assume, however, that all floors are identical and rectangular. Three questions will retain our attention.

1) Given a building whose lift location is known and whose floors are already partitioned into offices, where should common facilities be located to maximize accessibility (or, equivalently, minimize overall travel time)? And what will be the use of each lift?

2) Defining the catchment area of the facility as the population that can access it within r time units, where should p facilities be located in order to maximize the total catchment?

3) Given several lifts and a common facility, what will be the relative use of these lifts by people wishing to reach the facility?

The first question is largely operational in nature and, as will be seen, can readily be answered using the standard location theory and methodology. The second question can be used at the planning level since the relative position of offices can be arranged to maximize the population covered by a set of already located facilities. The third question also arises at the planning stage. All too often, lift usage in a building is uneven, resulting in overusage and excessive wear and tear of the most frequented lifts.

2 Multi-storey facility location

First consider the problem of locating p identical communal facilities (e.g., meeting rooms, photocopying machines, etc.) in an already partitioned multi-storey building. Objectives may be considered, like a median objective or a covering objective. In median problems, the objective is to minimize the sum of distances or traveling times from users to their closest facility. In covering problem, facilities must be located so as to ensure that a given proportion of users (sometimes all) are within r distance or time units from it or to ensure that a maximum number of users are within r units of their closest facility (see Daskin, 1995 for an overview of basic location problems).

These problems and others can readily be solved by means of classical location models and algorithms (see, e.g., Daskin, 1995). Indeed, the building may be modeled as an undirected graph $G = (V, E)$ where V is a vertex set and E is an edge set. In practice, V contains one element for each office door and one for each lift. An edge corresponds to a direct corridor link between two vertices. To solve location problems on G, the shortest travel time t_{ij} must be computed between each pair $\{i, j\}$ of vertices. Let $L \subset V$ be the set of lifts and let f_i be the floor of i. If $f_i = f_j$, then t_{ij} is simply computed as the length (in time units) of a shortest path between i and j. If $f_i \neq f_j$, then

$$t_{ij} = \min_{\ell \in L}\{t_{i\ell} + t_{\ell j}\} + |f_i - f_j|s, \tag{1}$$

where s is the travel time by lift between two consecutive floors. Here as in the remainder of our analysis, congestion and waiting times at lifts are disregarded.

In the latter case, the optimal lift ℓ^* used between i and j is given by

$$t_{i\ell^*} + t_{\ell^* j} = \min_{\ell \in L}\{t_{i\ell} + t_{\ell j}\}, \tag{2}$$

i.e., the right-hand side is constant c and the equation $t_{i\ell^*} + t_{\ell^* j} = c$ is that of an ellipse with foci i and j. Elementary analytical geometry allows for an interesting representation of (2). Order the lifts ℓ in non-decreasing order of $t_{i\ell} + t_{\ell j} = c_\ell$ and consider the family of concentric ellipses corresponding to each $c_\ell\,(\ell \in L)$. Then the optimal lift is that associated with the innermost ellipse. Since the travel times t_{ij} do not in general correspond to any classical metric, equation (2) is not that of an ellipse in the usual geometrical since, but it would be if a Euclidean metric was used (Figure 1).

Figure 2 shows “ellipses” obtained for a Manhattan metric, more representative of reality in a typical office building.

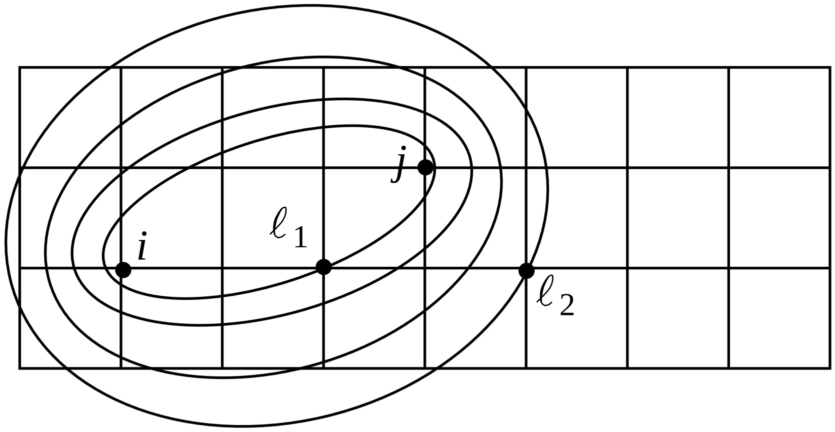

Figure 1: Concentric ellipses for a Euclidean metric.

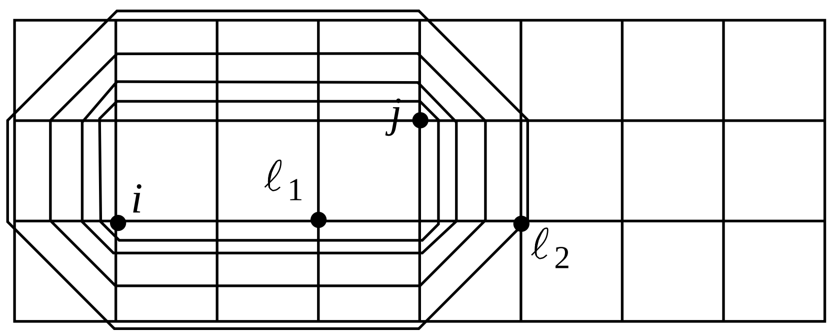

Figure 2: Concentric ellipses for a Manhattan metric.

Given an origin-destination matrix between all vertex pairs in a multi-storey building and the estimated frequency of travel between these points, the above analysis can help determine the relative lift utilization given that people usually travel using shortest paths. Embedding this information within a simulation model should help determine facility locations that will produce a balanced lift usage.

3 Maximizing the catchment area of a facility

The catchment area of a facility is the set of all points in the building that are within r time units from it. We are first interested here in locating a single facility i in order to maximize its catchment area. This situation is best understood by using a continuous representation of the building (i.e., no wall partitions) and a Euclidean metric. On floor f_i, the catchment area of i includes all points within a circle of radius r centered at i. On other floors, users will use the lift ℓ closest to i on floor f_i and walk to i. Thus the catchment area of i on floor f_j ($i \neq j$) is a circle of radius r_j around lift ℓ, where

$$r_j = \max\{0,\, r - t_{i\ell} - |f_i - f_j|s\} \tag{3}$$

(see Figure 3). For any $j \neq i$, the value of r_j is maximized if $t_{i\ell} = 0$, i.e., if facility i is

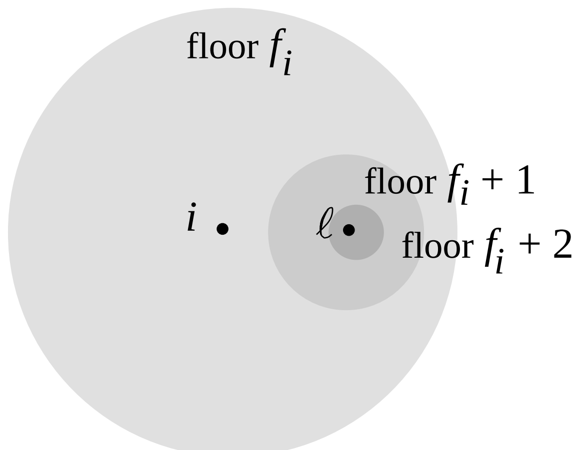

Figure 3: The catchment area of facility i.

located at lift ℓ. Hence, a simple geometrical analysis confirms what our intuition suggests: common facilities that cannot be located on each floor should be located as close to lifts as possible. This analysis also suggests that if p identical common facilities are to be located (where p is less than the number of floors), then these should be located near lifts on equally spaced floors since solutions of this type will maximize the total catchment areas of all facilities.

4 Determining the catchment area of a lift

Finally, we consider a facility i located on floor f_i and another floor f_j. The problem is to determine the relative usage of lifts by users of floor f_j on their way to facility i. In other words, how is floor j partitioned into catchment areas, one for each lift. Such a partition is often referred to as a Voronoi partition (Okabe, Boots and Sugihara, 1992). Again, we assume a continuous space.

This question is answered by considering two lifts k and ℓ and determining the condition under which lift k is preferred to lift ℓ. Performing this analysis for all pairs of lifts will yield the desired Voronoi partition. Given a point j floor f_j, lift k is equivalent to lift ℓ if

$$t_{jk} + t_{ki} = t_{j\ell} + t_{\ell i}, \tag{4}$$

in other words, if

$$t_{jk} - t_{j\ell} = t_{\ell i} - t_{ki}. \tag{5}$$

Thus the right-hand side of (5) is a constant. This equation is that of a hyperbola with foci k and ℓ, and with one branch passing through facility i. If i is closer to lift ℓ than it is to lift k, then lift ℓ is preferred by all users of facility i, except those located in the area bounded by the hyperbola branch associated with lift k (see Figure 4).

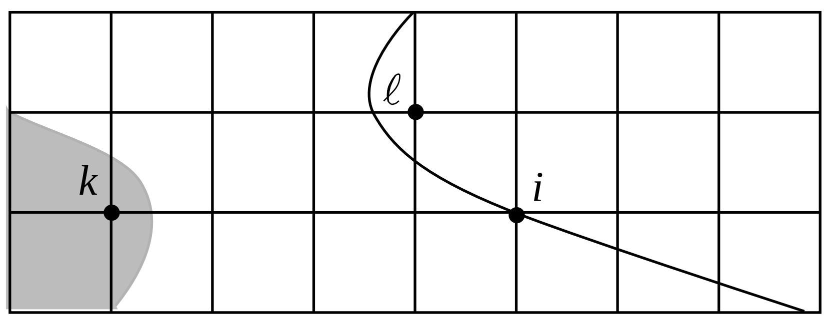

Figure 4: The catchment area of lift k.

Again, considering in turn several potential locations for i, a lift Voronoi partition can be computed to determine relative use and possibly arrive at a satisfactory solution.

Acknowledgments

This work was supported in part by a grant from the Spanish Research Project BFF-2000-1052-CO2-01, and by the Canadian natural Sciences and Engineering Research Council under grants OGP00039682. This support is gratefully acknowledged.

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