Gañán-Calvo replies The Comment claims that the physical arguments given in Ref. [1]—i.e., that all terms of the momentum equation for the liquid particles at the liquid surface become of the same order at the onset of ejection—do not take into account that the jet ejection is strongly influenced by the fastest capillary wave produced during the rim retraction process, which collapses at the bubble bottom. Following this insight, the author goes on to state that his Comment aims to reveal the *true physical mechanism* responsible for the ejection of the jet, which eventually recovers the scaling for the drop velocities in Ref. [1]. For the convenience of the reader, the original notation is used in this Reply.

The role of the capillary waves in the ejection is lengthly discussed in Ref. [2]. By virtue of the mechanisms involved [3–5], every capillary wave with wavelength λ collapsing at the axis produces a balance of all terms of the momentum equation at the instants of collapse and curvature reversal: see Eqs. (2)–(4) in Ref. [1] and the extensive study [2] for details. However, not the whole wave spectrum is effective for ejection. In fact, when those capillary waves are carefully observed {see, for example, Ref. [6], and the simulations in Ref. [7], Figs. 4(a) and 4(b)}, the fastest capillary wave *is not* the one responsible for the effective ejection. On the contrary, it can be said that, in general, the *slowest* but *strongest* capillary wave of the spectrum is the one that launches a liquid column with sufficient length and mechanical energy to sustain the ejection of a droplet. That strongest wave is precisely the one whose wavelength (L in the original work) is proportional to the radius of the parent bubble R_o (see Refs. [1,2]; note that constants of proportionality are not considered in dimensional and scaling analysis). The way to get to this conclusion is to formulate a global energy balance {Eq. (5) of the original work [1]}. The validity of this fundamental result and the scaling proposed in Ref. [1] has been independently demonstrated in Refs. [7–9].

The Comment provides a seemingly ingenious way to get to the correct scaling of the speed of ejection, circumventing the condition of sufficiency for that ejection. According to the Comment, the fundamental fact (missed by previous authors) is that the fastest capillary wave capable to reach the axis would be the one responsible for ejection. Unfortunately, the reasoning of the Comment involves two interesting flaws that, taken together, happen to lead to the scaling proposed in Ref. [1]. This is analyzed in the following.

First, while the Comment correctly states that the damping rate of any wave with wavelength δ is $\mu/(\rho\delta^2)$, it goes on to argue that the one responsible of the ejection should not be attenuated during the capillary time $t_{c,R_o} = (\rho R_o^3/\sigma)^{1/2}$, where R_o is the radius of the bubble. Thus, the author of the Comment demands $\mu/(\rho\delta^2) \sim (\rho R_o^3/\sigma)^{-1/2}$. However, if δ is smaller than R_o, the fastest wave with speed $[\sigma/(\rho\delta)]^{1/2}$ would effectively collapse in a time comparable to $t_{c,\delta} = R_o/[\sigma/(\rho\delta)]^{1/2} = (\rho\delta R_o^2/\sigma)^{1/2}$, which is *shorter* than t_{c,R_o}: consequently, the Comment author's demand is meaningless. In reality, stating $\mu/(\rho\delta^2) \sim t_{c,R_o}^{-1}$ is a way to forcefully introduce the needed length R_o that appears in the correct scaling.

The next dubious idea in the Comment is to state that $\rho V^2 \sim \mu V/\delta$ is valid for the whole spectrum, where V is the velocity induced by the wave with wavelength δ. This is simply to demand that the Reynolds $\mathrm{Re}_\delta = \rho V\delta/\mu \sim 1$, which would yield inconsistent results in the absence of the first debatable assumption $\mu/(\rho\delta^2) \sim (\rho R_o^3/\sigma)^{-1/2}$. In summary, if one forces δ to be the wavelength of the wave driving the ejection, the two flaws together yield the correct scaling $V \sim (\mu^2\sigma/\rho^3 R_o^3)^{1/4} \sim V_c \mathrm{Oh}^{1/2}$, where $V_c = (\sigma/\rho R_o)^{1/2}$ and $\mathrm{Oh} = \frac{\mu}{(\rho\sigma R_o)^{1/2}}$ is the Ohnesorge number.

In conclusion, in contrast to what is stated in the Comment, the mechanism hypothesized in Ref. [1] not only implicitly takes into account the role of capillary waves as shown in Ref. [2], it also goes far beyond proposing the key condition of sufficiency for ejection, unveils the physical mechanism leading to the scaling of the speed of ejection, provides the size of the ejected droplet, and explains the physics behind the singular behavior of the system close to a critical Ohnesorge number, as it is remarkably supported by available experimental and numerical data in the ample parameter range of validity. In contrast, exclusively focused on the jet velocity and owing to some questionable hypotheses, the Comment fails to fully apprehend the true physical mechanisms of the jet ejection as claimed.

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