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Assessing the Efficiency of Rapid Transit Configurations

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Abstract

Eight basic transit network configurations are analyzed with respect to two measures: passenger/network effectiveness and passenger/plane effectiveness. Assumptions are made with respect to trip distribution and competition with other transportation modes.

Key Words: rapid transit configurations, network effectiveness

1. Introduction

Over the last decades, several cities throughout the world have looked at the construction or extension of rapid transit systems as a partial answer to increased traffic congestion and urban sprawl. Such systems include traditional underground metros, surface rail and light rail transit networks, and monorails (see Jiménez Solano (1993) for a taxonomy). In a recent article, Gendreau, Laporte and Mesa (1995) have examined the criteria considered by decision makers in the planning of rapid transit systems. The main considerations include purposes, cost, network characteristics, coverage and utilization, and external attributes. Network design lies at the heart of the problem: how to design a network configuration capable of improving the population's mobility by providing shorter travel times. While the operations research literature on network design is rich and still developing (see Ahuja et al. (1995) for a survey), standard optimization

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methods will rarely be applicable to large civil engineering projects such as the construction of highways, airports and mass transit systems (Magnanti and Wong (1984)). This is partly because of the size of the integer programming models involved, but also because of the fact that these models typically involve non-linearities, stochastic elements, as well as multiple and conflicting objectives.

Traditionally, scenario analysis has been the favored planning tool. Several sensible network configurations are drawn up and assessed with respect to a number of criteria. From these, a "best" solution is selected, typically after a lengthy consultation process involving planners, engineers, politicians and citizen groups. While operations research tools can in no way be a substitute for a multi-player decision process, it can assist it in a number of ways. Two examples come to mind. One is the use of heuristics such as tabu search to locate an alignment maximizing population coverage, subject to station spacing constraints (Dufourd, Gendreau and Laporte (1996)). Here, the search space does not have to be an entire city; it can be restricted, for example, to one or several promising corridors targeted by planners. In such a context, optimization is used primarily to fine tune proposed alternatives. Another example is the analysis of proposed or prototype networks on the basis of their topological characteristics. This line of research is rooted in the work of Musso and Vuchic (1988) who analyzed a number of station travel paths, line overlapping, directness of travel, overall network connectivity, etc. This work was later extended by Laporte, Mesa and Ortega (1994) who proposed two efficiency measures. The first, "passenger/network effectiveness", compute in a idealized network the ratio λ of the sum of all O/D passenger travel times over the sum of all node to node travel times in the network. The second, "passenger/plane effectiveness", computes an index Q_p to compare passenger travel time on the network to what it would be to travel in the plane in which is embedded, using an l_p norm ($p = 1$ corresponds to a Manhattan distance, $p = 2$ corresponds to a Euclidean distance). These two measures were computed for several typical networks such as stars, cartwheels, triangles, grids, etc. under the assumption of *uniformity*. In other words, it was assumed that all node pairs on the network were equally likely O/D pairs. This limits in some respect the degree of realism of this study as in practice, the most likely trips are from the periphery to the center of the network, and competition with other modes is a function of trip length.

In this paper, we analyze the passenger/network effectiveness and the passenger/plane effectiveness of eight basic configurations by dropping the uniformity assumption and introducing mode competition. As we work on idealized networks and not on real data, a number of normative assumptions were made and some parameters were fixed to realistic values. Assumption had to be made between tradeoff functions between the use of public transit and private automobile, peak hour congestion was ignored, aversion for transfer between different lines (beyond added travel time) was not considered, etc. Sensitivity analyses were then conducted. We believe that in spite of the simplifications that were made, this type of analysis can help compare basic configurations and can easily be applied to real situations using proper parameter settings.

The remainder of this paper is organized as follows. Our model is developed in Section 2, followed by computational results in Section 3 and by the conclusion in Section 4.

2. The Model

We consider a circular city with a business core Z_1 and an outer annulus Z_2 corresponding to a residential area. In some cities with a natural barrier like a river, a semi-circular representation may be more appropiate: Here Z_1 is a semi-circular inner centre and Z_2 is an outer semi-annulus. Some typical circular and semi-circular configurations are illustred in figures 1 to 8 in the appendix. There are three classes of O/D trips, according to whether they are made inside Z_1, inside Z_2, or between Z_1 and Z_2.

The transit network is represented by an undirected graph $G = (V, E, t)$, where V is a vertex set representing stations, E is an edge set representing direct transit links between adjacent stations, and $t = (t_{ij})$ is the travel time matrix on the edges. This network is embedded in a plane. Each vertex v_i of V correspond to a point P_i of that plane. The catchment area of v_i is then

$$A_i = \{z \in \mathbb{R}^2 : \| z - P_i \| \leq \delta_i\}$$

where δ_i is a constant that could represent the maximum acceptable travel time for using station v_i. All catchment areas are assumed to be disjoint.

2.1. Passenger/Network effectiveness

To define passenger/network effectiveness, consider n_{ij}, the number of trips between v_i and v_j, and τ_{ij}, the shortest travel time between v_i and v_j, and let $\theta_{ij} = n_{ij}\,\tau_{ij}(T)$. Then the total cost incurred by passengers is

$$\theta = \sum_{i<j} \theta_{ij}$$

while the total network cost is

$$T = \sum_{(v_i,v_j)\in E} t_{ij}$$

The required passenger/network effectiveness coefficient is therefore

$$\lambda = \theta/T$$

2.2. Passenger/Plane effectiveness

The idea behind the passenger/plane effectiveness coefficient is to measure how well the network is embedded in the plane. Two networks with the same vertex set, but different edge sets will typically have a different coefficient. As mentionned in the introduction, the computation coefficient depends on the metric l_p used for travels in the plane. Denote by l_{pij} the travel time in the plane between v_i and v_j using an l_p norm. Let also $m_{pij} = n_{ij} l_{pij}$, $\Theta = (\theta_{ij})$ and $M_p = (m_{pij})$. Then the passenger/plane effectiveness coefficient is defined as

$$Q_p = \frac{\| \Theta - M_p \|}{| V |}$$

when $\| \cdot \|$ is the Frobenius norm, i.e., if $H = (h_{ij})$ is an $m \times n$ matrix, then

$$\| H \| = \Big(\sum_{i=1}^{m} \sum_{j=1}^{n} h_{ij}^2 \Big)^{1/2}$$

2.3. Trip distribution

To evaluate n_{ij} we consider the two zones to which v_i and v_j belong, the travel demand by any mode between points of A_i and A_j, and the fraction of the demand corresponding to the use of the transit network. More specifically,

$$n_{ij} = c_{ij}\, f_{ij}\, g_{ij} \ .$$

We now explain the three factors used in the determination of n_{ij}.

a) The coefficient c_{ij} takes one of three values c_1, c_2 or c_3, with $c_1 + c_2 + c_3 = 1$, according to whether v_i and v_j are both in Z_1, both in Z_2 or in two different zones. (One could use more refined coefficients to reflect the fraction of A_i and A_j intersecting with each zone).

b) The second factor, f_{ij}, is a "friction coefficient" representing the travel demand between A_i and A_j. This is a deterrence function that depends on the travel time x (called impedance) according to a relation of the form

$$f(x) = x^{\alpha}\, e^{-\beta x} \ ; \ \alpha, \beta > 0$$

(see Ortúzar and Willumsen (1990), p. 138). In our application, x is the travel time $d(P,Q)$ between P and Q. Thus,

$$f_{ij} = \int_{P \in A_i} \int_{Q \in A_j} d(P,Q)^{\alpha}\, e^{-\beta\, d(P,Q)}\, dP\, dQ \ .$$

To approximate this expression, we replace $d(P,Q)$ by its mean value $\overline{d}_{ij}$ over the integration domain, using the formula for the expected distance between two points belonging to disjoint circles of centres P_i and P_j and of radii ρ_i and ρ_j (Koshizuka and Kurita (1991)):

$$\overline{d}_{ij} = d(P_i, P_j) + \frac{{\rho_i}^2 + {\rho_j}^2}{8d(P_i, P_j)}$$

so that

$$f_{ij} \approx \pi^2 \rho_i^2\, \rho_j^2 (\overline{d}_{ij})^{\alpha}\, e^{-\beta\, \overline{d}_{ij}} \ .$$

c) Finally, we use a logit function for the modal split distribution. Assuming several modes of transportation, one of which being public transit, the proportion of trips between v_i and v_j using the transit network will be

$$g_{ij} = \left(1 + e^{-\gamma(\tau_{ij} - {\tau'}_{ij})}\right)^{-1} ,$$

where τ_{ij} and ${\tau'}_{ij}$ are the shortest travel time between v_i and v_j using public transit network and all the other transportation modes, respectively, and γ is a parameter.

3. Computational Results

In order to carry out the various computational tests, the following problem generation rules were used. The radii of Z_1 and Z_2 were taken as $\rho_1 = 2.5$ and $\rho_2 = 9$. The average distance between two adjacent vertices was set to approximately 1, and the radius δ_i of each catchment area A_i was set equal 0.5. We used average surface speeds of $v_1 = 20$ and $v_2 = 40$ for the zones Z_1 and Z_2, respectively. The speed in the transit network is 60 so that it takes one time unit to cross each edge. We assumed a train stopping time of 0.4 at each station and a transfer time of 4 at connecting stations. In addition, we used $c_1 = 1/3$, $c_2 = 1/2$ and $c_3 = 1/6$.

The values of the parameter α and β were obtained by an indirect method. The mean and variance of the trip length distribution are

$$\begin{cases} \mu = \frac{\alpha+1}{\beta} \\ \sigma^2 = \frac{\alpha+1}{\beta^2} \end{cases}$$

Using the experimental values $\mu = 3.551224$ and $\sigma^2 = 3.23154$ obtained by Blumenfeld, Shrager and Weiss (1975), we were able to compute

$$\begin{cases} \beta = \frac{\mu}{\sigma^2} = 1.09893 \\ \alpha = \mu \cdot \beta - 1 = 2.90257 \end{cases}$$

so that the value of f_{ij} would be

$$f_{ij} \leftarrow \pi^2 (0.5)^4 (\overline{d}_{ij})^{2.90257} e^{-1.09893 \overline{d}_{ij}}$$

The parameter γ of the logit function is calculated using the fact that the maximal interstation distance is 16, which corresponds to two stations located at both ends of a diameter. Traveling between two extreme stations requires crossing 16 edges and 15 vertices. Thus, $\max_{i<j} \tau_{ij} = 16 + 15 \times 0.4 = 22$, while the same trip across the urban network has a length of $\tau'_{ij} = 11 \times (60/40) + 5 \times (60/20) = 31.5$. Thus the maximum difference is $31.5 - 22 = 9.5$. Using this information, we obtain a reasonable value of $\gamma = 0.309941$.

The two effectiveness indices λ and Q_2 were first computed using $\alpha = 2.90257$, $\beta = 1.09893$ and $\gamma = 0.309941$ for each of the eight basic network configurations shown in Figures 1 to 8 (see Appendix). The results are presented in Table 1. In addition, we performed sensitivity analyses for $\alpha \in [0.5, 4]$ and $\beta \in [0.5, 2]$.

Circular cities			
PARAMETERS	CIRCUMF.	CARTWH.	TRIANG.
λ	0.0095	0.0124	0.0127
Q_2	0.0219	0.0220	0.0207

Circular cities			
PARAMETERS	GRID	STAR	U AND C.
λ	0.0156	0.0157	0.0194
Q_2	0.0228	0.0221	0.0281

Semi-circular cities		
PARAMETERS	HALF-WHEEL	HALF-RADIAL
λ	0.0201	0.0219
Q_2	0.0317	0.0303

Table 1: Values of the effectiveness indices for eight basic networks

Computational results indicate that for circular cities, the circumferential configuration by far offers the best passenger/network effectiveness. The cartwheel configuration is the second best except when $\alpha \in [2.25, 4]$ and $\beta = 0.5$, or $\alpha \in [3, 4]$ and $\beta = 0.75$, in which case the triangle configuration is the second best. The best passenger/plane effectiveness is obtained for the triangle and circumferential configurations. For a given value of α, the triangle configuration is best if β is low; as β becomes larger, the circumferential configuration is the best choice. For example, when $\alpha = 2$, triangle is best for $\beta \leq 1$; when $\alpha = 4$, triangle is best for $\beta \leq 1.5$. It should be noted that in the case of uniform trip distributions (Laporte, Mesa and Ortega (1994)), these three configurations (circumferential, triangle and cartwheel) also came out best. In that paper, the half-radial and half-wheel configurations always yield the best values of effectiveness for semicircular cities. Now, regarding to passenger/plane effectiveness, half-wheel is better than half-radial if α is low or β is high. Finally, the half-wheel always produces the best passenger/network effectiveness value.

4. Conclusion

We have analyzed a number of basic rapid transit network configurations with respect to two measures introduced in Laporte, Mesa and Ortega (1994), but under more realistic assumptions. Here, the hypothesis of uniform travel distribution between all station pairs is removed and replaced by a more realistic scenario: two concentric zones with different travel characteristics are used and, in addition, competition with an alternative travel mode is considered using simple modeling assumption and realistic parameter settings, we derive a comparative evaluation of several network designs. Sensitivity analyses point to the robustness of the results. We do not suggest that our modeling assumptions and choices of paramaters hold in all settings. We believe, however, that this type of analysis can help compare alternative network designs.

Appendix

Illustration of eight basic network configurations

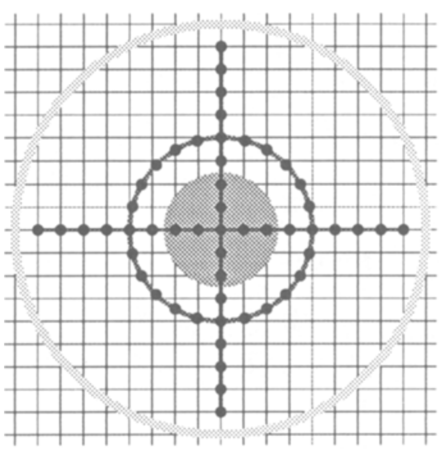

Figure 1: Cartwhell

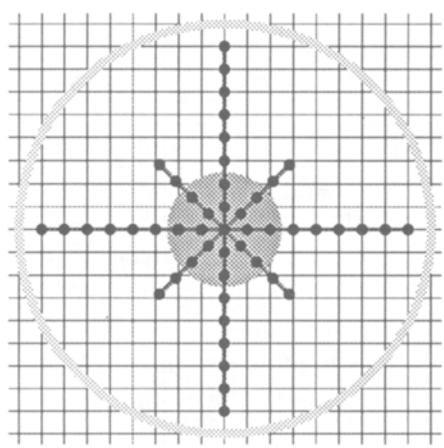

Figure 2: Start

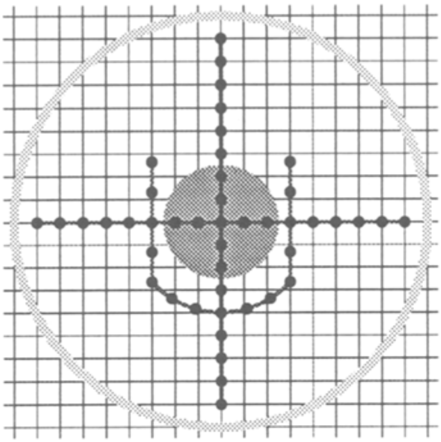

Figure 3: U and Cross

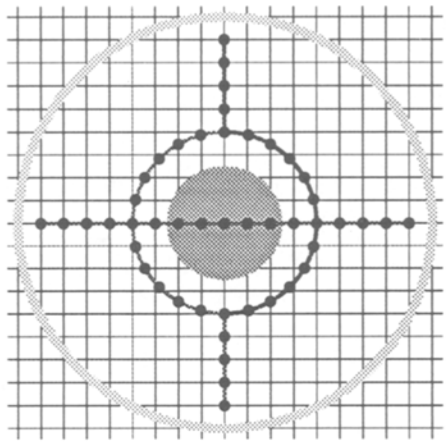

Figure 4: Circumferential

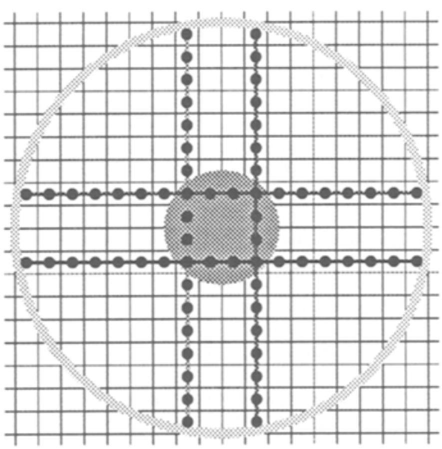

Figure 5: Grid

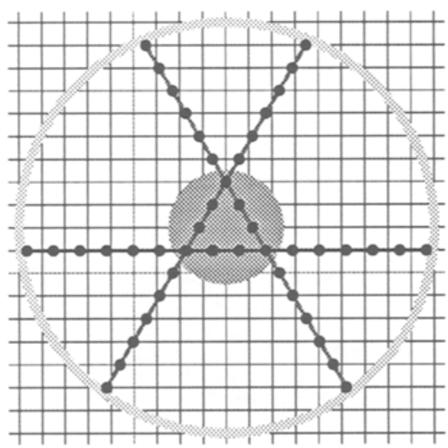

Figure 6: Triangle

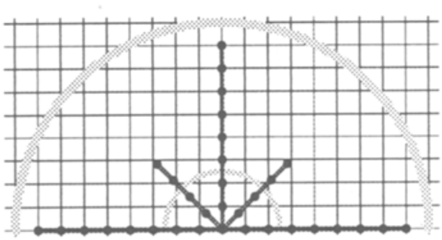

Figure 7: Half-Radial

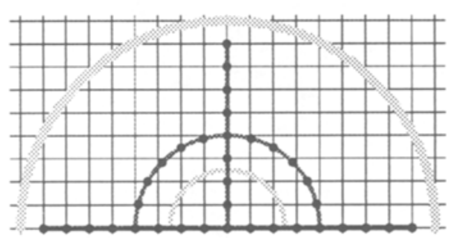

Figure 8: Half-Wheel

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