VOLTAGE SENSITIVITY BASED TECHNIQUE FOR OPTIMAL PLACEMENT OF SWITCHED CAPACITORS

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Abstract - This paper is about the development of a fast but exact enough technique to solve the optimal allocation and sizing of capacitors on power systems, no matter the voltage level. The proposed methodology is mainly characterized by assuming a linear behaviour for the reactive problem and having an objective function that minimizes the sum of the voltage magnitude deviations from the specified voltage limits squared, for all those buses with over/under voltages. The proposed approach has been tested on the IEEE 14-bus and 30-bus systems, and on an existing distribution system with actual data. Results are compared with those of using an exact calculation of voltages.

Keywords - Capacitor placement, capacitor sizing, VAR planning, sensitivity factor, voltage profile.

1 Introduction

POwer distribution from electric power generators to consumers is accomplished via the transmission, subtransmission, and distribution lines. Voltage profile improvement and system losses reduction by capacitor installation depend greatly on how capacitors are placed and operated in the system [1, 2]. The general capacitor placement problem consists of determining the optimal location and size of capacitors to be installed and the efficient control schemes in the buses of the system [3]-[8].

Much mathematical research on the capacitor placement has been studied in distribution systems. A nonlinear programming such as gradient search method is proposed in [1]. Baran and Wu [2] decomposed capacitor placement problem into a master problem and a slave problem by using mixed integer programming. The master problem determines the location of the capacitors and the slave problem determines the type and size of the capacitors. Simulated annealing is used in [3, 4] after formulating capacitor placement problem as a discrete combinational optimization problem. Huang et al. [5] proposes by tabu search (TS)-based solution algorithm and uses sensitivity analysis method to select the candidate installation locations of capacitors to reduce the search space. Also, many researchers proposed genetic algorithm (GA) application to search global optimal solution of capacitor placement problem [6, 7, 8]. Sundhararajan et al. [6] used sensitivity analysis to search the location of the capacitors and GA to determine the size of the capacitors, which somewhat depends on experiences in the selection of the probability parameters. Miu et al. [7] suggested the two-stage algorithms that combine the good qualities of GA and a fast sensitivity-based heuristic.

This paper presents a linear reasoning for determining the optimal locations and sizes for capacitor placement for voltage profile improvement. Voltage sensitivity indices have been usually used as indicators of voltage stability and several indices of this kind have been proposed [9, 10]. One such voltage sensitivity index, dV/dQ, called the reactive voltage sensitivity factor ($RPVS$), was the first index used to predict voltage control problems [11]. This index also provides a means of allocating capacitors to improve bus voltages as we shall show.

The analysis of the results after applying the proposed algorithm to different systems allow to conclude that the new methodology is quite fast and accurate so it can be used not only by the planner of the system, but also on real time to determine the best way to operate on all the switched capacitors being on the system to improve all voltages in a global way.

The papers is structured as follows: section 2 defines bus voltage sensitivity factors and how computing them. Next, in section 3 the mathematical problem of optimization is posed, focused on minimizing voltage magnitude deviations from the specified voltage limits by using the bus voltage sensitivity factors. Once the minimization problem is solved, a heuristic algorithm looking for establishing the minimum number of allocations and switches is described. Section 4 applies the new methodology to three different cases, two IEEE test systems and an actual one. Results are finally commented and analyzed.

2 Bus voltage sensitivity factors

A sensitivity which given the variation of node i voltage magnitude due to a unit reactive power injection to node j is called the reactive power voltage sensitivity ($RPVS_{ij}$) and denoted by ($\Delta V_i/\Delta Q_j$).

The linearized steady state system power voltage equations are given by,

$$\begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{1}$$

where ΔP and ΔQ are vectors of real and reactive bus power injection changes, while $\Delta\theta$ and ΔV are vectors of bus voltage angle and magnitude changes.

The Jacobian matrix in (1) is that used when the power flow equations are solved by the Newton-Raphson technique. Although system voltages are affected by both P and Q, P can be kept constant at each specified operating point and the voltage variations can be evaluated by considering the incremental relationship between Q and V. As a consequence, making null the incremental changes in P in (1), the following relation $\Delta Q - \Delta V$ results,

$$\Delta Q = \left[L - MH^{-1}N\right]\Delta V = J_R \Delta V \tag{2}$$

Load Buses	Linear method	Exact method	Absolute error	Linear method	Exact method	Absolute error	Linear method	Exact method	Absolute error
	$RPVS_{i4}$			$RPVS_{i9}$			$RPVS_{i12}$		
4	0,0445	0,0482	0,0037	0,0206	0,0239	0,0033	0,0014	0,0019	0,0005
5	0,0268	0,0294	0,0026	0,0124	0,0146	0,0022	0,0008	0,0011	0,0003
7	0,0207	0,0226	0,0019	0,0599	0,0672	0,0073	0,0040	0,0052	0,0012
9	0,0206	0,0240	0,0034	0,1178	0,1356	0,0178	0,0078	0,0106	0,0028
10	0,0171	0,0210	0,0039	0,0978	0,1186	0,0208	0,0065	0,0092	0,0027
11	0,0089	0,0112	0,0023	0,0507	0,0631	0,0124	0,0034	0,0049	0,0015
12	0,0014	0,0018	0,0004	0,0078	0,0100	0,0022	0,2178	0,2390	0,0212
13	0,0031	0,0041	0,001	0,0178	0,0230	0,0052	0,0518	0,0619	0,0101
14	0,0130	0,0173	0,0043	0,0745	0,0979	0,0234	0,0269	0,0366	0,0097

Tabla 1: RPVS factors in each load bus of the IEEE 14-bus test system

where J_R is called the reduced Jacobian matrix of the system and its inverse directly relates the bus voltage magnitude and bus reactive power injection.

Adopting the well-known strong interdependence between active powers and bus voltage angles, and between reactive powers and voltage magnitudes, the jacobian matrix in (1) is simplified to,

$$\begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (3)$$

and, in consequence, equation (2) is reduced to,

$$\Delta Q = L \quad \Delta V \qquad (4)$$

Finally, the matrix $RPVS$, which is associated to all the PQ nodes of the system, can be obtained from this last system,

$$RPVS = \frac{\Delta V}{\Delta Q} = L^{-1} \qquad (5)$$

The former way of reasoning allows to compute a quasi-exact $RPVS_{ij}$ factors . The procedure will be the following,

1. For a given load and generator power profile, load flow equations (1) are solved to know the state variables θ_F and V_F in all buses.
2. Matrix L is computed from θ_F and V_F.
3. Matrix $RPVS$ is determined by (5)

If the equations of the Fast Decoupled Load Flow method (FDLF) were the starting point instead of the Newton-Raphson power flow equations, only the linearized, decoupled, reactive power model would be required. This one is represented by,

$$B''\Delta V = \Delta Q \qquad (6)$$

where B'' is equal to the imaginary part of the nodal admittance matrix changed of sign.

Then, matrix $RPVS$ can be computed from the inverse of B'',

$$RPVS = (B'')^{-1} \qquad (7)$$

The main advantage from computing $RPVS$ matrix from (7) instead of (5) is that matrix B'' is constant while matrix L depends on the state variables. This implies a lot of saving of computational time when this issue becomes important. On the contrary, accuracy is reduced due to the assumed simplifications after adopting the FDLF method.

Following in table 1 the RPVS factors in each load bus of the IEEE 14-bus test system are listed, both using (5) ("Exact method") and (7) ("Linear method"). The bus voltage sensibilities regards to reactive power injections in buses 4, 9 and 12 are showed, that is, the factors $RPVS_{i4}$, $RPVS_{i9}$ and $RPVS_{i12}$ respectively, where i refers to any load bus. These three nodes have been chosen as there are capacitors banks in all of them, so it could be of interest to quantify how much voltage magnitudes change when the reactive power injection in any of these three buses is modified. Absolute errors between the exact and linear form of obtaining RPVS factors are also displayed. It can be concluded from these results, and also thanks to the final relative errors computed when the compensated voltage magnitudes are determined (see table 4 in section 4), that the error by using the matrix B'' is almost null. As a consequence, the linear methodolgy to determine the RPVS factors has been finally the implemented one. Nevertheless, the exact RPVS factors are saved to study the accuracy of the results after the whole problem is solved.

It must noted that the higher the load level of the system, the larger the absolute error between the exact and the linear RPVS factors. This result was expected since the exact method takes into account the actual voltage magnitudes while the linear supposes voltage magnitudes equal to nominal voltages. On the other hand, it always results that exact $RPVS$ factors are higher than the linear $RPVS$ ones in buses whose voltages are less than the nominal value, and just the opposite in nodes with voltages larger than the nominal. These results allow to conclude that the final solution with the linear RPVS factors is always enclosed by the exact solution.

All these considerations encourage to follow with the use of the linear RPVS factors to try to solve the proposed problem.

3 Optimization procedure

Following, the optimization problem is posed by using the former RPVS factors. The objective consists of deducing the best buses where the new capacitors must be allocated and how much reactive power must be injected by those components in order to get the following aims:

- Eliminating undervoltages
- Avoiding overvoltages in any bus due to the increased injected reactive power on the system
- Minimizing the number of capacitors to be installed and the number of switches on them.

If N denotes the total number of buses in the power system, and M the number of load nodes, that is, the usually named PQ nodes, C is the size of the set S_C that comprises all those nodes where capacitors can be allocated. This set is known in advance to the formulation of the problem and the inequality $C \leq M$ is always met. For example, if the problem to solve consisted of determining the best switches over already installed capacitors, set S_C would be perfectly defined by those nodes where shunt elements are allocated; in the contrast, if it were a planning problem, probably the best point of starting would be to define S_C with all the being PQ nodes.

Once the load flow equations have been solved and RPVS factors have been obtained, the voltage mismatch vector ΔV_i is computed for each load bus i as follows,

1. If $V_i > V_i^{max} \longrightarrow \Delta V_i = V_i^{max} - V_i$
2. If $V_i < V_i^{min} \longrightarrow \Delta V_i = V_i^{min} - V_i$
3. If $V_i^{min} < V_i < V_i^{max} \longrightarrow \Delta V_i = 0$

Computing the vector ΔV_{Mx1} in this way, the overvoltages and undervoltages in the system are being quantified. Then, this vector is used to define a first version of the objective function as,

$$f = \sum_{i}^{M} \left[\left(\sum_{j}^{C} RPVS_{ij} \Delta Q_j \right) - \Delta V_i \right]^2 \qquad (8)$$

where the vector ΔQ_{Cx1} is obtained from the RPVS matrix,

$$RPVS_{MxC} \Delta Q_{Cx1} = \Delta V_{Mx1} \qquad (9)$$

No consideration has been done yet in relation to minimize the investment in new capacitors. The following measures give answer to this question:

- Buses i with $\Delta V_i = 0$ are eliminated from (8) and the related equation i in system (9) is also removed. This implies that those buses whose voltages are in limits are not forced to hold their actual values.
- Due to the previous item, buses that initially have their voltages in limits could surpass the maximum specified voltage V^{max} after the reactive power compensation. This undesirable effect can be avoided by adopting a practical maximum voltage, V^{pmax}, lower than the real maximum specified one, that is, $V^{pmax} = V^{pmax} - tol^{max}$.
- Considering overvoltages into the optimization problem allows to penalize those capacitors more sensible to increase even more those overvoltages.

So, the final minimizing problem results,

$$f = \sum_{i}^{M'} \left[\left(\sum_{j}^{C} RPVS_{ij} \Delta Q_j \right) - \Delta V_i \right]^2 \qquad (10)$$

$$RPVS_{M'xC} \Delta Q_{Cx1} = \Delta V_{M'x1} \qquad (11)$$

where M' is less than M and results after removing nodes with voltage magnitudes in limits.

After solving this optimization problem, a vector of new injected reactive powers ΔQ_{Cx1} is deduced. This solution is arranged from large to lowest size and new switched capacitors are allocated sequentially (following the order deduced) until all voltages are in limits. This form of proceeding ensures a minimum number of new capacitors. Again, RPVS factors are used to recalculate voltage magnitudes each time a new capacitor is considered to be placed,

$$V_i^{comp} = V_i^F + RPVS_{ij} * \Delta Q_j \qquad (12)$$

There are some computational issues related to the solution of (10) and (11). Some negative solutions could result after the resolution of the minimization problem, which implies inductive shunts compensating capacitors. Obviously this solution is impracticable. When this situation occurs, the minimization problem is solved again after removing those buses $j \in S_C$ with negative solution. This is done until no negative solution results. This way to proceed guarantees the minimum number of switches to compensate undervoltages. As a consequence of this issue, again a practical minimum voltage higher than the real minimum specified one must be considered, $V^{pmin} = V^{pmin} + tol^{min}$. Other point is that switched capacitors banks instead of fixed capacitors could be the available compensation shunts. In this case, when the ΔQ_j solution is finally defined as the best reactive power injection, the number of switching times for the specified capacitor bank can be easily deduced.

Summarizing, the whole process is as follows,

1. Initially, load flow problem is solved to know the state of the system: complex voltages, taps of transformers and reactive power injected by shunt capacitors being into the system.
2. Matrix of sensitivities is computed and used to define the objective function, either by a linear or an exact technique as it has been discussed in section 2.
3. Optimization problem is solved, equations (10) and (11), so a vector of new injected reactive powers is deduced.
4. The former solution is arranged from large to lowest size.

5. New switched capacitors are allocated sequentially (following the order deduced in point 4) until all voltages, recomputed by (12), are in limits.

4 Case studies

Three different systems will be analyzed with the proposed methodology,

- [A] The IEEE 14-bus test system.
- [B] The IEEE 30-bus test system.
- [C] An actual 128-bus system from a Spanish utility.

The voltage limits in all the cases have been set up at $\pm 5\%$ of nominal voltage, and the adopted maximum and minimum tolerance have been defined in such a way that the allowed voltage interval is reduced to $1.0 < V_i < 1.03$ for all buses i. Also it is supposed any load bus is sensible to allocate a new capacitor.

The two IEEE test systems have been overloaded in order to have noticeable undervoltages. This allows to know more about the accuracy of the proposed methodology.

Load Buses	Initial Voltage	ΔV	ΔQ
4	0.94640	0.0535	-1.4040
5	0.94870	0.0512	2.0067
7	0.96620	0.0332	0.2160
9	0.94180	0.0579	-0.1832
10	0.91930	0.0804	0.2830
11	0.92330	0.0765	0.2707
12	0.95760	0.0424	0.0170
13	0.92970	0.0702	0.4242
14	0.88280	0.1169	0.2776

Tabla 2: Initial voltages, ΔV vector and ΔQ vector for the IEEE 14-bus test system

The initial load state for the A system results in undervoltage levels for all load buses as it is showed in column one of table 2. From these voltages and the commented practical voltage limits, the vector ΔV is computed, which is listed in the second column of this table 2. Then the optimization problem (10) and (11) is solved resulting the vector ΔQ depicted in the third column of the table 2.

Load Buses	Initial ΔV	ΔV			Final Voltage
		5	5,13	5,13,10	
4	0.0535	0	0.0009	0.0122	0.9878
5	0.0512	-0.018	-0.0023	0	1.0061
7	0.0332	0.0052	0.0022	0	1.0104
9	0.0579	0.0299	0.0193	0	1.0138
10	0.0804	0.0572	0.0484	0	1.0161
11	0.0765	0.0645	0.0599	0.0265	0.9735
12	0.0424	0.0405	0	0.0035	0.9965
13	0.0702	0.066	-0.0061	0	1.0183
14	0.1169	0.0993	0.049	0.038	0.9620

Tabla 3: Solution for the IEEE 14-bus test system by using the linear $RPVS$ factors

As it was argued previously, buses where a negative injected reactive power results are eliminated from set S_C, buses 4 and 9 in this case. Then the optimization problem is again solved but with a new number C of possible allocations for capacitors less than the initial one. Once no any negative elements of ΔQ is obtained, this vector is arranged from large to lowest size and new switched capacitors are allocated sequentially until all voltages are in limits. For this case this situation is reached after installing capacitors in buses 5, 13 and 10, and injecting a reactive power of 116.51, 67.40 and 46.24 Mvar respectively. The ΔV vector after each compensation, and the final compensated voltages in PQ buses, obtained by (12), are showed in table 3. As it can be noted, there are no any under/overvoltages.

In order to study the accuracy of the method when linear $RPVS$ factors are used instead of the exact ones, the whole problem for the A case has been solved by computing the exact $RPVS$ factors. In this case the optimal placement of capacitors are the buses 5, 13 and 11, with an injected reactive power of 126.89, 64.11 and 40.27 Mvar respectively. This new solution only differs from the previous one in the last bus where a new capacitor should be allocated, resulting bus 11 instead of bus 10. This difference is due to these nodes are very closed to each other. Table 4 illustrates about the two solutions. Moreover, the final voltages by using the $RPVS$ factors and by solving the exact load flow equations are compared in both cases. It can be concluded as the obtained solution with the linear methodology is exact enough not only to deduce the number of capacitors to be installed and their allocation, but also to compute the final voltages. The maximum relative error is not higher than a 2%, which is very low if we take into account how much low the initial voltages were. This relative error has been defined as follows,

$$\text{Relative error}(\%) = \frac{\text{Comp. voltage} - \text{Exact comp. voltage}}{\text{Exact comp. voltage}} 100$$

where "Comp. voltage" refers to those compensated voltages obtained from using RVPS factors (linear or exact ones), and "Exact comp. voltage" are those voltages computed from solving an exact load flow, both cases with the new capacitors already allocated.

Once the accuracy of the proposed methodology has been countersigned in a certain way by the former results, for the other two tested cases not such detailed results will be written, but only the final solution.

For the B case, with 24 PQ nodes, five buses have initially undervoltages lower tan 0.95. Figure 1 shows, among others, these initial voltages. The optimal solution, by using the linear factors, drives to allocate three new capacitors in buses 9, 21 and 30, being the injected reactive power, $\Delta Q_{9,21,30} = \left[\begin{array}{ccc} 24.97 & 2.82 & 8.47 \end{array}\right]$ Mvar. The compensated voltage magnitudes are also depicted in figure 1, those using the linear $RPVS$ factors as well as the exact ones computed by solving the load flow equations. Note as compensated voltages by using the proposed linear methodology are very closed to the exact ones once more.

The C case has 111 PQ buses, and the simulated load profile gives undervoltages in 27 of them. When the proposed algorithm is applied, the reached solution consists of installing 5 new capacitors injecting a reactive power detailed in table 5. The large amount of reactive power obtained in bus 14030 is justified by a high reactive load connected in this bus that produces an extremely low initial voltage of 0.823 in this bus. Figure 2 is the counterpart

Load	$\Delta Q_{5,13,10} = \begin{bmatrix} 1.1651 & 0.6740 & 0.4624 \end{bmatrix}$			$\Delta Q_{5,13,11} = \begin{bmatrix} 1.2689 & 0.6411 & 0.4027 \end{bmatrix}$		
	Final V_i		Relative	Final V_i		Relative
Buses	Linear $RVPS$	Exact Load Flow	error (%)	Exact $RVPS$	Exact Load Flow	error (%)
4	0,9878	0,9857	0.21	0,9901	0,9845	0.57
5	1,0061	1,0032	0.29	1,0130	1,0051	0.79
7	1,0104	1,0097	0.07	1,0034	0,9983	0.51
9	1,0138	1,0139	-0.01	0,9996	0,9902	0.95
10	1,0161	1,0115	0.45	0,9889	0,9746	1.47
11	0,9735	0,9717	0.19	1,0131	0,9910	2.23
12	0,9965	0,9981	-0.16	0,9979	0,9944	0.35
13	1,0183	1,0006	1.77	1,0224	0,9941	2.85
14	0,9620	0,9583	0.39	0,9645	0,9415	2.44

Tabla 4: Comparative study of final voltages for the IEEE 14-bus test system by using $RPVS$ factors or an exact load flow

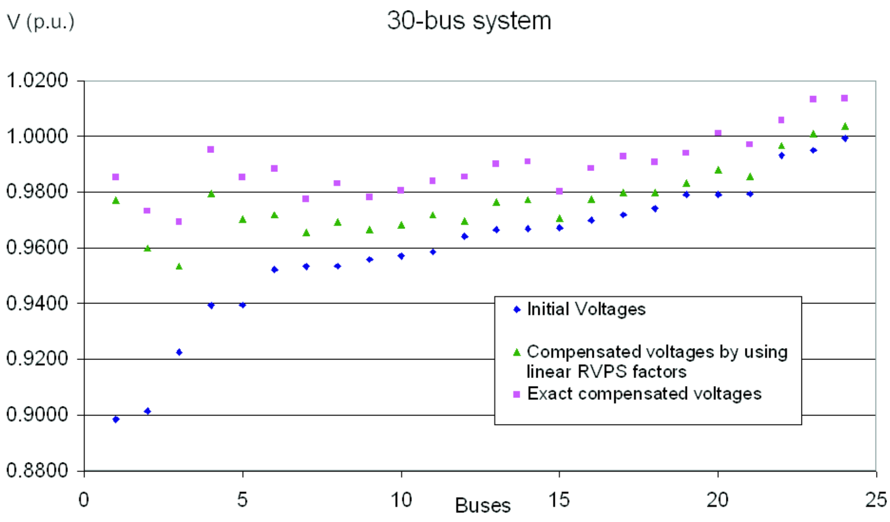

Figura 1: Voltage profiles for the 30-bus system

to 1, now for the C system. Again, compensated voltages are quite good, being the most relative error of 3.26 %. This result is quite good taking into account the high level of load of the system. As in the rest of the tested systems the upper limit of 1.05 is not exceeded. Moreover, the resulting few number of capacitors regards to the size of the system is emphasized, achieving the main proposed goal, that is, all the voltage magnitudes are in limits.

New Capacitors Allocation	Injected Reactive Power (Mvar)
4629	54.40
5126	22.57
5156	25.97
5183	43.05
5190	12.93
14030	214.45

Tabla 5: Solution for the actual 128-bus test system

5 Conclusion

In this paper, a procedure to solve the capacitor placement problem is considered. A linear programming method is employed which minimizes the bus voltage deviations from voltage limits. This linear methodology has been guaranteed to reach realistic solutions not only to solve the optimization problem but also to deduce the final voltage magnitudes, even for very overloaded systems. The implemented algorithm is simple and fast, so its use on real time would be of interest in order to determine the best way to operate on all the switched capacitors being on the system to improve all voltages in a global way.

6 Acknowledgments

Authors acknowledge the financial support provided by the Spanish Minister of Science and Technology under grant ENE 2004-03342.

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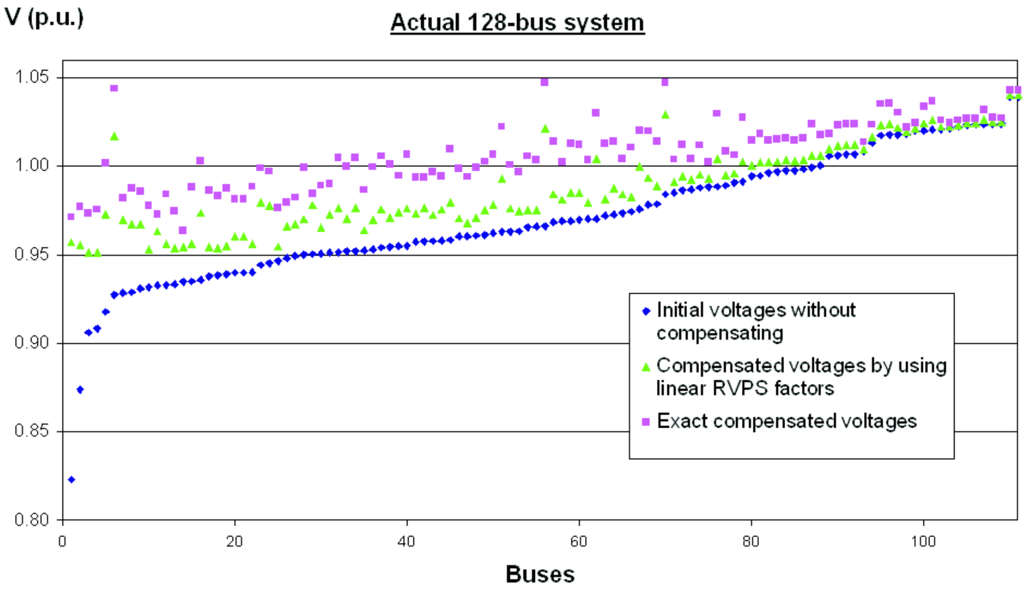

Figura 2: Voltage profiles for the 128-bus system

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