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Hydrodynamic description for ballistic annihilation systems

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Abstract. The problem of the validity of a hydrodynamic description for a system in which there are no collisional invariants is addressed. Hydrodynamic equations have been derived and successfully tested against simulation data for a system where particles annihilate with a probability p, or collide elastically otherwise. The response of the system to a linear perturbation is analyzed as well.

Keywords: Hydrodynamic, kinetic theory, dissipative system
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INDRODUCTION

The standard approach deriving hydrodynamic equations from a kinetic description fundamentally relies on the existence of collisional invariants. What happens for a system where there are no such invariants? To address this question, the model of probabilistic ballistic annihilation [1] is particularly convenient, where particles undergoing ballistic motion annihilate with probability p upon colliding, or are scattered elastically otherwise. Our motivation is to study the applicability of hydrodynamics to systems where scale separation is not granted *a priori* and in which there are no collisional invariants. We show that the hydrodynamic description arises in the appropriate time scale, when the kinetic modes are much slower than the hydrodynamic ones [2]. The existence of scale separation must be assumed. This property has not been proven for our model in general, but only for Maxwell molecules and for p smaller than a given value [2].

The Boltzmann equation for the system under consideration has the form

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_1\cdot\nabla\right)f(\mathbf{r},\mathbf{v}_1,t)=pJ_a[f|f]+(1-p)J_c[f|f], \tag{1}$$

where J_a is the annihilation operator and J_c is the elastic collision operator.

The Boltzmann equation for this system admits a homogeneous scaling solution in which all the time dependence is encoded in the density and temperature

$$f_H(\mathbf{v},t)=\frac{n_H(t)}{v_H(t)^d}\chi_H(c), \qquad \mathbf{c}=\mathbf{v}/v_H(t) \tag{2}$$

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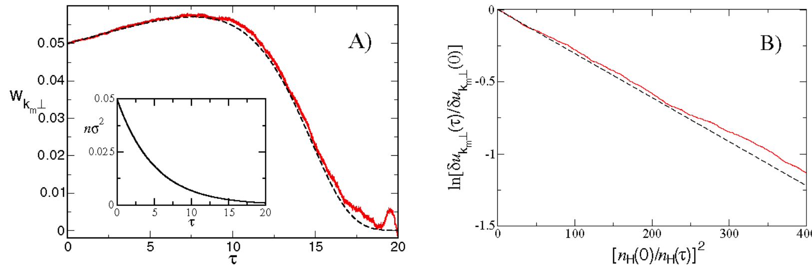

FIGURE 1. Molecular Dynamic results for the evolution of the transverse velocity for a system of hard disks as a function of the number of collision per particle, τ and as a function of $(n_H(0)/n_H(\tau))^2$. The coefficient of annihilation is $p = 0.1$. Dashed lines are the theoretical predictions.

where $v_H(t) = (2T_H(t)/m)^{1/2}$. The density and the temperature obey the equations

$$\frac{\partial n_H(t)}{\partial t} = -p\nu_H(t)\zeta_n n_H(t), \qquad \frac{\partial T_H(t)}{\partial t} = -p\nu_H(t)\zeta_T T_H(t), \tag{3}$$

where $\nu_H(t)$ is the collision frequency of the corresponding hard sphere fluid in equilibrium (but with a time dependent density and temperature), and the decay rates ζ_n and ζ_T are functionals of the distribution function (2).

HYDRODYNAMIC EQUATIONS FOR THE PROBABILISTIC BALLISTIC ANNIHILATION MODEL

We define the relative deviation of the hydrodynamic fields around the homogeneous decay state as

$$\rho(\mathbf{r},\tau) \equiv \frac{\delta n(\mathbf{r},\tau)}{n_H(\tau)}, \quad \mathbf{w}(\mathbf{r},\tau) \equiv \frac{\delta \mathbf{u}(\mathbf{r},\tau)}{v_H(\tau)}, \quad \theta(\mathbf{r},\tau) \equiv \frac{\delta T(\mathbf{r},\tau)}{T_H(\tau)}, \tag{4}$$

where τ is proportional to the number of collisions per particle.

If we assume scale separation, we obtain a closed set of equations for the linear deviation of the hydrodynamic fields

$$\begin{aligned}
&\left(\frac{\partial}{\partial\tau} + 2p\zeta_n\right)\rho_{\mathbf{k}} + i\ell_H(\tau)k w_{\mathbf{k}||} + p\zeta_n\theta_{\mathbf{k}} = 0,\\
&\left[\frac{\partial}{\partial\tau} - p\zeta_T + \ell_H^2(\tau)\tilde{\eta}k^2\right]\mathbf{w}_{\mathbf{k}\perp} = 0,\\
&\left[\frac{\partial}{\partial\tau} - p\zeta_T + \frac{2(d-1)}{d}\ell_H^2(\tau)\tilde{\eta}k^2\right] w_{\mathbf{k}||} + \frac{i}{2}\ell_H(\tau)k\left[(1-2p\zeta_{u,\rho})\rho_{\mathbf{k}} + (1-2p\zeta_{u,\theta})\theta_{\mathbf{k}}\right] = 0,\\
&\left[\frac{\partial}{\partial\tau} + p\zeta_T + \frac{2}{d}\ell_H^2(\tau)\tilde{\kappa}k^2\right]\theta_{\mathbf{k}} + \left[2p\zeta_T + \frac{2}{d}\ell_H^2(\tau)\tilde{\mu}k^2\right]\rho_{\mathbf{k}} + i\frac{2}{d}\ell_H(\tau)k w_{\mathbf{k}||} = 0,
\end{aligned} \tag{5}$$

with $w_{\mathbf{k}||}$ and $\mathbf{w}_{\mathbf{k}\perp}$ the longitudinal and transverse parts of the velocity field and $\ell_H(\tau)$ is proportional to the mean free path. The transport coefficents $\tilde{\eta}$, $\tilde{\kappa}$ and $\tilde{\mu}$ can be expressed as Green-Kubo formulas.

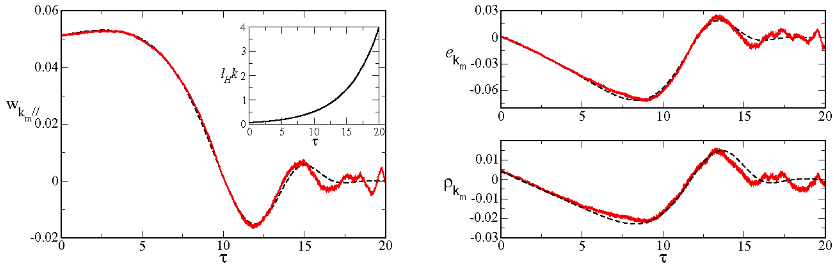

FIGURE 2. Evolution of the longitudinal velocity, density and energy ($e_{\mathbf{k_m}} = \theta_{\mathbf{k_m}} + \rho_{\mathbf{k_m}}$) for a system of hard disks as a function of the number of collision per particle, τ. The coefficient of annihilation is $p = 0.1$. Dashed lines are the theoretical predictions.

STUDY OF A LINEAR PERTURBATION

In order to study the validity of our predictions we have considered a linear perturbation in the transverse hydrodynamic field in real space of the form

$$u_x(\mathbf{r},0) = A\sin(k_m y), \tag{6}$$

with $A = 10^{-1} v_H(0)$ and $k_m = 2\pi/L$, where L is the linear size of the system. As the equation for the transverse velocity field is decoupled from the other ones, we have that the evolution of a perturbation is given by

$$\delta u_{\mathbf{k}\perp}(\tau) = \exp\left\{-\frac{\tilde{\eta}\tilde{k}^2}{4p\zeta_n}\left[\frac{n_H^2(0)}{n_H^2(\tau)} - 1\right]\right\}\delta u_{\mathbf{k}\perp}(0), \qquad \tilde{k} = \ell_H(0)k \tag{7}$$

The plot of $\delta u_{\mathbf{k}\perp}(\tau)/\delta u_{\mathbf{k}\perp}(0)$ as a function of $(n_H(0)/n_H(\tau))^2$ (Fig. 1) allows then by simple exponential fitting to extract $\tilde{\eta}/\zeta_n$. The equation for the longitudinal velocity is coupled with the equations of the density and the temperature. If we consider a perturbation of the longitudinal velocity

$$u_x(\mathbf{r},0) = A\sin(k_m x), \tag{8}$$

this perturbation induces a response of the other two fields, see Fig. 2.

As can be seen, there is an excellent agreement between our predictions and the results of Molecular Dynamics simulation, without fitting parameters, which gives strong support to the theoretical analysis developed.

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