Proceedings of the 18th World Congress
The International Federation of Automatic Control
Milano (Italy) August 28 - September 2, 2011

Self-triggered sampling selection based on quadratic programming ⋆

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Abstract:
This paper proposes a model-based control strategy for networked control systems subject to norm bounded disturbances. The communication channel is supposed to be shared with several processes and, therefore, the access to the network needs to be minimized to avoid collisions and packet losses. We propose to use a variable sample rate scheme in which the controller operates in open-loop between successive state measurements. The sampling time is decided on-line solving a sequence of quadratic optimization problems in order to minimize the access to the common network while guaranteeing closed-loop practical stability. Both discrete and continuous time schemes are considered.

1. INTRODUCTION

Networked control systems (NCSs) are those in which a shared communication network links the sensors, controllers and actuators of several control loops, Zampieri [2008]. In some cases the use of networks is motivated by the very nature of the problem, while in others, the network is introduced to exploit the advantages that this architecture provides. In general, introducing a communication network in the control loop may reduce the costs, increase the flexibility and maintenance of the system and facilitate system diagnosis. However, in many practical applications the inclusion of a network to interchange real time control information introduces a number of shortcomings that must be addressed. When a certain number of devices are sharing a communication channel, which in general is not intended for processes with real time requirements, different effects, such as data losses or time varying delays, may appear degrading the control performance and even unstabilizing the plant.

Therefore, NCS involves a number of challenging control problems that have been studied during the last decade. Some works address the problem of controlling a plant subject to these undesired network induced dynamics. For instance, the effects of the quantization phenomenon and new advanced quantization strategies have been investigated in Brockett and Liberzon [2000], Canudas-de Wit et al. [2006] and Elia and Mitter [2001]. Another important research direction deals with NCS in which the network induced delays, that are in general time-varying, becomes important in the control performance or stability, see e.g. Yue et al. [2005], Jiang et al. [2008] or Millán et al. [2009]. The problem of the network package dropouts has also received great attention and has been addressed using predictive control and buffering techniques in Muñoz de la Peña and Christofides [2008] and Millán et al. [2008].

Another approach consists on controlling the plan while minimizing the access to the network by using a variable sampling rate in which measurements are only sent when they are indeed necessary. Minimizing the network load is critical in large scale systems in which the amount of data transmitted may be very large. In this way, instead of using a constant sampling period, network access is scheduled and employed only when necessary.

There can be found in the literature two different frameworks for the problem of selecting the sampling times: event-based and self-triggered control. Under the former framework (Arzén [1999], Tabuada [2007], Hetel et al. [2008], Cogill [2009], Lunze and Lehmann [2010]) the controller execution is triggered according to the state or output of the plant, which requires a continuous monitoring of the state of the plant. This drawback does not appear in the latter approach (Anta and Tabuada [2008], Cogill [2009], Anta and Tabuada [2010]). Self-triggered systems try to emulate the event-based ideas, but avoiding the continuous measuring of the state and, hence, the implementation problems this incurs.

It is worth mentioning the difference existing between these approaches and other control schemes in the context of robust stability of NCS subject to time-varying sampling instants (Suh [2008], Fujioka [2009], Fujioka et al. [2010]) in which, although intervals between sampling time are also time-varying, there is no freedom in the choice of the following sampling instant.

Aiming at reducing communication rates, different control strategies resort to the idea of using a plant model in the controller side. This idea has proven its effectiveness not only for periodic sampling (Montestruque and Antsaklis [2003, 2004], Orihuela et al. [2009]), but also for event-based control of linear stable systems (Lunze and Lehmann [2010]).

In this work the problem of reducing the use of a bandwidth-limited channel is tackled in a different manner. A scenario with a communication network in the sensor-to-controller path is considered. The system is a linear time invariant plant, subject to bounded additive disturbances. Starting from the knowledge of a stabilizing feedback controller and an associated Lyapunov function, a model-based controller is proposed in which

⋆ The authors would like to acknowledge CICYT (Grant DPI2010-19154), and the European Commission (EC) (FeedNetBack Project, grant agreement 223866), for funding this work.

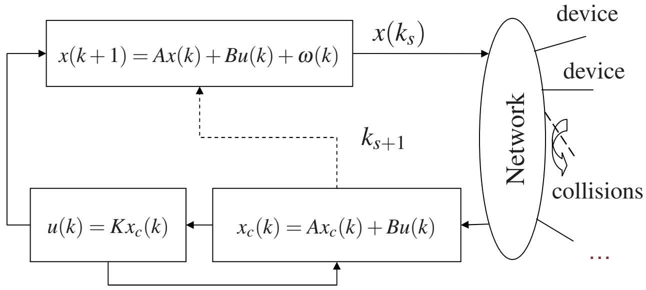

Fig. 1. Networked control system

the controller operates in open-loop between two consecutive samples. The following sampling time is decided by the controller, in such a way that practical stability is guaranteed while minimizing the number of access to the shared network, that is, while maximizing the time between successive samples. In order to decide the following sampling time the controller must solve on-line several quadratic optimization problems (QP). This Lyapunov-based sampling policy results in a self-triggered sampling strategy as in Mazo et al. [2009]. The main difference with that work is the use of a model to minimize the access to the shared network. The problem is solved in discrete time, but it is also included an extension for continuous systems with sampled state measurements.

The following section presents a description of the system, controller and network. Section 3 describes the Lyapunov-based sampling policy. The extension for continuous plants is given in Section 4. A simulation example is shown in Section 5. Conclusions and future research proposal are summarized in Section 6.

2. PROBLEM FORMULATION

Consider the following discrete time linear system given by:

$$x(k+1) = Ax(k) + Bu(k) + \omega(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$, and $u(k) \in \mathbb{R}^m$ are the state vector and control input vector respectively. The process disturbance is $\omega(k) \in \mathbb{R}^n$, and satisfies $\omega(k) \subseteq \mathscr{W}$, where:

$$\mathscr{W} = \{\omega \in \mathbb{R}^n : \|\omega(k)\|_\infty \leq \gamma, \gamma > 0\}. \tag{2}$$

It is assumed that a feedback local controller K, associated with a discrete Lyapunov function $V(x) = x^T Px$, has been designed for system (1) so that the control law $u(k) = Kx(k)$ ensures practical stability of the closed-loop system.

Consider system (1) being controlled through a network. The inclusion of such a network in the control loop induces collisions and packet dropouts. This problem becomes more important as the number of devices connected to the network and the sampling frequency of such devices grow. In order to control the system while minimizing the network traffic load, we resort to a model-based controller given by the following equations:

$$x_c(k+1) = Ax_c(k) + Bu(k), \tag{3}$$
$$x_c(k_s) = x(k_s), \quad s = 0, 1, 2... \tag{4}$$
$$u(k) = Kx_c(k), \tag{5}$$

where k_s are the discrete time instants in which the sensors measure the state of the plant and send it to the controller. Figure 1 shows an scheme of the proposed control system. The model state is updated whenever a new sample arrives. Then, the model evolves in open-loop until another measure reaches the controller. The main difference between this approach and the one of Montestruque and Antsaklis [2003] is that, here, the following sampling time is decided on-line by the controller. As Figure 1 suggests, the controller is close to the plant, hence, the same control signal is being applied to the system and is being fed to the model.

A communication protocol between the sensors and the controller is assumed to be operating, in such a way that it is possible for the controller to decide the sampling instants. This could be performed, for instance, if the controller sends a packet to the sensors which contains the following sampling instant. The arrival of this packet triggers a sensor event-based protocol that samples and sends the state of the plant at the appropriate time.

Under these considerations closed-loop equations of system (1) and controller (3)-(5) while the latter is evolving in open-loop are given by:

$$x(k+1) = Ax(k) + BKx_c(k) + \omega(k), \tag{6}$$
$$x_c(k+1) = (A+BK)x_c(k), \quad \forall k \in [k_s, k_{s+1}), \tag{7}$$
$$x_c(k_s) = x(k_s), \quad s = 1, 2, \ldots \tag{8}$$
$$k_{s+1} = f(x(k_s)), \tag{9}$$

where time k_s, with $s = 1, 2, ...$, are the time instants in which the controller receives the measurements form the sensor. The sampling instants k_s are calculated by the controller based on the state measurements received.

In the next section, we present a method to decide the next sampling instant k_{s+1} based on the system model, the controller gain K, the Lyapunov function V and the latest state measurement $x(k_s)$ in order to minimize the access to the network while guaranteing closed-loop practical stability.

3. LYAPUNOV-BASED SAMPLING PROCEDURE

This section describes the proposed procedure to minimize the access to the network while preserving closed-loop practical stability.

In view of equation (1) and equations (3)-(5), the model error $\delta(k)$ can be defined as:

$$\delta(k) \triangleq x(k) - x_c(k), \tag{10}$$

where $\delta(k_s) = 0$, $\forall k_s$. A possible evolution of the state of the system $x(k)$, the controller $x_c(k)$ and the error $\delta(k)$ is drawn in Figure 2.

The dynamics of the controller state and the model error between two consecutive sampling times can be written as follows:

$$x_c(k_s+j) = (A+BK)^j x(k_s), \forall j \in \mathbb{N} : \{k_s + j < k_{s+1}\} \tag{11}$$
$$\delta(k_s+j) = \sum_{i=1}^{j} A^{i-1}\omega(k_s+j-i), \forall j \in \mathbb{N} : \{k_s + j < k_{s+1}\} \tag{12}$$

Obviously, from equations (2) and (12), one can see that:

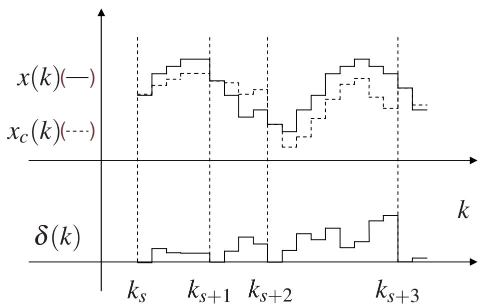

Fig. 2. Possible evolution of the state and the model error

$$\|\delta(k_s+j)\|_\infty < \gamma \sum_{i=1}^{j} \|A^{i-1}\|_\infty, \tag{13}$$

for all $\omega(k_s+i)$, with $i=0,...,j-1$. In what follows, the Lyapunov-based sampling procedure is developed starting from the available pair controller-Lyapunov function and taking into account the closed-loop equations (6)-(9). The forward difference of the Lyapunov function for $k \in [k_s, k_{s+1})$ yields:

$$\Delta V(k_s, k_s+j) = V(x(k_s+j)) - V(x(k_s)) = x^T(k_s+j)Px(k_s+j) - x^T(k_s)Px(k_s),\ \forall j \in \mathbb{N} : \{k_s+j < k_{s+1}\} \tag{14}$$

Now, substituting $x(k)$ from equation (10),

$$\Delta V(k_s, k_s+j) = \delta^T(k_s+j)P\delta(k_s+j) + 2x_c^T(k_s+j)P\delta(k_s+j) + x_c^T(k_s+j)Px_c(k_s+j) - x_c^T(k_s)Px_c(k_s). \tag{15}$$

The controller's goal is to maximize the next sampling instant k_{s+1} while guaranteeing that the forward difference is negative for all possible disturbances in order to ensure practical stability. To this end, the controller solves the following optimization problem.

$$\max\ k_{s+1} \tag{16}$$
$$\text{subject to:}$$
$$\Delta V(k_s, k_s+j) \le 0,\ \forall j \in \mathbb{N} : \{k_s+j < k_{s+1}\}$$
$$\forall \omega(k_s+i),\ i=0,...,j-1$$

Next, we present an algorithm which solves (16) to determine the next sampling time in an iterative manner:

Algorithm 1.

(1) Set $j=1$.
(2) Solve the problem

$$\min_{\delta}\ -\Delta V(k_s, k_s+j) \tag{17}$$
$$\text{subject to:}$$
$$\|\delta\|_\infty < \gamma \sum_{i=1}^{j} \|A^{i-1}\|_\infty.$$

(3) If $\Delta V(k_s, k_s+j) \le 0$, increase $j=j+1$ and go to Step 2. Otherwise, choose $k_{s+1} = k_s + j$.

Algorithm 1 increases k_{s+1} iteratively while a worst case bound on the difference between the value of the Lyapunov function of the current state and the state corresponding to the next sampling time is negative. Once this constraint does not hold, the algorithm stops and decides the next sampling time. This implies, that the $V(x(k_s))$ is a decreasing sequence of values with a lower bound (given by the size of the uncertainty), and hence that the closed-loop system is practically stable.

Next, will prove that this optimization problem can be stated as a QP problem. First of all, the standard QP problem is introduced, see Nocedal and Wright [2006].

Quadratic programming problem. Assume ξ belongs to $\mathbb{R}^p$ space. The $p \times p$ matrix H is symmetric, and f is any $f \times 1$ vector. The QP problem is stated as

$$\min_{\xi}\ g(\xi) = \frac{1}{2}\xi^T H \xi + f^T \xi + c, \tag{18}$$
$$\text{subject to}$$
$$D\xi \le b \text{ (inequality constraint)} \tag{19}$$

We prove next that problem (17) can be stated as a QP.

Proposition 1. Problem (17) can be formulated as a QP if the elements of equations (18)-(19) are chosen as

$$\begin{aligned} \xi &= \delta, \\ H &= -2P, \\ f^T &= -2\left[(A+BK)^j x_c(k_s)\right]^T P, \\ c &= -\left[(A+BK)^j x_c(k_s)\right]^T P \left[(A+BK)^j x_c(k_s)\right] \\ &\quad + x_c^T(k_s)Px_c(k_s), \end{aligned} \tag{20}$$

and for the inequality constraint

$$D = \begin{bmatrix} I_n \\ -I_n \end{bmatrix}, \quad b = \gamma \sum_{i=1}^{j} \|A^{i-1}\|_\infty \begin{bmatrix} \bar{1}_n \\ -\bar{1}_n \end{bmatrix}. \tag{21}$$

where $\bar{1}_n$ is a column vector of dimension n whose components are ones and I_n is the identity matrix with dimension n.

Proof. The proof is straightforward from equations (15) and (18)-(19). □

In view of Proposition 1, the controller needs to solve several QP problems in Algorithm 1 to find the next sampling time.

Remark. As one can see, in Algorithm 1 the minimum sampling time is one. It is not possible to ensure that the Lyapunov function decreases for all k because of the presence of bounded disturbances $\omega(k)$, which can make $\Delta V(k, k+1)$ strictly positive in a neighborhood of the origin. However, it is worth reminding that, by assumption, the system practical stability is guaranteed for the controller K with sampling time equal to one.

It is important to remark that the QP problem that needs to be solved in order to solve (17) is a multi-parametric QP problem (mpQP), for which the explicit solution can be obtained, see Bemporad et al. [2002]. In particular, the parameter $\theta \in R^{n+1}$ of the mpQP problem is

$$\theta(k_s, j) = \begin{bmatrix} x_c(k_s+j) \\ \sum_{i=1}^{j} \|A^{i-1}\|_\infty \end{bmatrix}.$$

This allows implementing the proposed variable sample control scheme efficiently.

4. EXTENSION TO CONTINUOUS-TIME SYSTEMS

The control scheme presented in the previous section can be readily extended to continuous-time systems under the following assumptions. Consider the following continuous time linear system subject to bounded disturbances.

$$\dot{x}(t) = Ax(t) + Bu(t) + \omega(t), \tag{22}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector and $\omega(t) \in \mathscr{W} \subset \mathbb{R}^n$ is the process disturbance where:

$$\mathscr{W} = \{\omega \in \mathbb{R}^n : \|\omega(t)\|_\infty \leq \gamma, \gamma > 0\}. \tag{23}$$

We assume that there exists a linear controller $u(t) = Kx(t)$ that asymptotically stabilizes the nominal system (system (22) with $\omega(t) = 0$) with a corresponding Lyapunov function $V(x) = x^T Px$.

System (22) is controlled through a network with the same structure as that in Figure 1, which implements the following model-based controller (which is the continuous time version of (3)-(5)):

$$\dot{x}_c(t) = Ax_c(t) + Bu(t), \quad \forall t \in [t_k, t_{k+1}) \tag{24}$$

$$u(t) = Kx_c(t), \tag{25}$$

$$x_c(t_k) = x(t_k), \quad k = 0, 1, 2... \tag{26}$$

where t_k is the sampling time (equivalent to k_s) in which the sensors send new information to the controller. A Lyapunov-based control design procedure can now be followed similarly to that in Section 3. We define the model error variable $\delta(t)$ as

$$\delta(t) \triangleq x(t) - x_c(t). \tag{27}$$

The dynamic of the error equation now becomes

$$\begin{aligned}\dot{\delta}(t) &= \dot{x}(t) - \dot{x}_c(t)\\ &= Ax(t) + Bu(t) + \omega(t) - Ax_c(t) - Bu(t),\\ &= A\delta(t) + \omega(t), \quad \forall t \in [t_k, t_{k+1}).\end{aligned} \tag{28}$$

Thus, the dynamics of the controller state and the model error between two consecutive sampling times evolves as:

$$x_c(t) = e^{(A+BK)(t-t_k)} x_c(t_k), \quad \forall t \in [t_k, t_{k+1}) \tag{29}$$

$$\begin{aligned}\delta(t) &= e^{A(t-t_k)}\delta(t_k) + \int_{t_k}^{t} e^{A(t-\tau)}\omega(\tau)d\tau =\\ &= \int_{t_k}^{t} e^{A(t-\tau)}\omega(\tau)d\tau, \quad \forall t \in [t_k, t_{k+1}).\end{aligned} \tag{30}$$

The following proposition is needed for further developments.

Proposition 2. If the dynamics of the error variable is given by (30), the error can be bounded as follows:

$$\|\delta(t)\|_\infty \leq \gamma\phi(t, t_k) \tag{31}$$

where $\phi(t, t_k) = \frac{1}{\|A\|_\infty}(e^{\|A\|_\infty(t-t_k)} - 1)$ and $\|A\|_\infty$ is the infinite norm of A.

Proof. Taking into account equation (30), the norm of the error can be bounded as follows:

$$\begin{aligned}\|\delta(t)\|_\infty &= \|\int_{t_k}^{t} e^{A(t-\tau)}\omega(\tau)d\tau\|_\infty \leq \int_{t_k}^{t} \|e^{A(t-\tau)}\|_\infty \|\omega(\tau)\|_\infty d\tau\\ &\leq \gamma \int_{t_k}^{t} e^{\|A\|_\infty(t-\tau)} d\tau = \gamma \frac{1}{\|A\|_\infty}(e^{\|A\|_\infty(t-t_k)} - 1).\end{aligned}$$

□

In what follows, the Lyapunov-based sampling procedure is developed. The controller's goal is to maximize the next sampling instant t_{k+1}, while guaranteeing that the derivative of the Lyapunov function is negative for all possible disturbances. Taking the time derivative of the Lyapunov function for $t \in [t_k, t_{k+1})$ yields

$$\frac{d}{dt}V(t) = x^T(t)P\dot{x}(t) + \dot{x}^T(t)Px(t) = 2x^T(t)P\dot{x}(t). \tag{32}$$

Now, substituting $x(t)$ from equation (27),

$$\begin{aligned}\dot{V}(x(t)) &= 2(\delta^T(t) + x_c^T(t))P(\dot{\delta}(t) + \dot{x}_c(t))\\ &= 2(\delta^T(t) + x_c^T(t))P(A\delta(t) + \omega(t) + Ax_c(t) + Bu(t))\\ &= \delta^T(t)(PA + A^T P)\delta(t) + 2\delta^T(t)P\omega(t) + 2x_c^T(t)P\omega(t)\\ &+ 2\delta^T(t)(PA + A^T P + PBK)x_c(t) +\\ &+ x_c^T(t)\big(P(A+BK) + (A+BK)^T P\big)x_c(t), \; \forall t \in [t_k, t_{k+1}).\end{aligned} \tag{33}$$

In the following algorithm we will ensure the negative definiteness of an upper bound on the time derivative of the Lyapunov function (33) at time t for all possible uncertainty trajectories.

The objective of the controller is to maximize t_{k+1} while guaranteeing that the time derivative of the Lyapunov function is negative for all possible disturbances; that is,

$$\begin{aligned}&\max \; t_{k+1}\\ &\text{subject to:}\\ &\frac{d}{dt}V(x(t)) \leq 0, \quad \forall t \in [t_k, t_{k+1})\\ &(22) - (26).\end{aligned} \tag{34}$$

This optimization problem is very difficult to solve. The parameter to be optimize, i.e. t_{k+1}, is involved in a nonlinear equation and there are an infinite number of constraints, because they must be satisfied for all $t \in [t_k, t_{k+1})$. In order to obtain the next sampling time, we propose to define $t_{k+1} = T_{min} + n\Delta$ and find the maximum n such that the time derivative of the Lyapunov function is negative at those time instants for all possible disturbances.

We present next an iterative algorithm that under mild assumptions provide an approximate solution to (34).

Algorithm 2.

(1) Set $t_{k+1} = t_k + T_{min}$.
(2) Solve the problem

$$\begin{aligned}&\min_{\delta(t_{k+1}), \omega(t_{k+1})} \; -\frac{d}{dt}V(x(t_{k+1}))\\ &\text{subject to:}\\ &\|\omega(t_{k+1})\|_\infty \leq \gamma\\ &\|\delta(t_{k+1})\|_\infty \leq \gamma\phi(t_{k+1}, t_k)\end{aligned} \tag{35}$$

(3) If $\dot{V}(x(t_{k+1})) \leq 0$, increase $t_{k+1} = t_{k+1} + \Delta$ and go to Step 2. Otherwise, choose t_{k+1}.

where T_{min} is lower bound for the following sampling time.

Next proposition shows that problem (35) can be stated as a QP.

Proposition 3. Problem (35) can be formulated as a QP if the elements of equations (18)-(19) are chosen as

$$\begin{aligned}\xi &= \begin{bmatrix} \delta(t) \\ \omega(t) \end{bmatrix}, \\ H &= -2\begin{bmatrix} PA + A^T P & P \\ P & 0 \end{bmatrix}, \\ f^T &= -2x_c^T(t_{k+1})\begin{bmatrix} PA + A^T P + K^T B^T P & P \end{bmatrix}, \\ c &= -x_c^T(t_{k+1})\left(P(A+BK) + (A+BK)^T P\right)x_c(t_{k+1}),\end{aligned}$$

and for the inequality constraint

$$D = \begin{bmatrix} I_n & 0 \\ -I_n & 0 \\ 0 & I_n \\ 0 & -I_n \end{bmatrix}, \quad b = \begin{bmatrix} \gamma\phi(t_{k+1}, t_k)\bar{1}_n \\ \gamma\phi(t_{k+1}, t_k)\bar{1}_n \\ \gamma\bar{1}_n \\ \gamma\bar{1}_n \end{bmatrix}. \tag{36}$$

where $\bar{1}_n$ is a column vector of dimension n whose components are ones and I_n is the identity matrix with dimension n.

Proof. The proof is straightforward from equations (33) and (18)-(19). □

The value of Δ must be chosen small enough in a way such that the dynamics of the controller state, and hence of the Lyapunov function, are smooth between two consecutive sampling, avoiding unexpected sign changes of the derivative of the Lyapunov function from t_k to t_{k+1}. In general T_{min} is chosen according with the minimum sampling time of the sensors.

As in the discrete time case, the resulting QP problem is a mpQP, and hence, an explicit solution can be obtained. In this case the parameter of the QP problem is:

$$\theta(t_k, j) = \begin{bmatrix} x_c(t_k + j\Delta) \\ \phi(t_k + j\Delta, t_k) \end{bmatrix}.$$

Assuming that the sign of the time derivative of the Lyapunov function does not change between two consecutive times, Algorithm 1 provides a suboptimal solution to problem (34). Note that this assumption will be satisfied for a sufficiently small Δ.

The constraint on the upper bound of the time derivative of the Lyapunov function is more restrictive than the constraint on the difference of the Lyapunov function imposed in the discrete time controller studied in the previous section. Note that, in continuous time, a constraint on the difference of the Lyapunov function does not yield a QP problem and hence is more difficult to implement in real time.

5. NUMERICAL EXAMPLE

In this section, we are going to apply the previous result to an unstable plant in order to show how the controller manages to reduce the traffic load maintaining the practical stability of the system.

Consider the following discrete time LTI system:

$$x(k+1) = \begin{bmatrix} 2.72 & 2.70 \\ 0 & 2.69 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1.7 \end{bmatrix} u(k) + \omega(k), \tag{37}$$

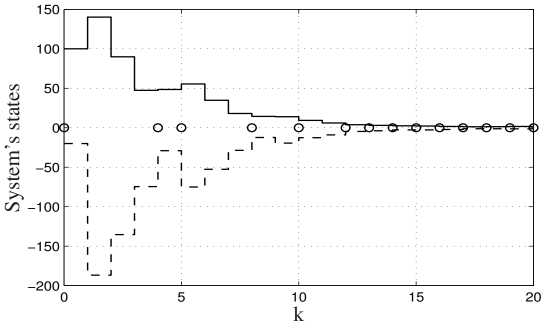

Fig. 3. Evolution of the system's states

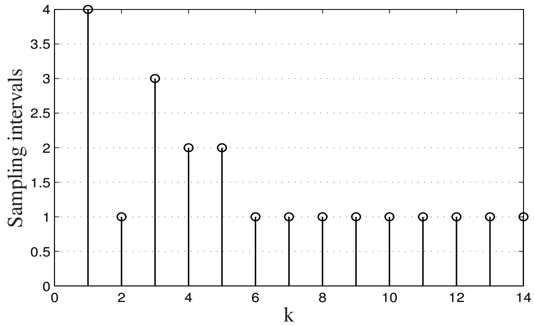

Fig. 4. Evolution of the sampling intervals

The initial condition for the system and the controller is $x(0)^T = [\,100 \;\; -20\,]$.

The following stabilizing controller has been designed for the discrete system:

$$K = [\,-1.1868 \;\; -2.0415\,].$$

The Lyapunov function is defined by

$$P = 10^6 \begin{bmatrix} 1.67 & 0.67 \\ 0.67 & 0.60 \end{bmatrix}.$$

Suppose that this system is controlled using a shared network so it is interesting to reduce the number of access to the shared medium.

Supposing that the disturbances are bounded $\|\omega(k)\|_\infty \leq 1$, the evolution of the system is illustrated in Figure 3, where the sampling instant are indicated with circles.

In Figure 4 the sampling instants obtained using the proposed method are shown. One can see that when the system is far of the equilibrium point it is possible to enlarge the sampling period still assuring asymptotic stability.

Finally, the evolution of the Lyapunov function is drawn in Figure 5.

6. CONCLUSIONS

We have presented a novel model-based controller for networked systems, aimed at reducing the number of accesses to a shared network. It has been shown that, using predictions based on a nominal model of the system, adequate asynchronous sampling times can be found by solving several QP problems which take explicitly into account the disturbances of the model. We

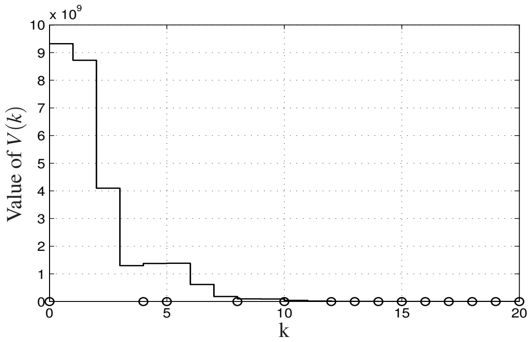

Fig. 5. Evolution of the Lyapunov function

have considered both discrete and continuous time controllers. The results have been demonstrated through simulation.

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