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NONLINEAR $\mathcal{H}_\infty$ MEASUREMENT FEEDBACK CONTROL OF EULER-LAGRANGE SYSTEMS

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Abstract: This paper considers the problem of designing explicit measurement feedback $\mathcal{H}_\infty$ control laws for a class of Euler-Lagrange systems. For these systems the joint positions are assumed as outputs of the system, while velocity measures are to be estimated from an *observer+controller* structure. The main contribution of this work lies in the explicit formulation of the dynamic structure of a joined *observer+controller* that guarantees local asymptotic stability as well as attenuation of disturbances according to an $\mathcal{H}_\infty$ framework. In order to illustrate this methodology, experimental results are shown on a 2 dof gyrostabilized platform. *Copyright ©2005 IFAC.*

Keywords: Non-linear $\mathcal{H}_\infty$ Control, Measurement feedback, Hamilton-Jacobi-Isaacs equation, Attraction basin.

1. INTRODUCTION

The vast majority of current control techniques for electromechanical systems are based on complete feedback of the system variables. Actually, even simple control laws as PD control, require measurement of all the state variables, positions and velocities.

Nonetheless, most practical electromechanical systems frequently omit velocity sensor due to savings in costs, volume or weight that can be obtained in this way. As a consequence, only measures of the joint displacements are usually available. This fact typically yields high-precision low-noise position signals, and by contrast, the velocity must be obtained from numerical estimation from these position signals. This results in a noisy velocity signal that must be carefully filtered to be used as feedback to the controller. Moreover, even in the case where velocity sensors are present (tachometers), these often provide low-quality noisy signals because of its manufacturing technology. Typically, discontinuities in the magnetic field of the tachometer stator at low frequencies and other high frequency phenomena reduce the quality of the measured velocity signal.

In practice, this circumstances may degrade the dynamic performance of the controlled system since noisy signals impose limits on the maximum attainable bandwidth of the controlled systems, hence reducing the values of the maximum controller gains that can be used.

Thus, this paper addresses the problem of designing a combined *observer+controller* structure for a class of Euler-Lagrange systems, such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the penalty output is minimized, or guaranteed to be less than or equal to a prescribed

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value, γ. As it is well known, this problem can expressed according to a nonlinear $\mathcal{H}_\infty$ framework.

The design of observers for electromechanical systems is very complex, due to the nonlinear and coupled structure of the associated dynamic models. So far in the literature, there have been a number of approaches to this problem. Some relevant results can be found in (Krener, A.J. and Isidori, A., 1983; Walcott, B.A.; Corless, M.J. and Zak, S.H., 1987). Most of these methods provide conditions under which the original system can be transformed, via nonlinear change of coordinates, into special canonical forms where the observer can be designed. Nonetheless, these conditions are somehow restrictive and are not met by many physical systems, as is the case of electromechanical systems.

It is possible to obtain less restrictive conditions when local estimation of the state vector is considered (Baumann, W.T. and Rugh, W.J., 1986; Nicosia, S. and Tomei, P. and Tornambe, A., 1989). The main drawback of these approaches is that the estimator can be used only in the neighborhood of the design operating point, and moreover, complex inverse transformations are required to get the state vector expressed in physical variables. These observers are somehow universal in the sense that are designed regardless of the underlying control strategy implemented. This often causes that the estimated state, when used in conjunction with a conventional state feedback controller, does not guarantee stability of the overall controlled system.

This fact motivated the development of combined control-observer design strategies, such that the stability of the system is guaranteed. Remarkable result on this respect are, for example, (Canudas de Wit, C. and Fixot, N. and Åström, K.J., 1992), where a modified computed torque technique with an embedded observer structure is proposed, or (Tomei, P., 1989; Nicosia, S. and Tomei, P., 1990), where a control structure for flexible joints robot is proposed, taking into account the dynamics of the observer, such that the joined *controller+observer* system guarantees stability assuming the observer gains satisfy certain restrictions. Nonetheless, these results do not take performance of the system into consideration, and assume a perfect knowledge of the system dynamics, so robustness is not considered either.

More recently, the so-called passivity-based approach, (Ortega and Spong, 1989), has gained much attention. This methodology exploits the system's physical structure to reshape its natural energy function, such that the control objective is achieved. This control philosophy is adopted in (Berguis, H. and Nijmeijer, H., 1994), where a passivity based approach that embeds the observer dynamics in the control structure is proposed .

The design of observer structures within the nonlinear $\mathcal{H}_\infty$ framework was initiated in (Isidori and Astolfi, 1992; Van der Schaft, 1991) with later developments in (Reif *et al.*, 1999; Kiriakidis, 2002). The main drawback of this approach lies in the difficulty of finding explicit solutions to the set of coupled PDE Hamilton-Jacobi-Isaacs equations (HJIE) inherent to the problem formulation. This has motivated few applications of this methodology to real problems, despite its potential good properties in terms of disturbance rejection or robustness.

In this paper the problem of designing explicit measurement feedback $\mathcal{H}_\infty$ control laws for a class of Euler-Lagrange systems is considered. For these systems the joint positions are assumed accessible as outputs of the system, while velocity measures are to be estimated from a combined *observer+controller* structure. This work extends previous results (Isidori and Astolfi, 1992) on the topic to the case of time-varying systems, with applications to reference tracking problems for Euler-Lagrange systems. For these systems, an explicit formulation of the dynamic structure of a combined *observer+controller* is given, while attenuation of disturbances is guaranteed according to the $\mathcal{H}_\infty$ formalism.

2. GENERAL FORMULATION

Consider a dynamical system in the form

$$\dot{x} = f(x,t) + g_1(x,t)\omega + g_2(x,t)u \qquad (1)$$
$$z = h_1(x,t) + k_{12}(x,t)u \qquad (2)$$
$$y = h_2(x,t) \qquad (3)$$

where the equation (1) describes the nonlinear plant dynamics in $\mathbb{R}^n$ with state vector $x(t)$. $u(t) \in \mathbb{R}^{m_u}$ represents the control action and $\omega \in \mathbb{R}^{m_\omega}$ is an exogenous disturbance acting on the system.

Additionally, equation (2) defines a penalizing function $z \in \mathbb{R}^{m_z}$, and $y \in \mathbb{R}^{m_p}$ in (3), is considered the accessible output of the system. Additionally, $x = 0$ is assumed to be an equilibrium point of the unperturbed unactuated system (1), which implies $f(0,t) = 0$, $h_1(0,t) = 0$ y $h_2(0,t) = 0$. Similarly, the functions $f(x,t)$, $g_1(x,t)$, $g_2(x,t)$, $h_1(x,t)$, $h_2(x,t)$ y $k_{12}(x,t)$ are assumed to be sufficiently smooth.

The dynamic control structure considered in this paper takes the form

$$\dot{\xi} = \eta(\xi, y) \qquad (4)$$
$$u = \theta(\xi)$$

where ξ is the controller state in a neighborhood Ξ of the origin of the system in $\mathbb{R}^v$ and $\eta : \Xi \times \mathbb{R}^{m_p} \to \mathbb{R}^{m_v}$, $\theta : \Xi \to \mathbb{R}^{m_u}$ are smooth functions. Additionally, $\eta(0,0) = 0$ and $\theta(0) = 0$ is satisfied to guarantee that the origin is an equilibrium point of the system as required.

In order to simplify the expressions of the controller the following hypothesis are also assumed

- (H1) $h_1^T(x,t)k_{12}(x,t) = 0$
- (H2) $k_{12}^T(x,t)k_{12}(x,t) = R = R^T \geq 0$

With these definitions, the control objective can be expressed as:*given a dynamical system in the form (1)-(3), obtain a dynamic output feedback control law, or equivalently, the functions η y θ in (4), that locally asymptotically stabilize the origin, satisfying an $\mathcal{L}_2$-gain attenuation less than γ for the mapping $\omega \mapsto z$.*

That is, the control law u must satisfy the dissipativity inequality

$$J_\infty(u,\gamma) = \frac{1}{2}\int_0^\infty \|z(x,u,t)\|^2 dt - \frac{\gamma^2}{2}\int_0^\infty \|\omega(t)\|^2 dt \leq 0 \qquad (5)$$

3. SOLUTION FOR THE GENERAL CASE

In order to formulate a general solution to the proposed problem, first a standard result (Van der Schaft, 1991) on the full state feedback solution is summarized.

3.1 The state feedback case

Theorem 1: Assume there exist a positive definite function $V(x,t)$, defined in a neighborhood of $x = 0$, such that satisfies de HJIE

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f - \frac{1}{2}u^{*T}Ru^* + \frac{\gamma^2}{2}\omega^{*T}\omega^* + \frac{1}{2}h_1^T h_1 \leq 0 \quad (6)$$

where

$$\omega^*(x,t) = \frac{1}{\gamma^2} g_1^T \frac{\partial V^T}{\partial x} \qquad u^*(x,t) = -R^{-1} g_2^T \frac{\partial V^T}{\partial x} \quad (7)$$

then, the state feedback control law $u(x,t) = u^*(x,t)$ locally asymptotically stabilizes system (1), verifying the $\mathcal{L}_2$-gain attenuation (5) for the mapping $\omega \mapsto z$.

3.2 Nonlinear $\mathcal{H}_\infty$ measurement feedback

In this section, the previous result on state feedback $\mathcal{H}_\infty$ control is used, as well as some additional results, to extend previous results (Isidori and Astolfi, 1992) on the topic to the case of time-varying systems.

First, let's introduce some fairly standard notation in this context. Thus, $\tilde{y} = y - \hat{y}$ is the *output observation error*, with y the measured output according to (3), and $\hat{y}$ the observer estimated output. If the estimated system state is denoted $\hat{x}$, the *observer error dynamics* can be expressed in terms of the variable $\xi = x - \hat{x}$. With these definitions, the following result can be stated

Theorem 2: Assume two positive definite functions, $V(x,t)$ and $W(x,\xi,t)$, defined in a neighborhood of $x = 0$ and $(x,\xi) = (0,0)$ respectively. If the following conditions are satisfied

- (i) $V(x,t)$ satisfies equation (6)
- (ii) $W(x,\xi,t)$ satisfies

$$\frac{\partial W}{\partial t} + \frac{\partial W}{\partial x} f_{e1} + \frac{\partial W}{\partial \xi} f_{e2} + \frac{1}{2}h_e^T h_e + \frac{\gamma^2}{2}\Phi^T\Phi \leq 0 \quad (8)$$

  where

$$f_e(x,\xi,t) = \begin{pmatrix} f_{e1}(x,\xi,t) \\ f_{e2}(x,\xi,t) \end{pmatrix} = \qquad (9)$$
$$= \begin{pmatrix} f(x,t) + g_1(x,t)\omega^*(x,t) + g_2(x,t)\upsilon^*(\xi,t) \\ f_o(\xi,t) + g_o(\xi,\hat{y},u,t,\Gamma) \end{pmatrix}$$
$$h_e(x,\xi,t) = \upsilon^*(\xi,t) - u^*(x,t) \qquad (10)$$
$$\Phi(x,\xi,t) = \frac{1}{\gamma^2}\left(\frac{\partial W(x,\xi,t)}{\partial x} g_1(x,t)\right)^T (11)$$

  with $\omega^*(x,t)$ defined as in (7), and $\upsilon^*(\xi,t)$, a realizable approximation to $u^*(x,t)$ in (7), and Γ a constant matrix value.
- (iii) The subsystem

$$\dot{x} = f(x,t)$$
$$\dot{\xi} = f_o(\xi,t) + g_o(\xi,\hat{y},0,t,\Gamma)$$

  is locally asymptotically stable.

Then, the control law u given by

$$\dot{\xi} = f_o(\xi,t) + g_o(\xi,\hat{y},u,t,\Gamma)$$
$$u = \upsilon^*(\xi,t) \qquad (12)$$

locally asymptotically stabilizes system (1) verifying the attenuation relation in (5).

Proof: Due to space limitations, the proof must be unfortunately omitted here. This result can nonetheless be proved following similar arguments to those used in (Isidori and Astolfi, 1992) for time invariant systems. For time-varying systems, as is the present case, the key argument of the proof lies on an appropriate application of the well known Barbalat theorem.

4. PARTICULARIZATION FOR EULER-LAGRANGE SYSTEMS

The result in theorem 2 requires obtaining the solutions to two coupled HJIE, or equivalently, finding functions $V(x,t)$ and $W(x,\xi,t)$ that satisfies partial differential inequalities (6) and (8). This is a hard problem in the general case, so in order to provide with explicit solutions, the problem is particularized to Euler-Lagrange systems as is described in the following sections

4.1 Euler-Lagrange systems

Let's consider in this section Euler-Lagrange systems that can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \omega(t) \qquad (13)$$

where, as is usual notation, $M(q)$ is the positive definite inertia matrix, $C(q,\dot{q})$ represents the Coriolis-centrifugal terms, and $G(q)$ is the potential energy term. The system is actuated by generalized force-torque vector τ, under the influence of exogenous disturbances $\omega(t)$.

If the state vector is taken to be $\tilde{x} \in \mathbb{R}^n$ as

$$\tilde{x} = \begin{pmatrix} \dot{q} - \dot{q}_r \\ q - q_r \end{pmatrix}$$

and assuming that $q_r(t)$ is a time varying reference to be followed, it can be easily interpreted as an stacked measure of the tracking position and velocity errors.

Using the following transformation from (Johansson, 1990)

$$z = T_0\tilde{x} \quad T_0 = \begin{pmatrix} \rho I & T_{12} \\ 0 & I \end{pmatrix} \qquad (14)$$

and applying the control action change

$$\tau = M(q)\ddot{q}_r + C(q,\dot{\hat{q}})\dot{q}_r + G(q) - \frac{1}{\rho}M(q)T_{12}\dot{\tilde{\hat{q}}}$$
$$-\frac{1}{\rho}C(q,\dot{\hat{q}})T_{12}\tilde{q} + \frac{1}{\rho}u \qquad (15)$$

where $T_1 = (\rho I \quad T_{12})$, system (13) can be expressed as

$$\dot{\tilde{x}} = f(\tilde{x},t) + g_1(\tilde{x},t)\omega + g_2(\tilde{x},t)(u + u_r) \qquad (16)$$

with

$$u_r = (\rho C(q,\dot{q}_r) - M(q)T_{12} - C(q,T_{12}\tilde{q}))\,\dot{\tilde{y}} \qquad (17)$$

It is worth to mention that this transformation yields an applicable control law since (15) only depends on accessible magnitudes, while nonaccesible components ($\dot{q}$) are lumped on the u_r term in (17).

The rest of terms in (16) can be easily proven to take the form

$$f(\tilde{x},t) = T_0^{-1}\begin{pmatrix} -M^{-1}(q)(\frac{1}{2}\dot{M}(q,\dot{q}) + N(q,\dot{q})) & 0 \\ \frac{1}{\rho}I & -\frac{1}{\rho}T_{12} \end{pmatrix} T_0\tilde{x} \qquad (18)$$

and

$$g_1(\tilde{x},t) = g_2(\tilde{x},t) = T_0^{-1}\begin{pmatrix} M^{-1}(q) \\ 0 \end{pmatrix} \qquad (19)$$

If additionally, equations (2) and (3) of the general formulation are particularized as

$$z(\tilde{x},u) = \frac{1}{2}\tilde{x}^T Q\tilde{x} + \frac{1}{2}u^T Ru$$

with Q and R positive definite matrices of appropriate dimensions, and

$$y = q - q_r = \begin{pmatrix} 0 & I \end{pmatrix}\tilde{x}$$

the following observer structure can be stated

4.2 Observer structure

With these definitions, the generic observer structure in (4) can take the form

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{pmatrix} = \begin{pmatrix} -M^{-1}C(q,\hat{x}_1)\hat{x}_1 - M^{-1}G + M^{-1} \quad + \\ +\frac{1}{\rho}M^{-1}C(q,\hat{x}_1)T_{12}\tilde{y} + \Gamma_2\tilde{y} \\ \\ \hat{x}_1 - \frac{1}{\rho}T_{12}\tilde{y} + \gamma_1\tilde{y} \end{pmatrix}$$
$$\hat{q} = \hat{x}_2 \qquad (20)$$

where functional dependencies on $M(q)$ and $G(q)$ have been omitted for the sake of compactness, and $\Gamma_1 = \gamma_1 I_{n \quad n}$ and $\Gamma_2 \in \mathbb{R}^{n \quad n}$ is a positive definite matrix.

Thus, if $\tilde{\xi} = \begin{pmatrix} \dot{\tilde{y}}^T & \tilde{y}^T \end{pmatrix}^T$ is defined, the *observer error dynamics* can be expressed

$$\dot{\tilde{\xi}} = T_0^{-1}\begin{pmatrix} -M^{-1}(q)C(q,\dot{q}) & 0 \\ \frac{1}{\rho} & -\frac{1}{\rho}T_{12} \end{pmatrix} T_0\tilde{\xi} + \qquad (21)$$
$$+\begin{pmatrix} \frac{1}{\rho}M^{-1}(q)C(q,\dot{\tilde{y}})T_{12}\tilde{y} - M^{-1}(q)\Gamma_2\tilde{y} - \gamma_1\dot{\tilde{y}} - \\ -M^{-1}(q)C(q,\dot{\hat{q}})\dot{\tilde{y}} \\ 0 \end{pmatrix}$$

4.3 Observer explicit formulation

Expression (21) gives the dynamical structure of system's observer provided appropriate matrices Γ_2, T_{12} and scalars $\rho \geq 0$, γ_1 can be found. The following result makes use of the generic structure developed in theorems 1 and 2, to provide analytical conditions for this unknowns to be found.

Theorem 3 : Assume matrices $K_1 \geq 0$, T_{12} and scalar $\rho \geq 0$ can be found such that

- $\begin{pmatrix} 0 & K_1 \\ K_1 & 0 \end{pmatrix} - T_1^T(R^{-1} - \frac{1}{\gamma^2}I)T_1 + Q \leq 0$
- $A_1 = \begin{pmatrix} \rho^2\bar{R}I & \frac{1}{2}K_1 + \rho\bar{R}T_{12} \\ \frac{1}{2}K_1 + \rho\bar{R}T_{12}^T & \bar{R}T_{12}^T T_{12} \end{pmatrix} \leq 0$
- $\Gamma_2 > \frac{T_{12M}^2 k_c \|\dot{\tilde{y}}\|}{\rho T_{12m}} I; \quad \gamma_1 > \frac{k_c k_r}{M_m}$ for $\|\dot{q}_r(t)\| \leq k_r \ \forall t \geq 0$

with $\bar{R} = \frac{1}{\gamma^2}I - R^{-1}$ and k_c satisfying, as is inherent to Euler-Lagrange systems, the property $\|C(q,\dot{q})\| \leq k_c\|\dot{q}\|$. Additionally, $(\cdot)_M$ and $(\cdot)_m$ denote respectively the maximum and minimum eigenvalue of the corresponding matrix.

If these conditions are satisfied, the control law $v^*(\xi,t) = \rho\dot{\tilde{\dot{y}}} + T_{12}\tilde{y}$ locally asymptotically stabilizes the *controller+observer* system satisfying the required $\mathcal{L}_2$-gain attenuation relation associated to the $\mathcal{H}_\infty$ problem.

Moreover, it is possible to give an estimation of the attraction basin of the combined *controller+observer* system as

$$S = \left\{ \|\chi\| < \kappa min\{\frac{1}{k_c}(\rho^2(M_m\gamma_1 - k_c k_r)), \frac{\rho T_{12m}k_{pm}}{T_{12M}^2 k_c} \right\}$$

with $\chi = \begin{pmatrix} \tilde{x}^T & \tilde{\xi}^T \end{pmatrix}^T$ and $= \frac{1}{\sqrt{2}}\sqrt{\frac{L_m}{L_M}}$

Proof: Due to space limitations, only a brief sketch of the proof is given. Theorem 3 is the result of particularizing the functions $V(x,t)$ and $W(x,\xi,t)$ in theorem 2, to the following expressions

$$V(\tilde{x},t) = \frac{1}{2}\tilde{x}^T T_0^T \begin{pmatrix} M(q) & 0 \\ 0 & K_1 \end{pmatrix} T_0\tilde{x} \qquad (22)$$

and

$$W(\tilde{x},\tilde{\xi},t) = \frac{1}{2}\tilde{x}^T T_0^T \begin{pmatrix} M(q) & 0 \\ 0 & K_1 \end{pmatrix} T_0\tilde{x} + $$
$$+ \frac{1}{2}\tilde{\xi}^T T_0^T \begin{pmatrix} M(q) & 0 \\ 0 & K_2 \end{pmatrix} T_0\tilde{\xi} \qquad (23)$$

Thus, the first condition of the theorem is obtained by direct substitution of (22) in (6).

Taking now

$$U(x,\xi,t) = V(x,t) + W(x,\xi,t)$$

as joined Lyapunov function for the *controller + observer* system, it is easy to show that

$$\frac{dU(x,\xi,t)}{dt} \leq -\frac{1}{2}\|h_1(x)\|^2 - \frac{1}{2}v^{*T}(\xi,t)Rv^*(\xi,t) -$$
$$-\frac{1}{2}\gamma^2\|\omega^*(x) + \Phi\|^2 + \left(\frac{\partial V}{\partial \tilde{x}} + \frac{\partial W}{\partial \tilde{x}}\right) g_1 u_r + \frac{\partial W}{\partial \tilde{\xi}} g_o (24)$$

expression which has been conveniently simplified by using the relations obtained from substituting (22) and (23) in (6) and (8) respectively.

Expression (24) can be forced to be negative if the rightmost terms satisfy

$$\left(\frac{\partial V}{\partial \tilde{x}} + \frac{\partial W}{\partial \tilde{x}}\right) g_1 u_r + \frac{\partial W}{\partial \tilde{\xi}} g_o \leq 0 \qquad (25)$$

This expression can be expanded by using the definitions in (22) and (23) such that it is transformed in an expression of the form

$$\begin{pmatrix} \tilde{x}^T & \tilde{\xi}^T \end{pmatrix} \begin{pmatrix} A_1 & A_3 \\ 0 & A_2 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{\xi} \end{pmatrix} \leq 0 \qquad (26)$$

where A_1, A_2, and A_3 functional matrices of $\tilde{x}$, $\tilde{\xi}$ and t. Applying at this point the well known Schur complement result, it is possible to obtain the second and third conditions of the theorem from imposing $A_1 \leq 0$ and $A_2 \leq 0$ respectively.

The additional result on the attraction basin can be obtained by computing an upper bound on expression (26), and assuming bounded reference velocity ($\|\dot{q}_r(t)\| \leq k_r \ \forall t \geq 0$).

5. EXPERIMENTAL RESULTS

To verify the theoretical analysis, a series of experiments were performed on a gyrostabilized platform. The platform has two degrees of freedom, such that can orientate any attached device according to a range of *orientation* and *elevation* coordinates. The system has a couple of gyroscopic devices to provide attitude relevant information for feedback control.

Figure 1 shows the tracking performance on the elevation axis of the platform, for a reference trajectory consisting in a series of steps linked by smooth fifth order polynomial interpolations. The

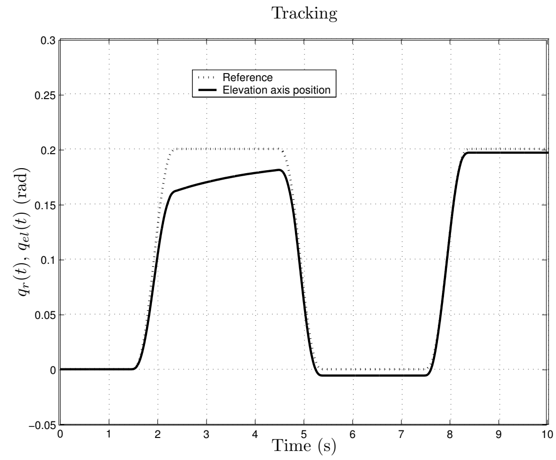

Fig. 1. Tracking behavior of the controlled platform

velocity estimator was given initially a perturbed estimation of velocity. This causes the tracking to be rather poor for the first few seconds. Nonetheless, it can be observed how the control structure gradually corrects the initial error, driving the system satisfactorily after the second step. It is worth to mention that the slight residual tracking error that can be observed in the graphics is not a consequence of the control technique employed, but of the unmodeled friction phenomena. Interestingly,

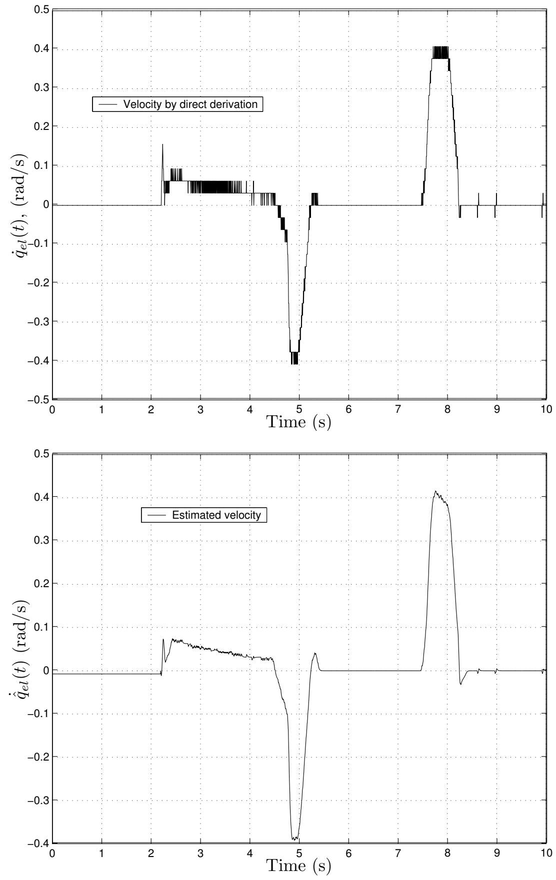

Fig. 2. Estimated velocity vs. measured velocity

figure 2 shows the behavior of both, the estimated velocity obtained from the observer structure, and the velocity obtained from first order derivation of the position information of the system, which results in more noisy signal.

6. CONCLUSIONS

This paper has presented an approach to design nonlinear measurement feedback $\mathcal{H}_\infty$ control laws. The paper generalizes previous results on the topic for the case of time-varying systems, such that a combined *controller+observer* structure can be designed that guarantees local asymptotic stability of the overall system while keeping bounded effects of disturbances acting on the system. More precisely, the control verifies an $\mathcal{L}_2$-gain attenuation for the mapping $\omega(t) \mapsto z$ less than a given constant value, γ.

Additionally, a solution for the particular case of the reference tracking problem in Euler-Lagrange systems is provided. For this kind of systems the above mentioned results are particularized, such that explicit conditions for the existence of the controller are given.

Finally, experimental results of the proposed technique are presented with application to a gyrostabilized platform, showing good results for the tracking problem proposed.

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