Proceedings of the 18th World Congress
The International Federation of Automatic Control
Milano (Italy) August 28 - September 2, 2011

Mixed H_2/H_∞ robust control approach for NCS with uncertainties and data dropouts ⋆

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Abstract: In this paper, a Robust Networked Control System (RNCS) subject to data losses constraints is considered. These data losses are modelled as an independent sequence of i.i.d. Bernoulli random variable. This random variable is replaced by an additive noise plus a gain, which is equal to the successful transmission probability in the feedback loop. Also, structural uncertainties in the model of the plant are considered.
To cope with this problem, a mixed H_2/H_∞ control technique is proposed in this work. In this way, the H_2 approach is used to stabilize the NCS taking into account the probability of data dropouts, while the H_∞ approach is in charge of making the closed-loop system robust enough against structural uncertainties of the nominal model.

Keywords: Control under communication constraints, control and estimation with data loss and networked embedded control systems.

1. INTRODUCTION

Nowadays, control systems wherein a communication network exits are getting importance more and more. Usually, the communication network connects some elements of the control system. This kind of systems and their characteristics are widely described in Hespanha et al. (2007) and Zhang et al. (2001).

There are a lot of studies in the literature about the main problems associated with Networked Control Systems. One of these problems is the related with the variable delays in the data transmission (see, for example, Gu and Chen (2003)). One way to approach this issue is to resort Lyapunov-Krasovskii functionals (see, for example, Millan et al. (2010) and Yue et al. (2005)).

Another important topic to be studied in NCSs is the network-induced data loss. This kind of problem occurs when the communication channel is not able to transmit the data and it get lost. This may occurs, for instance, due to colissions or low SNR (signal to noise ratio). There are differents ways to deal with these kind of NCSs. One way is the use of predictive control, which makes possible to calculate future model-based data and to use them to compute the control actions. Some examples of network control based on MPC for linear and non-linear systems can be found in Zhang et al. (2006), Millan et al. (2008) and in Muñoz and Christofides (2008).

A different way to deal with NCS subject to data dropouts constraints consists in modeling the dropouts by means of a switched system, i.e., a Markov jump linear system (MJLS). Related with this approach, Ling and Lemmon (2004) presents a result which shows that, for a specific NCS architecture subject to data dropouts constraints, the resulting MJLS is equivalent to a linear system with an external noise source. This noise has the particularity of having a variance that is proportional to the variance of another signal within the initial control loop. This result is used in Silva et al. (2009) to show that there exits a *second order moments equivalence* between the considered NCS and an auxiliary control system. In this auxiliary control system, the unreliable control channel has been replace by an additive i.i.d. noise channel that has a Signal to Noise Ratio (SNR) constraint. In that paper, the probability of data losses is a fixed value that is used in the control synthesis. The objective in Silva et al. (2009) is to minimize the error covariance designing the controller via Youla parametrization. However, in that work only a perfect LTI nominal model is considered, and therefore robust properties are not guaranteed.

In this paper, an NCS wherein a communication channel with a data dropouts source exists is considered, as well as structural uncertainties in the plant. Therefore, the main goal of this work is to find a robust controller for the plant with uncertainties and with data losses in the transmission; also finding out the minimal probability of success in the transmission such that *mean square stability (MSS)* and robustness properties can be guaranteed. A mixed H_2/H_∞ control approach is proposed in such a way that both structural uncertainties in the plant and data losses can be tolerated.

The remainder of the paper is organized as follows: In Section 2, a brief summary of the mixed H_2/H_∞ control problem theory is exposed. In Section 3 the control problem to be solved is presented. Section 4 shows the

⋆ The authors would like to acknowledge MCyT (Grant DPI2010-19154.), and the European Commission(EC) (FeedNetBack Project, grant agreement 223866), for funding this work.

architecture of the control scheme. Section 5 includes the obtained results with an example. Finally, Section 6 draws the main conclusions of the paper.

2. MIXED H_2/H_∞ CONTROL PROBLEM

In this section, a brief mixed H_2/H_∞ control approach is described. Further information can be found in Zhou et al. (1996) and Doyle et al. (1994). The control system described in Figure 1 is considered, where the generalized plant $P(z)$ and the controller $C(z)$ are both assumed to be real-rational and proper. The signals involved in the diagram are the following: $w' \in \mathbf{R}^{m_1}$ represents the disturbance vector, $u \in \mathbf{R}^{m_2}$ is the control input, $z_\infty \in \mathbf{R}^{p_1}$ and $z_2 \in \mathbf{R}^{p_2}$ are the error vectors, the first one for the measurement of the H_∞ performance, and the second one for the H_2 performance. The measurement supplied to the controller is represented by $m \in \mathbf{R}^{p_3}$.

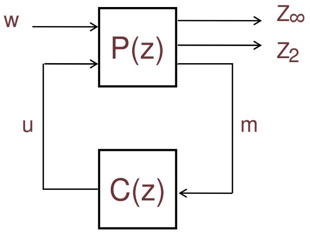

Fig. 1. Mixed H_2/H_∞ synthesis

The synthesis problem considered in this approach consists in finding a suboptimal LTI controller $C(z)$ that minimizes the following mixed H_2/H_∞ criterion:

$$Min \quad \alpha \|T_\infty\|_\infty^2 + \beta \|T_2\|_2^2, \tag{1}$$

subject to:

- $\|T_\infty\|_\infty < \gamma_0$
- $\|T_2\|_2 < \nu_0$

where $T_\infty(z)$ and $T_2(z)$ denote the closed-loop transfer functions from w' to z_∞ and z_2, respectively; and γ_0, $\nu_0 \in \mathbf{R}^+$.

As will be shown, the minimization of $\|T_2\|_2$ implies the minimization of the lower bound of the success probability in the data transmission.

In order to find out a controller by means of this control technique, it is necessary to put the original system into the form of the block diagram shown in Figure 1. To do this, the original system is changed with a *lower linear fractional transformation*.

In this case, T_∞ is chosen to represent a mixed-sensitivity H_∞ control problem, which is widely explained in Skogestad and Postlethwaite (1996). So two weighting functions are chosen: $W_s(z)$ to weight the sensitivity function $S(z)$ and $W_t(z)$ to weight the complementary sensitivity function $T(z)$. These weighting functions allow to specify the range of frequencies of relevance for the corresponding closed-loop transfer matrix. As it is known, an appropriate shaping of $T(z)$ is desirable for tracking problems, noise attenuation and for robust stability with respect to multiplicative output uncertainties. On the other hand, a convenient shaping of $S(z)$ will allow to improve the performance of the system. So, this approach is useful to have an appropriate performance on tracking problems, as well as for the system robustification against noises and uncertainties.

3. PROBLEM DEFINITION

This paper is focused on a RNCS wherein the main problems are the uncertainties in the model of the plant and the packets dropouts. So, the aim is to design a controller that stabilize a system subject to these two problems together.

The uncertainties under consideration will be represented by the following equation:

$$G^*(z) = G(z)(I + W_m(z)\Delta(z)),$$

where $G^*(z)$ represents all the possible plants, $G(z)$ is the nominal plant and $W_m(z)\Delta(z)$ is the multiplicative uncertainty, with $\|\Delta(z)\|_\infty < 1$.

In the following the way to deal with the information losses is presented. After that, a more realistic case is considered including the plant uncertainties.

The packets dropouts imply that there is an unreliable channel in the feedback path. This situation is illustrated in Figure 2, where $G^*(z)$ is the plant transfer function, $C(z)$ is the controller, r is the reference and y is the plant output. The relation between the channel input v and the channel output w is:

$$w(k) \doteq (1 - d_r(k))v(k), \quad \forall k \in \mathbf{N}_0, \quad \forall v(k) \in \mathbf{N}, \tag{2}$$

where d_r models data losses, so $d_r(k) \in \{0, 1\} \quad \forall k \in \mathbf{N}_0$.

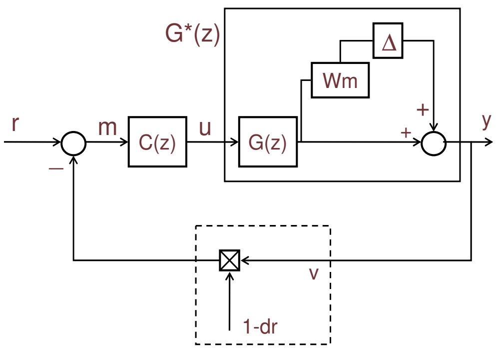

Fig. 2. RNCS with packets dropouts

The notion of stability used for this kind of systems is described in the following definition.

Definition 1 (Mean square stability) Silva et al. (2009) Consider a system described by $x(k+1) = f(x(k), w(k))$, where $k \in \mathbf{N}_0$, $f : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$, $x(k) \in \mathbf{R}^n$ is the system state at time instant k, $x(0) = x_0$, where x_0 is a second order random variable, and the input w is a second order wss process independent of x_0. The system is said to be mean square stable (MSS) if and only if there exit finite $\mu \in \mathbf{R}^n$ and finite $M \in \mathbf{R}^{n\times n}$, $M \geq 0$, such that

$$\lim_{k\to\infty} \mathbf{E}\{x(k)\} = \mu,$$
$$\lim_{k\to\infty} \mathbf{E}\{x(k)x(k)^T\} = M, \tag{3}$$

regardless of the initial state x_0.

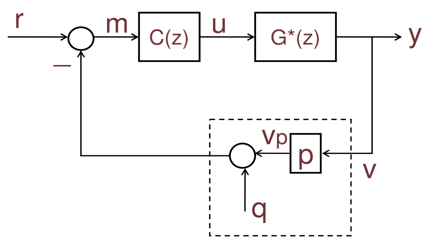

Fig. 3. RNCS with packets dropouts

The uncertainties will be included in the system given by the following theorem. This result makes possible to change the original system into another equivalent one.

Theorem 1. (Equivalence) Silva et al. (2009), Ling and Lemmon (2004) Consider the feedback loop in Figures 2 and 3. It is supossed that $p \in (0,1)$ and Assumptions 1 and 2 from Silva et al. (2009) hold. Then:

(1) If the feedback system depicted in Figure 2 is MSS and the feedback system in Figure 3 is internally stable, then the stationary PSDs of the error ($e \doteq r - y$) and of all the signals in the loops are the same in both situations.

(2) The networked system in Figure 2 is MSS if and only if the feedback loop in Figure 3 is asymptotically stable and

$$\frac{p}{1-p} > \|T_p(z)\|_2^2, \tag{4}$$

where $T_p(z)$ is the transfer function from q to v_p in Figure 3, namely

$$T_p(z) \doteq -pG^*(z)C(z)(1+pG^*(z)C(z))^{-1}. \tag{5}$$

Proof. The proof goes as the same lines as the proofs in Ling and Lemmon (2004).

As a consequence of Theorem 1, it is known that studying the MSS of the system in Figure 2 is equivalent to achieve the stability of the system in Figure 3 while condition (4) holds. Thereby, the problem can be posed as to find a controller $C(z)$ that stabilizes the system in Figure 3 and satisfies the equation (4), taking into account that the plant $G(z)$ is the nominal plant model and that the closed-loop system must be robust against the uncertainties in the plant model.

As before mentioned, structural uncertainties are going to be considered in the model of the plant $G^*(z)$. Due to this fact, the mixed sensitivity approach within the H_∞ scope allows to impose robust performance by means of appropriate design of weighting functions. In particular, it is well known that robust stability can be imposed by weighting the complementary sensitivity function if structural multiplicative uncertainty is considered (Ortega and Rubio (2004), Ortega et al. (2006)), while performance can be imposed by means of a reasonable weight on the sensitivity function.

On the other hand, it is necessary that condition (4) holds. Then, by solving an H_2 control problem it is possible to find the minimal probability of success in the transmission (p). Therefore, by mixing these two techniques, a mixed H_2/H_∞ control problem is formulated, with the following cost function to minimize: $\alpha \|T_\infty\|_\infty^2 + \beta \|T_2\|_2^2$, where $\|T_\infty\|_\infty$ includes some weighting functions to achieve the system robustification and $\|T_2\|_2$ will be $\|T_p(z)\|_2$, to impose condition (4).

Problem 1 Consider the RNCS in Figure 2 where the plant $G(z)$ has bounded structural multiplicative uncertainties. Then, the problem consists in finding a robust controller $C(z)$, using the RNCS in Figure 3, that achieves the following conditions simultaneously:

- Minimize $\|T_\infty\|_\infty$ to achieve a good performance on tracking problems and the system robustification against the plant uncertainties.
- Minimize $\|T_2\|_2$ to calculate the minimal successful probability of data losses possible for the NCS, imposing condition (4), so the systems in the Figures 2 and 3 are equivalents.

4. CONTROLLER DESIGN

In this section the controller synthesis will be performed by means of the described mixed H_2/H_∞ control technique. Some weighting transfer functions will be introduced in the system to deal with the uncertainties of the plant model. The augmented system is represented in Figure 4. The weighting transfer functions $W_s(z)$ and $W_t(z)$ weight the sensitivity function ($S(z)$) and the complementary sensitivity function ($T(z)$), respectively. The outputs of these weighting transfer functions are the signals z_s and z_t respectively, and they represent the components of the vector z_∞ in Figure 1.

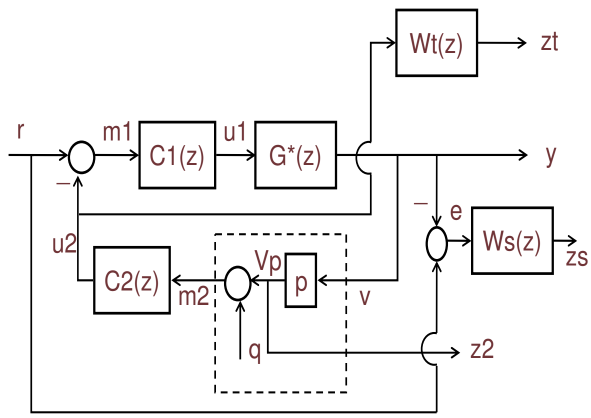

Fig. 4. RNCS and the weighting transfer functions

It is important to note that the system under consideration (Figure 3), is a non-unitary feedback system. So, in order to eliminate the steady state errors, a two-degrees-of-freedom controller is proposed. Therefore, the controller will be formed by two transfer functions, $C_1(z)$ and $C_2(z)$. Also, the sensitivity function ($S(z)$) and the complementary sensitivity function ($T(z)$) expressions will change. These expressions will be:

$$S(z) = \frac{1 + C_1(z)G(z)\,(C_2(z)p - 1)}{1 + C_1(z)G(z)C_2(z)p}$$

$$T(z) = \frac{C_1(z)C_2(z)G(z)p}{1 + C_1(z)C_2(z)G(z)p}$$

The sensitivity function ($S(z)$) represents the transfer function from the reference to the error signal. The complementary sensitivity function ($T(z)$) depends on the open-loop transfer function of the system, which is: $L(z) = C_1(z)C_2(z)G(z)p$, so the control signal u_2 should be the input of the weighting transfer function $W_t(z)$, as it is represented in Figure 4.

The objectives of the controller are the following:

(1) Minimize the H_∞ norm of the closed loop from the exogenous disturbances vector to the vector z_∞.
(2) Minimize the H_2 norm of the closed loop signal from that vector to the signal z_2.

So, as mentioned before, the mixed H_2/H_∞ control problem will be solved to find a suboptimal controller which achieves a trade-off between the minimum of the two norms under consideration. To carry out the synthesis, the system in Figure 4 has to be expressed, by means of a *lower linear fractional transformation*, as in Figure 1. It is easy to see that, by identifying the terms, the followings equations hold:

$$z_\infty = [z_s \quad z_t]^T, \quad w' = [r \quad q]^T,$$

$$P(z) = \left[\begin{array}{cc|cc} W_s(z) & 0 & -W_s(z)G(z) & 0 \\ 0 & 0 & 0 & W_{ks}(z) \\ \hline 0 & 0 & pG(z) & 0 \\ \hline I & 0 & 0 & -I \\ 0 & I & pG(z) & 0 \end{array}\right]$$

With respect to the minimization problem in (1), $T_\infty(z)$ and $T_2(z)$ are chosen as follows:

$$\|T_2(z)\|_2 = \|T_p(z)\|_2$$

$$\|T_\infty(z)\|_\infty = \left\| \begin{bmatrix} W_s(z)S(z) \\ W_t(z)T(z) \end{bmatrix} \right\|_\infty$$

The parameters will be chosen in such a way that the condition (4) holds. This means that:

$$\nu_0 = \frac{p}{1-p}$$

At this point, it's worth mentioning some comments in relation to the choice of the others parameters. It is interesting to note that, if the priority is to achieve the minimal possible p, it is important to obtain a controller that provides an H_2 norm of $T_2(z)$ very close to its minimum. Then, for this case, the parameter β should be greater than α. On the contrary, if the interest lies on achieving the best performance and robustness against noises and uncertainties, it is better to choose the parameter α greater than β. This means that the resulting controller will provide a very small H_∞ norm of $T_\infty(z)$.

The probability of success in the transmission p is assumed to be fixed in the controller synthesis. This is possible if the network requirements are well-known. In any case, if the value of p changes, the stability of the closed-loop system is guaranteed if p is greater than the minimal probability of success in the transmission obtained.

5. NUMERICAL RESULTS

To illustrate the methodology proposed in this paper, this section shows the obtained results when the control strategy is applied to a particular example. In this example the following unstable nominal plant will be considered:

$$G(z) = \frac{z - 0.5}{z(z - 1.1)}$$

The sampling time will be $t_m = 0.05s$.

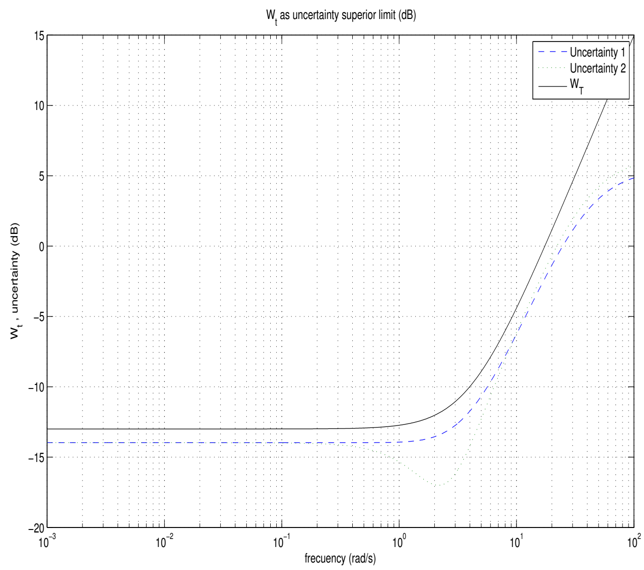

Fig. 5. Uncertainties and W_t

To take into account the uncertainties in the plant, two non-nominal models have been also considered. To obtain these two other models, the real plant is supposed to have unmodelled dynamics, so, high frequency poles are include. Also a percentage of uncertainty in the model gain has been considered. From these two systems and the nominal plant, the multiplicative uncertainties can be computed. The frequency response of these uncertainties have been plotted in Figure 5.

From this estimation of the uncertainty, the weighting transfer function $W_t(z)$ for the complementary sensitivity function is designed in such way that its modulus must be greater than the modulus of the uncertainties for all frequency. The frequency response of $W_t(z)$ has been also represented in Figure 5.

By solving the mixed H_2/H_∞ control problem for this case using some functions of the *μ−Analysis and Synthesis Toolbox* for MATLAB and considering a success probability $p = 0.7$, a robust controller is obtained yielding the following results:

$$\|T_\infty\|_\infty = 0.8441, \quad \|T_2\|_2 = 1.3615$$

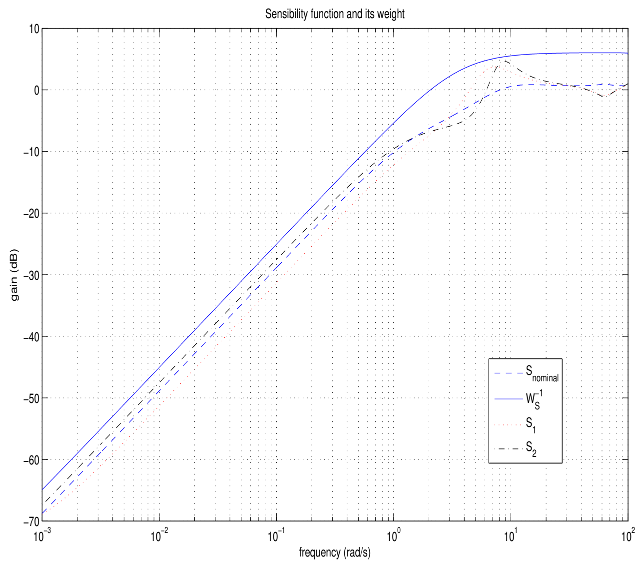

Fig. 6. $S(z)$ of the nominal plant model and $W_s(z)$

This means that the system can afford a success probability p equal to or greater than 0.65, to guarantee MSS and to preserve the demanded robustness properties.

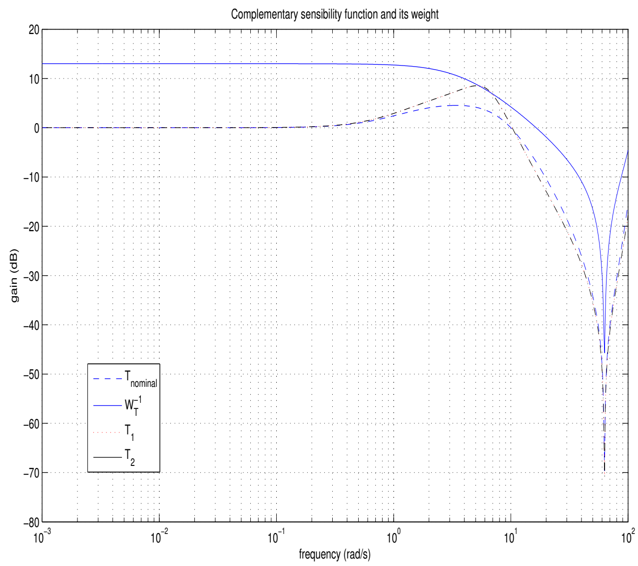

Fig. 7. $T(z)$ of the nominal plant model and $W_t(z)$

In Figure 6 the sensitivity functions of the nominal and non-nominal plants models and the inverse of the weighting transfer function $W_s(z)$ are represented. This graphic shows how all the sensitivity functions, of the nominal system and systems with uncertainties, are below the inverse of the weighting function $W_s(z)$. This fact indicates that the output y can follow the reference r for all the plant models under consideration, that is, a tracking problem can be solved although the plant model is not exactly known.

Figure 7 represents the complementary sensitivity functions of the nominal and non-nominal plants models and the inverse of the weighting transfer function $W_t(z)$. From this graphic it is possible to see that all the complementary sensitivity functions, of the nominal system and systems with uncertainties, are below the inverse of the weighting function $W_t(z)$, so the obtained controller is robust against the uncertainties in the plant model.

To corroborate these results, some simulations have been carried out with the proposed example. Figure 8 shows how the system follows the reference with a successful transmission probability $p = 0.7$, which is greater than the minimal p that can provide MSS and robustness properties for this system. This graphic represents the outputs of the closed-loop system with the nominal plant, with the plant with the uncertainties 1 and with the plant with the uncertainties 2. The results are very similar because the robustness of the system. However, there exist some differences between the differents outputs. For example, the output with the uncertainties 1 has an overshoot that is greater than the overshoot when the nominal model is used. With respect to the output with the uncertainties 2, the overshoot is reduced with respect the other cases, but the stationary performance is worse.

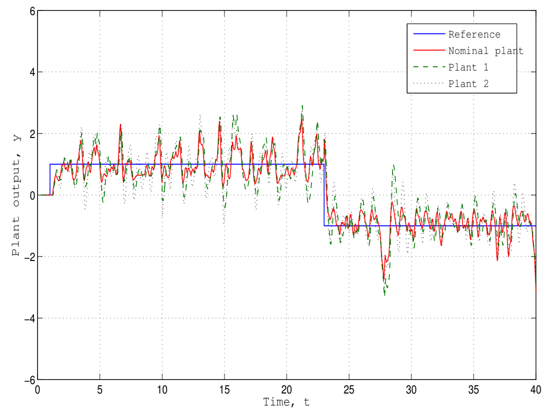

Fig. 8. Simulation results with p = 0.7

The outputs of the different systems for a value of $p = 0.9$ are shown in Figure 9. In this case, the probability of success in the transmission has been increased, although the controller used in these simulations is the one calculated for $p = 0.7$. Obviously, the results are better than in the ones presented in Figure 8, but the differences between the performance with the different systems is the same as in the case of $p = 0.7$. Also, there are steady state errors because the controller is the calculated for $p = 0.7$ so the feedback is non-unitary. These steady state errors might be avoid by calculating the controller using $p = 0.9$, but the objective is to compare the results with the same controller, supposing that p have changed in the network.

Finally, Figure 10 presents the outputs of all the systems imposing $p = 0.4$, while using the same controller as in the precedings simulations. Obviously, the performances get worse for all the systems, and in the case of the plant with the uncertainties 1 and 2, the closed-loop system becomes unstable. So, with $p = 0.4$, the robust stability is lost.

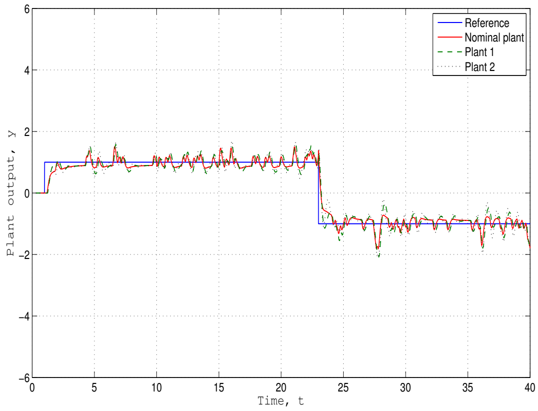

Fig. 9. Simulation results with p = 0.90

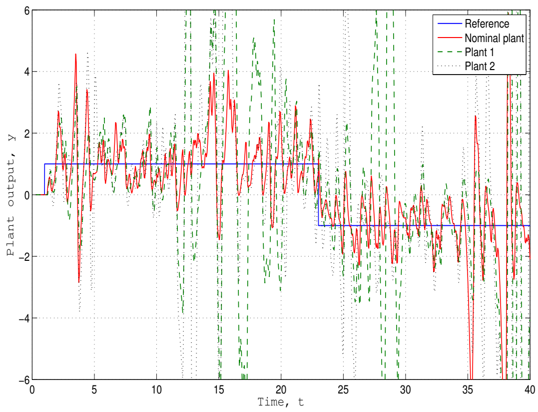

Fig. 10. Simulation results with p = 0.40

6. CONCLUSIONS

The paper has focused on a NCS subject to data dropouts constraints. In particular, control loops for SISO LTI plants, where the feedback path comprises a communication channel that produces data losses, are considered. This system has been studied as an equivalent one wherein the unreliable channel has been replaced by an additive i.i.d. noise channel, plus a gain.

The objective of this paper has been the synthesis of a controller that avoid the model uncertainties and support the failed transmissions. Also, the lower bound of the success probability in the transmission has been found. To perform this task, a mixed H_2/H_∞ control problem has been proposed. To obtain a robust controller, some functions have been chosen to weight some sensitivity functions. Moreover, from this control problem, the minimal successful transmission probability is obtained such that MSS and robustness properties for the closed-loop system are guaranteed.

Finally, an example has been exposed to obtain some numerical results that illustrate the closed-loop system performance. These simulation results corroborated that robust performance is achieved if the successful probability transmission is higher than the minimum computed, while the differents systems performances get worse, until the robust stability is lost, as the successful probability transmission decreases.

ACKNOWLEDGEMENTS

The authors would like to acknowledge MCyT (Grant DPI2010-19154.), and the European Commission(EC) (FeedNetBack Project, grant agreement 223866), for funding this work.

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