ADAPTIVE OPTIMAL CONTROL OF SHIP STEERING AUTOPILOTS

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Abstract: This paper describes the application of an adaptive LQG/LTR controller to automatic steering of ships. The controller is based on Nomoto's model and on the innovation model to identify the discrete time system. The benefits of this controller are demonstrated by simulation.

Keywords: Ship Control, Adaptive Control, Loop Transfer Recovery, Robust Control.

1. INTRODUCTION

A conventional autopilot for ship steering is based on the PID algorithm. In several cases manual adjustments of the regulator are necessary because the dynamics of a ship vary with speed, trim and loading. Also disturbances in term of wind, waves, currents, etc., must be take into account. For all this, it is of interest to have adaptive autopilots.

One of the possible solutions to controlling a plant with parameters which vary with time is by means of a self-tuning regulator. The basic self-tuning controller consists of a suitable combination of a recursive parameter estimation algorithm combined with a linear controller whose parameters are computed from the process parameter estimates. For this it is possible to implement a wide variety of self-tuning algorithms (Astrom 1983).

This article presents a self-tuning regulator in which the controller is an LQG/LTR. That is a LQG controller with a mechanism of Loop Transfer Recovery (LTR), to leads a more robust controller (Doyle 1979).

The control structure that we used is decribed in section 2. In section 3 we ilustrated the proposed method with an example by simulation and finally the major conclusions to be drawn are given.

2. CONTROL STRUCTURE

There are two different basic operations for controlling a ship: course changing and course keeping and in general two different controller structures will be necessary. For the second structure (course keeping) with small variation about the operating point, it is possible to obtain a 3^{rd} order model of the system and in most cases this model can be simplified to a second order system (Nomoto's model). It make the relation between the heading (ψ), and the rudder (δ).

It is convenient to add a pure time delay to model the steering engine dynamics and unmodelled dynamics. With this, the transfer function of the system is,

$$G(s) = \frac{\psi(s)}{\delta(s)} = \frac{K}{s(s+1/T)}\, e^{-\tau s} \qquad (1)$$

The sampled version of the system with the model of the disturbances due to wind, waves and measurement errors, described as random process, corresponds to the following equations (Astrom 1983).

$$A(z^{-1})y(k) = B(z^{-1})u(k) + C(z^{-1})e(k)$$

$$\begin{aligned} \text{with:}\quad A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} \\ B(z^{-1}) &= b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} \qquad (2) \\ C(z^{-1}) &= 1 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3} \end{aligned}$$

This model of the system is used for control propose and it is identifier by ELS in each sampling period. It has been verified by extensive experiments on many ships, that the model can indeed capture the essencial characteristics of ship steering dynamics (Astrom 1983).

The adaptive control structure used, as we can be seen in figure 1, corresponds to that of a self-tuning regulator (STR) which, in brief, consists in calculating the parameters of the regulator supposing that the plant parameters are those given by means of an identification algorithm. Recursive extended least square as identification algorithm has been used. In each sampling period the self-tuning regulator consists of the following steps: a)

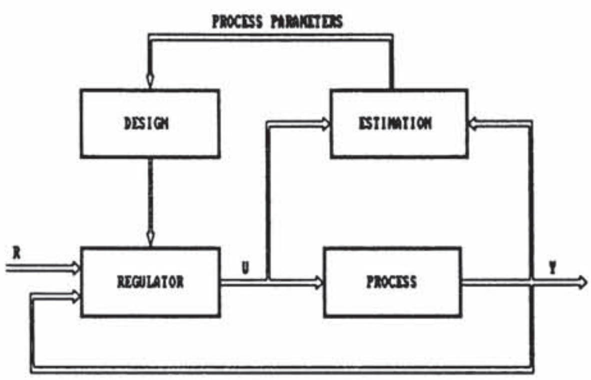

Figure 1: Diagram of the self-tuning controller

An estimation of the parameters of a linear model by measuring the inlet and outlet values of the process. b) The adjustment of the parameters of the regulator. c) The calculation of the control signal.

In this case, the regulator correspond to a LQG/LTR controller. The controller parameters are calculated on the basis that the estimated parameters are the true parameters. This implies that closed loop process behavior depends, to a great extent, on the accuracy of the parameter estimate and on the robustness of the controller to changes in the parameters of the system.

Linear Quadratic Gaussian Controller

Considering the process model,

$$A(z^{-1})y(k) = B(z^{-1})u(k) + C(z^{-1})e(k) \quad (3)$$

It can be written (Astrom 1989), in state space form as,

$$\begin{aligned} \hat{x}(k+1) &= \overline{A}\hat{x}(k) + \overline{B}u(k) + \underbrace{K_{op}e(k)}_{v_1(k)} \\ y(k) &= \overline{C}\hat{x}(k) + \underbrace{e(k)}_{v_2(k)} \end{aligned} \quad (4)$$

$$\overline{A} = \begin{pmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{pmatrix}; \overline{B} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$$\overline{C} = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \quad (5)$$

$$K_{op}^T = \begin{pmatrix} c_1 - a_1 & \cdots & c_n - a_n \end{pmatrix}$$

K_{op} is the optimal steady-state gain in the Kalman filter. This model is called the innovation model. This structure of the model has the advantage of obtaining K_{op}, directly, without having to solve the Riccati equation. Introducing the loss function,

$$\mathcal{J} = \sum_{k=0}^{\infty} \left(y^2(k) + \rho u^2(k) \right) \quad (6)$$

the optimal regulator is given by,

$$u(k) = -K_c\hat{x}(k)$$

where $K_c = (R_c + B^T PB)^{-1} B^T PA$ and P is obtained from the well known Riccati equation:

$$A^T PA - P - A^T PB(R_c + B^T PB)^{-1} B^T PA + Q_c = 0$$

with $Q_c = C^T C$ and $R_c = \rho$

Considering plant model equation 4, we have,

$$\begin{aligned} M_o = var[v_1(k), v_2(k)] &= K_{op}\sigma_e^2 \\ Q_o = var[v_1(k), v_1(k)] &= K_{op}\sigma_e^2 K_{op}^T \\ R_o = var[v_2(k), v_2(k)] &= \sigma_e^2 \end{aligned} \quad (7)$$

with $e(k) = y(k) - \hat{y}(k)$ and $\sigma_e^2 = var[e(k), e(k)]$

The problem of finding optimal K_{op} brings about the so called *discrete-time optimal observer problem* or the *KBF problem* predictive version (Kwakernaak 1972.) This problem is solved by a recurrancy.

The filter version can be used with matrix K_{of} being obtained from the relationship: $K_{of} = A^{-1}K_{op}$, so that the complete solution of the LQG problem, is given by,

$$\begin{aligned} \hat{x}(k+1) &= \overline{A}\hat{x}(k) + \overline{B}u(k) + K_{op}[y(k) - \hat{y}(k)] \\ y(k) &= \overline{C}\hat{x}(k) \\ \xi(k) &= \hat{x}(k) + K_{of}[y(k) - \hat{y}(k)] \\ u(k) &= -K_c\xi(k) \end{aligned} \quad (8)$$

It has been observed that the *Linear Quadratic Gaussian Controller* (LQG) method worked well when very precise mathematical models were used, but the method was extremely sensitive to imprecisions in the parameters and to structural modifications.

The idea is to try to recuperate the open loop transfer function (LTR) which is provided by the application of the control law alone, because in this way stability is assured, there is little sensitivity, and the temporary specifications are fulfilled. This can be achieved, in theory, acting on the parameters of the Kalman filter so that the open loop transfer function, when the Kalman filter has been introduced, approximates the original open loop transfer function.

For continuous time systems, this can be achieved by making the *Kalman filter* gains depend on a determined parameter q and the open loop LQG asymptotically approximates the open loop LQR (Doyle 1979). It is also possible to make the recuperation, in a dual form, by acting on the parameters of the loss function in the LQR problem in accordance with a *sensitivity recovery* procedure due to Kwakernaak (1972).

In the case of discrete systems (Maciejowski 1985), it can be shown that for minimum phase systems, $det(CB) \neq 0$ and using the filtered version of the Kalman filter as observer, perfect recuperation is obtained acting on the parameters of the LQR.

In continuous systems there is a complete duality between the state vector feedback LQR and the

state estimation KBF, which allows for the recuperation of the two options, given the duality of both problems. However in the case of discrete systems this duality is not complete. The state vector feedback scheme is the dual one of the predictive version of the observer. Therefore, it is not possible to achieve an exact recuperation in the case of the Kalman filter modifications. However the use of the Kalman filter recuperation option frequently yields useful results.
Then the LQG/LTR method works well for minimum phase systems but cannot be relied on for non-minimum phase ones. In several cases it works well also for non-minimun phase systems, that has been proved by simulation studies (Maciejowski 1985, López 1992).
Based in this idea, we propose to modify the Kalman filter covariance matrices (that is, to change K_{op} and K_{of}), in order to obtain the recuperation. To do this the *Kalman filter* is designed with some fictitious covariance matrices. The following are used:

$$Q = Q_o + qBB^T \quad ; \quad R = R_o$$

where Q_o and R_o (eq. 7), are nominal covariance matrices and q a parameter.
The *Kalman filter* is calculated from the modified covariance matrix. In this way the greater value of the parameter q the nearer it is to the open loop transfer function of the LQR. By doing this, precision in the state estimation is lost because the *Kalman filter* is calculated by fictitious covariances. However robustness is gained.
Self-tuning Controller
An explicit self-tuning control algorithm was developed incorporating the LGQ/LTR design method of the previous section, the parameters of the system model being determined on-line via recursive extended least squares estimation.
For this, consider the process model of the equation 3, that for estimation purposes, it can be expressed as:

$$y(k) = \varphi^T(k)\theta(k)$$

$$\begin{aligned}\varphi^T(k) \quad = \quad & [-y(k-1), -y(k-2), \cdots, -y(k-n), \\ & u(k-1), u(k-2), \cdots, u(k-n), \\ & e(k-1), e(k-2), \cdots, e(k-n)]\end{aligned}$$

$$\theta^T(k) = [a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n, c_1, c_2, \cdots, c_n]$$

The parameters identifier is a very important part of the self-tuning controllers. There are various types of identifiers which are dealt with in the relative literature. Generally, the most used methods is the recursive extended least squares are, because of its simplicity and its good convergence characteristics. The algorithm is performed by the following steps:

1. Select the initial values of $P(k)$ and $\theta(k)$.
2. Read the new values of $y(k+1)$ and $u(k+1)$.
3. Calculate the a priori error:

$$e(k+1) = y(k+1) - \varphi^T(k+1)\theta(k)$$

4. Calculate $L(k+1)$ given by the expression:

$$L(k+1) = \frac{P(k)\varphi(k+1)}{c(k) + \varphi^T(k+1)P(k)\varphi(k+1)}$$

5. Calculate the new parameter estimated given by:

$$\theta(k+1) = \theta(k) + L(k+1)e(k+1)$$

6. Actualize the covariance matrix.

$$P(k+1) = \left(I - L(k+1)\varphi^T(k+1)\right)\frac{P(k)}{c(k)}$$

7. Calculate the new forgetting factor $c(k+1)$.

$$c(k+1) = 1-(1-\varphi^T(k+1)L(k+1))\frac{e(k+1)^2}{So}$$

If $c(k+1) < c_{min}$ Then $c(k+1) = c_{min}$

8. Actualize the measurements vector $\varphi(k+2)$.
9. Make $k = k+1$ and return to step 2.

In order to reduce the memory of the identifier we use a variable forgetting factor ($c(k)$) (Fortescou 1981), as has been described previously.
It is known that the adaptive control is nonlinear and time varying. But it is desirable to make the system as linear time invariant as possible (Lamaire 1991), because in this way, the control system is more robust to disturbances than a highly nonlinear adaptive controller.
If the controller is calculated infrequently, the adaptive controller reduces to a robust control law, that is, the adaptive controller becomes simply the best robust linear time invariant control law that one could design based only on a priori information.
For all this, the implementation of the method described in the previous section, has been make in two time scales. Every sampling time, we calculate the control law and the identifier is runnig, but the controller are redesigned only every Nc sampling time.

3. SIMULATION STUDIES

In order to study the proposed method the theoretic model of a large ship, for two different condition of speed, has been chosen, where for small deviations of the angle of the rudder it can be approximated by Nomoto's second order model so that:

$$G_1(s) = \frac{0.001880}{s(s+0.006450)}\; e^{-5s} \qquad (9)$$

$$G_2(s) = \frac{0.011085}{s(s+0.01567)}\ e^{-2s} \qquad (10)$$

These models corresponds to a cargo of 161 m with two different speeds and it is obtained from (Astrom 1989). They have been identified by ELS using a sampling time of 10 seconds.

In the starting phase the system is identified by ELS algorithm, based on a non-recursive method (robust multistep algorithm), and with this, the controller is calculated. After then, the plant is identified by recursive ELS method and is changed from G_1 to G_2. In figure 2, where the fixed controller calculated in the starting phase is used, we can see that the control effort is high and the output of the system is nearly unstable. Figure 3 shows the results of applying the adaptive controller explained in the previous section and, as can be seen, it is quite satisfactory.

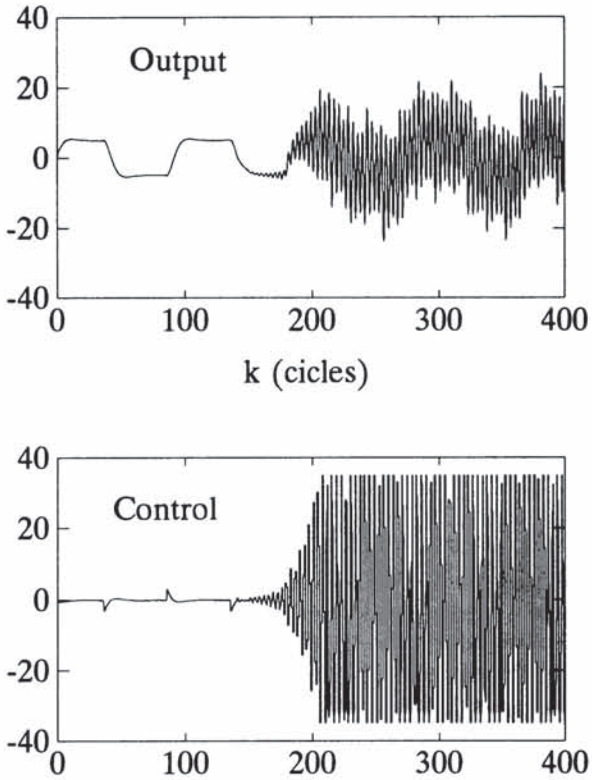

Figure 2: System response with fixed regulator

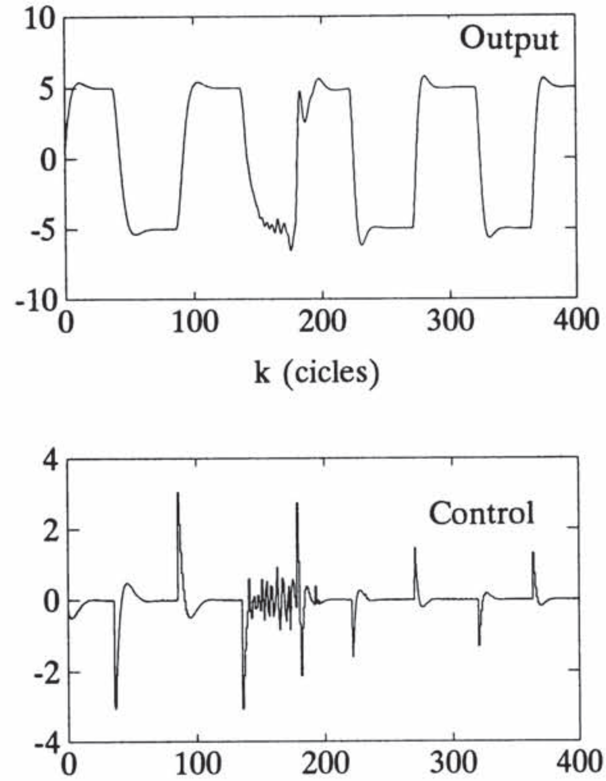

Figure 3: System response with adaptive regulator

4. CONCLUSIONS

A self-tuning regulator with a LQG/LTR controller for non-minimum phase systems with no prior knowlege of existing noise has been developed. The ELS identification method has been used and the innovations model has been considered to be the system model. The proposed method has been applied to the model of a ship and the advantages of said methods have been shown by the simulations carried out.

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