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On the general scaling theory for electrospraying

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A systematic dimensional rationale is proposed here to analyse the electrohydrodynamic equations governing liquid electrospraying phenomena in the well-known steady cone-jet mode with no ambient discharges. As a result, a general, unified description of the complete parametrical space for the emitted current and droplet size is given. Four main distinct subspaces, their relevant boundaries and corresponding scaling laws are identified. Laws already proposed fit in their appropriate region, and previously unknown laws are found. A closed solution for the electric current I when inertia and polarization forces dominate is obtained, in agreement with published experimental results.

1. Introduction

The use of electrohydrodynamic forces to disintegrate liquids from the micron down to the nanometer range in an orderly way, e.g. by so-called cone-jet electrospraying (Zeleny 1917; Taylor 1964; Cloupeau & Prunet-Foch 1989), has great relevance in the field of liquid atomization, with thousands of publications per year and commercial devices making use of it. Furthermore, since the droplets produced are highly charged, it has been applied with much success to the mass spectrometry of large biomolecules. Although electrospray is a robust and controllable phenomenon, many aspects remain not completely understood, stirring much controversy.

This work aims to propose a general dimensional description of the entire working parameter space for steady cone-jet electrospraying. As a result, we have established a parametrical two-dimensional ‘chart’ with four distinct parametrical regions and corresponding scaling laws for the droplet size and the emitted electric current, to guide electrospray users for any given liquid and working conditions. Although two of them have been already identified, two are new. These laws are compared with available published experiments to show their validity.

Some numerical solutions were recently presented (Higuera 2003) to describe the complete transition region between an infinite Taylor cone and an infinite asymptotic jet (Gañán-Calvo 1997), which solve the eigenvalue problem of the emitted electric current as a function of the liquid properties and the emitted flow rate. However, a complete systematic parametrical study of the phenomenon, including the asymptotic limits and regions of interest, has never been theoretically attempted. This work is focused on the physical and mathematical modelling formalism of the cone-jet electrospraying phenomenon in order to investigate whether a complete parametrical description to identify all physically possible regimes and asymptotic limits can be established (Barenblatt 1987, 1996).

2. Analysis rationale

Consider the cone-jet configuration of figure 1. A cone-like meniscus is attached to a feeding tube with diameter D, from whose apex a thin liquid jet is emitted. The

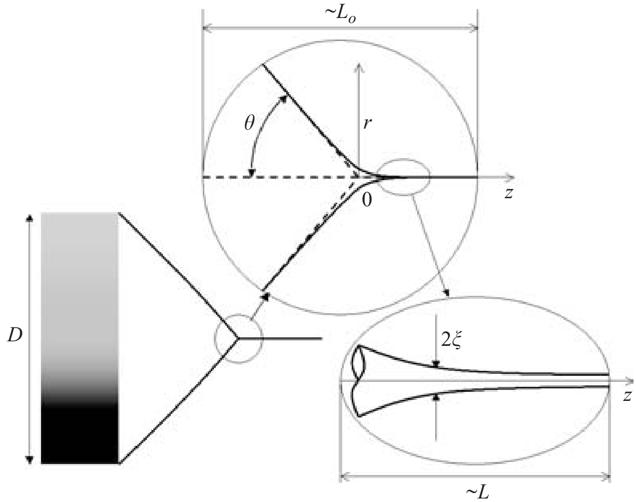

FIGURE 1. The cone-jet geometry and coordinates.

problem variables z, r, ξ, v, E_n, E_n^i, E_s, and $\tau_s = \varepsilon_o(E_n - \beta E_n^i)E_s$ are respectively the axial coordinate along the jet, radial coordinate, the jet radius, liquid velocity, normal outer and inner electric fields on the jet surface, the surface electric field in the axial direction, and the tangential surface stress (Melcher & Warren 1971; Gañán-Calvo 1997, 1999; Hohman *et al.* 2001*b*). The problem parameters σ, K, ρ, μ and Q are the liquid–gas surface tension, liquid electric conductivity, density, viscosity and emitted flow rate, respectively. β is the ratio of the liquid to vacuum permittivities $\beta = \varepsilon_i/\varepsilon_o$.

Using Coulomb's law, one may express the potential $\Phi(r, z)$ due to the cone-jet charged surface as that given by a charge line distribution $A(z)$ at the axis:

$$\Phi(z, r) = \int_{-\infty}^{\infty} \frac{A(z')\,\mathrm{d}z'}{4[(z-z')^2 + r^2]^{1/2}} \tag{2.1}$$

where E_n and E_s must be equal to the negative of the normal and tangential partial derivatives of Φ, respectively, at the cone-jet surface given by $r = \xi(z)$ (Hohman *et al.* 2001*b*). The electric problem so stated is left undetermined unless the appropriate boundary conditions are given, which include the upstream applied electric potential at the liquid feeding tube, and the downstream spray structure or electrode geometry ahead of the issuing jet, together with the appropriate chain of electrostatic 'images' from $-\infty$ to ∞ (Hohman *et al.* 2001*b*). In a virtual problem with no liquid emission (Pantano, Gañán-Calvo & Barrero 1994), and therefore with an equipotential cone surface, each applied electric potential within a narrow range would give a particular cone-like meniscus geometry satisfying all boundary conditions. The local structure of the electric field in the vicinity of the cone tip (characteristic length L_o, figure 1) is shown to be Taylor's (Taylor 1964; Pantano *et al.* 1994), where electrostatic and surface tension forces alone balance. Locally, and assuming $z = 0$ at the cone tip, Taylor's electric field is equivalent to that given by the following charge line distribution:

$$A(z) = (\sigma/\varepsilon_o)^{1/2}\mathscr{D}_o 2^{1/2}(-z)^{1/2} \tag{2.2}$$

for negative z values, where $\mathscr{D}_o = [\tan(\theta_T)Q_{1/2}'^2(\theta_T)]^{-1/2}$, and θ_T and $Q_{1/2}'$ are the Taylor angle in the absence of emission (see Pantano *et al.* 1994; Gañán-Calvo 1997) and the derivative of the Legendre function of order 1/2, respectively. Note that this expression is independent of the applied potential. Imagine now that liquid emission is in the form of a steady, extremely thin liquid jet of radius ξ, and that the local size L_o of the tip region is sufficiently large compared to the typical jet radius. In this case, the problem will be nearly independent of the outer, far boundary conditions and the applied potential at the scale D (the small influences of the applied voltage and the presence of the charged spray are not dealt with in this work). Thus, following previous studies (Higuera 2003; Gañán-Calvo 1997), we will assume that the cone-jet transition with typical dimension L is sufficiently local ($L \lesssim L_o$) to neglect the role of the applied potential at the scale D, as long as this potential is sufficient to maintain the steady cone-jet regime.

To describe the steady cone-jet electrospray phenomenon, we make the common, well-established assumption of a 'leaky dielectric' (Saville 1997), which allows bulk free charges to relax to the liquid surface in times t_e smaller than any other characteristic time t_h of the process, i.e.

$$t_e = \varepsilon_i / K \ll t_h = d^2 L Q^{-1}, \tag{2.3}$$

thus defining a quasi-steady state (Saville 1997; Gañán-Calvo 1997), where d and L are the characteristic transversal and axial distances, respectively (figure 1). This condition will subsequently be verified and can be expressed as $t_e/t_h = \beta \varepsilon_o Q (K d^2 L)^{-1} \ll 1$. Under this condition, $\beta E_n^i \ll E_n$, and thus the surface charge can be expressed as $\sigma_e \simeq \varepsilon_o E_n$. The jet slenderness (Melcher & Warren 1971; Eggers & Dupont 1994; Gañán-Calvo, Dávila & Barrero 1997; Gañán-Calvo 1997, 1999; Hohman *et al.* 2001*b*) also allows some important simplifications. First, owing to the smallness of the jet diameter, taking the limit of (2.1) for $O(r = \xi) \ll O(z)$ one can write

$$E_n \simeq A/\xi \tag{2.4}$$

at the outer jet surface. Secondly, since the surface stress is readily diffused into the whole liquid jet section, we can assume a plug-flow-field axial velocity written by continuity as

$$v = Q(\pi \xi^2)^{-1}. \tag{2.5}$$

Thus, making use of cylindrical coordinates centred at the cone-jet necking (see figure 1), the slender approximation of the liquid momentum equation in the z-direction can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\sigma}{\xi} + \frac{1}{2\pi^2}\frac{\rho Q^2}{\xi^4}\right) + \frac{6\mu Q}{\pi \xi^2}\frac{\mathrm{d}}{\mathrm{d}z}\left(\xi^{-1}\frac{\mathrm{d}\xi}{\mathrm{d}z}\right) = \frac{2\varepsilon_o E_n E_s}{\xi} + \frac{\varepsilon_o}{2}\frac{\mathrm{d}}{\mathrm{d}z}\left[E_n^2 + (\beta - 1)E_s^2\right]. \tag{2.6}$$

The three terms on the left-hand side stand for the axial resultant of the surface tension force, the liquid inertia, and the resultant of the viscous resistance in the axial direction. The two terms on the right-hand side are the axial component of the tangential electrostatic surface stress, and the axial resultant of the normal electrostatic surface stress (comprising the electrostatic 'suction' and the polarization force). The normal and tangential dynamical conditions at the liquid surface are included in (2.6). Furthermore, the formal asymptotic boundary conditions at $z \to \infty$ require $\mathrm{d}\xi/\mathrm{d}z \to 0$ (assuming that the jet breakup zone is far away), while $\mathrm{d}\xi/\mathrm{d}z \sim O(1)$ for $z < 0$ (conical region), with $\mathrm{d}\xi/\mathrm{d}z < 0$ in the whole z domain.

Finally, the charge continuity can be expressed as

$$I = \frac{2Q\varepsilon_o}{\xi} E_n + \pi \xi^2 K E_s \tag{2.7}$$

where I stands for the total emitted electric current, which results for a given set of liquid properties and emitted flow rate. Formally, the emitted electric current I is an eigenvalue of the problem (Gañán-Calvo 1997).

In order to systematically search for possible asymptotically self-similar solutions confirming the scalings sought, we will search for generalized affine transformations of the problem equations with asymptotic invariance respect to the governing parameters identified (Barenblatt 1987, 1996). To carry out this central task of this work, we introduce the five characteristic dimensions L, d, E_1, E_2, associated with z, ξ, E_n, E_s, respectively, and I. We emphasize central because we seek generalized self-similar solutions of the equations (or invariant solutions with respect to the parameters) in the intermediate region between the cone and the developed jet, since it is in this intermediate region where the eigenvalue of the problem (the emitted electric current) is fixed.

2.1. *Dimensional arguments and derivation of the asymptotic scales*

We emphasize here that the intermediate region under analysis is where the transition from a dominant electric bulk conduction to a dominant surface charge convection takes place. Thus, from equation (2.7), one can consistently define

$$\frac{Id}{Q\varepsilon_o E_1} = 1, \quad \frac{I}{d^2 K E_2} = 1. \tag{2.8}$$

It is also essential to note that the dominant part of the integral in (2.1) is due to the presence of the conical meniscus (2.2). Thus, from (2.1) and (2.2) one can also define

$$E_2 = \left(\frac{\sigma}{\varepsilon_o L}\right)^{1/2}. \tag{2.9}$$

The condition that the self-induction electric field of the jet is never dominant leads, from (2.4), to the condition

$$\left(\frac{\sigma L}{\varepsilon_o}\right)^{1/2} \gtrsim E_1 d. \tag{2.10}$$

Finally, the momentum equation establishing a global balance between applied forces (motors) and resistance forces provides the two closing dimensional arguments to find the five characteristic dimensions (L, d, E_1, E_2, I). A consistent analysis of this balance involves the following two domain extremes:

2.1.1. *Developed jet*

In this region extreme,

$$I \to 2Q\varepsilon_o E_n/\xi, \quad E_s \to (\sigma/\varepsilon_o)^{1/2}\mathscr{D}_o 2^{1/2}\pi z^{-1/2}/4. \tag{2.11}$$

Since $\mathrm{d}\xi/\mathrm{d}z < 0$ everywhere, the only positive motor left in this region is the axial component of the tangential electrostatic surface stress (note that both E_n^2 and E_s^2 decrease with z) with a limiting value

$$\frac{2\varepsilon_o E_n E_s}{\xi} \to \left(\frac{\sigma}{\varepsilon_o}\right)^{1/2} \frac{I}{Q}\mathscr{D}_o 2^{1/2}\pi z^{-1/2}/4. \tag{2.12}$$

Therefore, this term must be always dominant in the analysis of our transition scale L. Thus, to compare surface tension, inertia and viscous forces to the dominant motor (2.12), we define the non-dimensional numbers

$$R_\sigma = \frac{\rho Q^3 \varepsilon_o^{1/2}}{I\sigma^{1/2} d^4 L^{1/2}}, \quad R_\rho = \frac{\mu Q^2 \varepsilon_o^{1/2}}{d^2 L^{3/2} I \sigma^{1/2}}, \quad R_\mu = \frac{\sigma^{1/2} Q \varepsilon_o^{1/2}}{I d L^{1/2}}. \tag{2.13}$$

There are three possibilities:

(I) Dominance of surface tension force. Defining $R_\sigma = 1$, one must have $R_\rho \lesssim 1$ and $R_\mu \lesssim 1$.

(II) Dominance of inertia. Defining $R_\rho = 1$, one must have $R_\sigma \lesssim 1$ and $R_\mu \lesssim 1$.

(III) Dominance of viscous force. Defining $R_\mu = 1$, one must have $R_\sigma \lesssim 1$ and $R_\rho \lesssim 1$.

2.1.2. *Cone-jet necking*

In this opposite region extreme, $I \to \pi\xi^2 K E_s$. Since in this region the tangential electrostatic surface stress decays faster than the axial component of the normal electrostatic stress (note that the cone is eventually a pure balance between surface tension and normal electrostatic stress), the electrostatic suction or the polarization force may dominate. Thus, if one seeks the region joining the two extremes, one is left with two possible definitions:

$$\frac{I}{Q} = \frac{\beta - 1}{L}\left(\frac{\sigma\varepsilon_o}{L}\right)^{1/2} \quad \text{or} \quad \frac{I}{Q} = \frac{1}{L}\left(\frac{\sigma\varepsilon_o}{d}\right)^{1/2}, \tag{2.14}$$

representing the dominance of either the electrostatic suction or the polarization force, respectively (obviously, in either case the alternative expression must be a limiting condition consistently with sub-dominance). The value of the liquid polarity parameter β will determine which scenario prevails for a given Q.

2.2. *The six fundamental asymptotic scales*

The six possible condition pairs between the three possibilities described in § 2.1.1 and the two described in § 2.1.2, together with equations (2.8) and (2.9), provide the six different sets of parametrical equations defining the six candidate generalized affine transformations of the problem equations which may yield asymptotically self-similar or invariant solutions with respect to the problem parameters, when the limiting conditions are considered asymptotically, i.e. assuming total dominance of the terms considered. To save space, we will only give the resulting expressions for the electric current I and jet diameter d. It is important to note that the non-dimensional equations resulting from the proposed scalings (affine transformations) are asymptotically independent of all governing parameters, and therefore their solutions (should they exist) are generalized self-similar.

2.2.1. *IE-scaling: dominance of inertia and electrostatic suction*

This is the most common parametrical regime encountered in the electrospray literature, which yields an invariant formulation whose scaling of the electric current and jet diameter is

$$I = (\sigma K Q)^{1/2}, \quad d = \left(\frac{\rho\varepsilon_o Q^3}{\sigma K}\right)^{1/6}. \tag{2.15}$$

Defining the two non-dimensional parameters

$$\alpha_\rho = \frac{\rho K Q}{\sigma\varepsilon_o}, \quad \alpha_\mu = \frac{K^2\mu^3 Q}{\varepsilon_o^2\sigma^3}, \tag{2.16}$$

the validity limits of this asymptotic invariant formulation given by all the limiting conditions may be finally expressed as

$$\alpha_\rho \gg \alpha_\mu^{1/4}, \quad \alpha_\rho/(\beta - 1) \gg 1. \tag{2.17}$$

This scaling was originally proposed by Gañán-Calvo (1999) and Hartman *et al.* (1999) without mentioning its limits of validity. Since it has been widely verified in numerous experimental works (e.g. Gañán-Calvo 1999; Hartman *et al.* 1999; Gamero-Castaño & Hruby 2002) and extensively calculated by Higuera (2003), we omit any further analysis. The scaling proposed in Gañán-Calvo (1997) lies within this region, but in that work the asymptotic analysis was closed in the neck region by an approximate patching that missed the physical details in that region described by the complete, non-dimensional equations studied here. Although the scaling for the jet diameter was correct, that approximation resulted in the appearance of a logarithm of δ_ρ in the scaling of the emitted current not satisfactorily supported by experiments.

2.2.2. *IP-scaling: dominance of inertia and polarization forces*

In this case, we obtain

$$I = \left(\frac{\rho K^2 Q^2}{(\beta-1)\varepsilon_o}\right)^{1/2}, \quad d = \left(\frac{\rho\varepsilon_o Q^3}{\sigma K}\right)^{1/6} \tag{2.18}$$

with required conditions expressed asymptotically as

$$1 \gg \frac{\alpha_\rho}{(\beta-1)} \gg \frac{\alpha_\mu}{(\beta-1)^4}. \tag{2.19}$$

This new asymptotic scaling holds for polar liquids within the limits given above. Note that the expression for the jet diameter is the same as in the previous IE-scale. Thus, droplet size measurements should be invariant with respect to whether electrostatic or polarization forces are dominant.

Furthermore, reducing this formulation to its limits, the momentum equation yields a beautiful, uniformly valid equality at the cone-jet necking (where $I \simeq \pi\xi^2 K E_s$) for sufficiently large liquid polarities:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{2\pi^2}\frac{\rho Q^2}{\xi^4}\right) \simeq \frac{\varepsilon_o}{2}\frac{\mathrm{d}}{\mathrm{d}z}\left[(\beta-1)E_s^2\right] \Longrightarrow I \simeq \left(\frac{\rho K^2 Q^2}{\varepsilon_o(\beta-1)}\right)^{1/2}. \tag{2.20}$$

For consistency and for the specialized reader, it is important to mention here that our transition region in this regime is preceded by a slender region around $z=0$ (figure 1) with characteristic diameter $d^* = (\rho Q^2/\sigma)^{1/3}$ and length $L^* = (\beta-1)d^* \gg d^*$ where one can easily verify that, given the characteristic axial electric field proportional to Taylor's ($E_2^* = (\sigma\varepsilon_o^{-1}L^{*-1})^{1/2}$), then (i) the electric conduction is dominant ($I = \pi\xi^2 K E_s$), (ii) the electrostatic suction is of the order of the polarization force, of inertia, and of the surface tension force, i.e. $\sigma/d^* \sim \rho Q^2 d^{*-4} \sim \varepsilon_o E_n^2 \sim \varepsilon_o(\beta-1)E_s^2$, and (iii) the term $2\varepsilon_o E_s E_n \xi^{-1}$ is not dominant. This intermediate region provides the consistent grounds for matching the cone to our transition region downstream.

In figure 2, we have plotted the electric current, I/I_G (where $I_G = (\sigma K Q)^{1/2}$), versus the parameter $\alpha_\rho/(\beta-1)$, from several available data sets. We have used Fernández de la Mora & Loscertales's (1994) data for formamide, water, and octanol. Octanol data agree with IE-scaling (horizontal line), while the trends for water and formamide tend to match the IP-scaling solution (inclined straight line) within its limits of validity. We have also plotted a set of interesting data for water (López-Herrera *et al.* 2004), which together with Fernandez de la Mora's data illustrate two possible solutions ('long' and 'short' cones), already reported in Chen, Pui & Kaufman (1995) for two very slightly different applied voltages (less than 0.1%). These two experimental branches suggest the existence of a solution bifurcation (in fact, a bi-stable solution is experimentally found), where the branch with larger current might involve local gas ionization effects

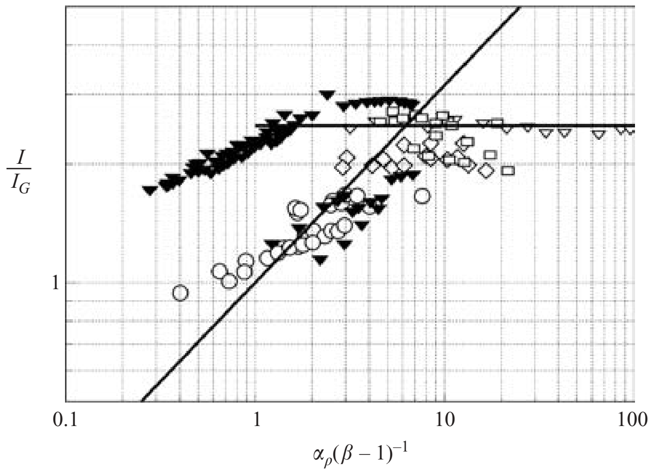

FIGURE 2. Comparison of experimental data with IE- and IP-scalings of the electric current made non dimensional with I_G. Horizontal line: IE scaling. Line of slope 1: IP scaling. Fernandez de la Mora's data: ○, formamide; □, ethylene glycol; ◇, water; ▽, octanol. López-Herrera's data: ▼.

around the cone tip (López-Herrera *et al.* 2004). Note that $t_e/t_h \ll 1$ corresponds here to $\alpha_\rho \gg 1$, which is verified for all published experimental data within this region ($\beta \gg 1$).

2.2.3. *VE-scaling: dominance of viscous force and electrostatic suction*

This regime is found in electrospinning conditions when the liquid has a sufficiently large electric conductivity. Its characteristic emitted electric current and jet diameter scale as

$$I = (\sigma K Q)^{1/2}, \quad d = \left(\frac{\mu \varepsilon_o^2 Q^3}{\sigma K^2}\right)^{1/8} \tag{2.21}$$

with validity limits

$$\alpha_\rho \ll \alpha_\mu^{1/4}, \quad \frac{\alpha_\mu}{(\beta-1)^4} \gg 1. \tag{2.22}$$

Here, $t_e/t_h \ll 1$ consistently gives $\alpha_\rho \ll \alpha_\mu^{1/4}$. This scaling, which holds in a variety of experimental situations for high-viscosity liquids, was recently and independently proposed by Higuera (2003). Figure 3 shows many different results from the literature (Gamero-Castaño & Hruby 2002; Chen *et al.* 1995; Gañán-Calvo *et al.* 1997) which agree with IE scaling ($\alpha_\rho \alpha_\mu^{-1/4} > 1$) for the droplet diameters, while the recent results by Ku *et al.* (2001) using glycerol with different electrical conductivities confirm the trend of this VE-scaling ($\alpha_\rho \alpha_\mu^{-1/4} \ll 1$). The plotted straight lines illustrate the slope of the two jet diameters from IE- and VE-scalings. However, care is required when using the VE-scaling for the jet diameter, since high-viscosity liquids usually yield very long jets, with a strong thinning far downstream of our transition L scale. One could end up with almost any droplet size dependence (increasing or decreasing) on the liquid viscosity (see for example Jayasinghe & Edirisinghe 2002), depending on the location of the breakup point. Typically, there is a larger jet-to-drop diameter ratio in a viscous breakup compared to a nearly inviscid one, but this problem is outside the scope of the present work.

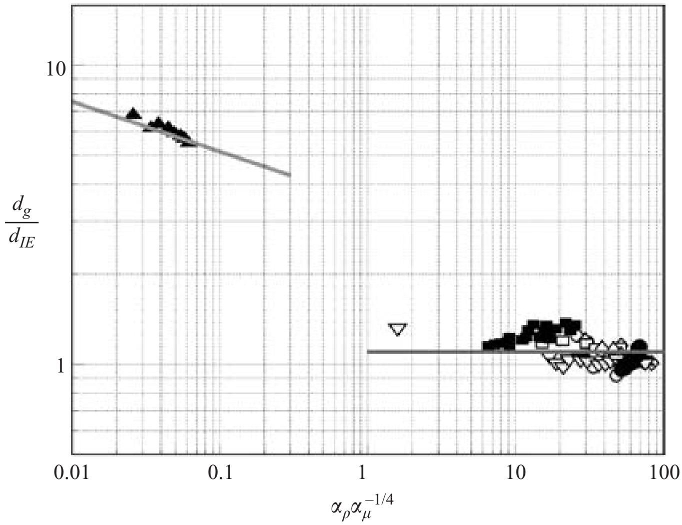

FIGURE 3. Data to the left (▲), droplet diameters d_g from Ku *et al.* (2001) using one liquid only, compared to the VE-scaling for the jet diameter (continuous line). Data to the right, droplet diameters taken from the literature (Gamero-Castaño & Hruby 2002; Chen *et al.* 1995; Gañán-Calvo *et al.* 1997) for different liquids, compared to the IE-scaling (horizontal line). Data made non-dimensional with d from the IE scaling.

2.2.4. *VP-scaling: dominance of viscous force and polarization force*

This scaling is given by

$$I = \left(\frac{\mu^3 K^3 Q^2}{(\beta-1)^4 \sigma^2 \varepsilon_o^2}\right)^{1/2}, \quad d = \left(\frac{\mu Q}{(\beta-1)\sigma}\right)^{1/2} \tag{2.23}$$

with consistent validity limits

$$\frac{\alpha_\rho}{(\beta-1)} \ll \frac{\alpha_\mu}{(\beta-1)^4} \ll 1. \tag{2.24}$$

The requirement $t_e/t_h \ll 1$ now gives $\alpha_\mu \gg 1$. This is also a new scaling which holds for very viscous polar liquids within the conditions given above. However, the author has not found published experiments under these conditions reporting the existence of a capillary conical meniscus, i.e. all situations belonging to this asymptotic regime show a very elongated meniscus shape which hardly resembles a well-defined cone region.

2.2.5. *Marginal scalings: dominance of surface tension forces*

The dominance of surface tension is a marginal situation that some authors have already theoretically considered (Cherney 1999; Higuera 2003). We mention it here for formal completeness, although we have never observed a well-defined range of experimental data belonging to a regime where surface tension dominates in absolutely stable conditions, and therefore we omit a detailed analysis. There are two possibilities:

(i) *Dominance of electrostatic suction:* If $\beta - 1$ is small enough and surface tension force dominates, the resulting invariant formulation gives

$$I = (\sigma K Q)^{1/2}, \quad d = \left(\frac{\varepsilon_o Q}{K}\right)^{1/2} \tag{2.25}$$

which is valid under conditions $\alpha_\rho \lesssim 1, \alpha_\mu \lesssim 1, (\beta-1) \lesssim 1$. This obviously holds for

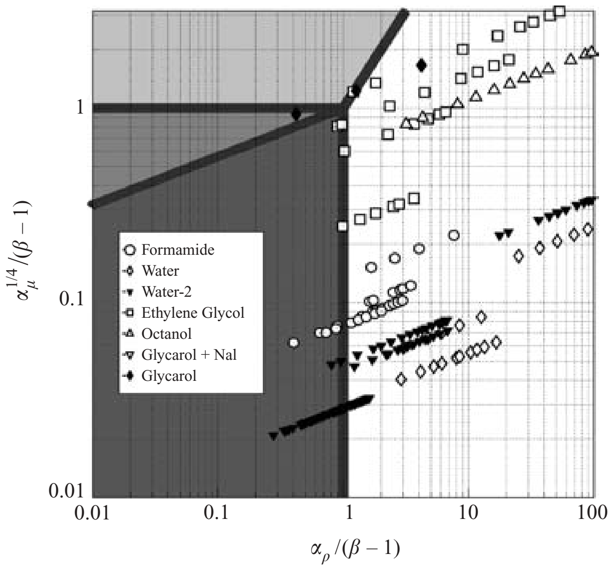

FIGURE 4. The four main parameter subspaces: white (IE-scaling), dark grey (IP-scaling), light grey (VE-scaling) and medium grey (VP-scaling). Note that the data in the IP region and close to its vertical boundary consistently tend to follow the IP-scaling in figure 2 for the current, while data in the VE region tend to follow the VE-scaling in figure 3 for the jet size.

non-polar liquids only, in the admisible limits of stability. In this scaling $d=L$, which is a limiting situation (d must be $d \lesssim L$).

(ii) *Dominance of polarization force:* If β is large enough and surface tension force dominates, the resulting asymptotic invariant formulation yields

$$I=\left(\frac{\sigma K Q}{\beta-1}\right)^{1/2}, \quad d=\left(\frac{\varepsilon_o Q}{K}\right)^{1/3}, \tag{2.26}$$

under conditions $\alpha_\rho \lesssim 1, (\beta-1) \gtrsim 1, \alpha_\mu \lesssim (\beta-1)^3$. This last scaling was originally proposed by Fernández de la Mora & Loscertales (1994), and subsequently studied by Cherney (1999). However, experimental data reported in Fernández de la Mora & Loscertales (1994) belong to any of the other four parameter subspaces (IE, IP, VE, VP), and do not seem to follow their proposed marginal scaling (see figure 2).

To summarize, the parameter space resulting from this analysis is plotted in figure 4. We show how the dimensional variables should scale with the problem parameters within each region of asymptotic validity. In each of the four parameter subspaces of interest, the set of resulting non-dimensional equations is, sufficiently far from the boundaries, independent of the problem parameters. We have proposed a closed solution for the electric current I when inertia and polarization forces dominate (IP-scaling) in agreement with published results.

Finally, it should be emphasized that the absence of applied voltage difference between the liquid and the electrode in this analysis is applicable in experimental conditions when there is an almost conical static meniscus shape held by electrostatic forces. This is typical when IE- and IP-scalings hold. When viscous forces are dominant (most electrospinning situations) the cone deforms by viscous forces at large upstream distances from the jet, making the cone hardly distinguishable from the jet in some cases (Hohman *et al.* 2001*a*; Feng 2002), which may invalidate our assumptions.

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Note added to online version

There are typographical errors in equations (2.13) and (2.14) in the paper as printed. The correct equations are

$$R_\rho = \frac{\rho Q^3 \varepsilon_o^{1/2}}{I\sigma^{1/2} d^4 L^{1/2}}, \quad R_\mu = \frac{\mu Q^2 \varepsilon_o^{1/2}}{d^2 L^{3/2} I \sigma^{1/2}}, \quad R_\sigma = \frac{\sigma^{1/2} Q \varepsilon_o^{1/2}}{I d L^{1/2}}, \tag{2.13}$$

$$\frac{I}{Q} = \frac{L}{d^2}\left(\frac{\sigma\varepsilon_o}{L}\right)^{1/2} \quad \text{or} \quad \frac{I}{Q} = \frac{\beta - 1}{L}\left(\frac{\sigma\varepsilon_o}{L}\right)^{1/2}. \tag{2.14}$$