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EXISTENCE AND EXPONENTIAL STABILITY FOR NEUTRAL STOCHASTIC INTEGRO–DIFFERENTIAL EQUATIONS WITH IMPULSES DRIVEN BY A ROSENBLATT PROCESS

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Dedicated to Prof. Dr. Juan J. Nieto on the occasion of his 60th birthday

ABSTRACT. The existence and uniqueness of mild solution of an impulsive stochastic system driven by a Rosenblatt process is analyzed in this work by using the Banach fixed point theorem and the theory of resolvent operator developed by R. Grimmer in [12]. Furthermore, the exponential stability in mean square for the mild solution to neutral stochastic integro–differential equations with Rosenblatt process is obtained by establishing an integral inequality. Finally, an example is exhibited to illustrate the abstract theory.

1. **Introduction.** Integro-differential equations are of great importance in the modeling of several physical phenomena. Their resolution can be done through the theory of resolvent operators (see Grimmer [12]). However, the resolvent operator does not satisfy semigroup properties. The study of the quantitative and qualitative properties of solutions to stochastic neutral differential equations like existence, uniqueness and stability, have been widely examined by many researchers by analyzing various mathematical models in different areas such as mechanics, electronics, control theory, engineering and economics, etc (see [3, 5, 7, 8, 25]). There exist many works dealing with several theoretical aspects of the Rosenblatt process. For example, Leonenko and Ahn [17] proved the rate of convergence to the Rosenblatt process in the Non-Central Limit Theorem. Recently, Sakthivel et al.[26] investigated a class of abstract functional second-order non autonomous stochastic evolution equations driven by Rosenblatt process with index $H \in (\frac{1}{2}, 1)$.

Several processes of evolution systems have impulsive effects at certain moments of time. This phenomenon is observed in some fields as biology, mechanics and physics

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(see [16, 27]). The theory of impulsive stochastic integro–differential equations has been receiving much attention recently. However, very few researchers have been interested in the stability of the mild solutions to stochastic integro–differential equations impulsive (see [6, 11, 13]).

In this paper, we consider the following neutral stochastic integro–differential equation with impulses of the form

$$\begin{aligned} & d\left[x(t)-g(t,x_t,\int_0^t a_1(t,s,x_s)ds)\right] \qquad (1)\\ &= A\left[x(t)-g(t,x_t,\int_0^t a_1(t,s,x_s)ds)\right]dt \\ &\quad + \int_0^t \Upsilon(t-s)\left[x(s)-g(s,x_s,\int_0^s a_1(t,\tau,x_\tau)d\tau)\right]ds\,dt \\ &\quad + f(t,x_t,\int_0^t a_2(t,s,x_s)ds)\,dt+\tilde{F}(t)dZ_Q^H(t), t\in J, t\neq t_i, \end{aligned}$$

$$\Delta x(t_i) = I_i(x(t_i^-)),\ t=t_i,\ i=1,2,..., \qquad (2)$$

$$x_0(t)=\phi(t),\ -r\leq t\leq 0, \qquad (3)$$

where $J:=[0,b]$ and A is the infinitesimal generator of a strongly continuous semigroup $(T(t))_{t\geq 0}$ of bounded linear operators in a Hilbert space X with domain $D(A)$, $\Upsilon(t)$ is a family of closed linear operators on X with domain $D(\Upsilon(t))\supset D(A)$, Z_Q^H is a fractional Brownian motion, $\phi\in\mathscr{PC}:=\mathscr{PC}([-r,0],X)=\{\psi:[-r,0]\to X,\ \psi(\cdot)$ is continuous everywhere except in a finite number of points $\tilde{t}$ at which $\psi(\tilde{t}^-)$ and $\psi(\tilde{t}^+)$ exist and $\psi(\tilde{t}^-)=\psi(\tilde{t})\}$. For $\psi\in\mathscr{PC}$, $\|\psi\|_{\mathscr{PC}}=\sup_{s\in[-r,0]}\|\psi(s)\|<+\infty$. The mappings $g,f:[0,+\infty)\times\mathscr{PC}\times X\to X$, a_1 , $a_2:[0,+\infty)\times[0,+\infty)\times\mathscr{PC}\to X$, $\tilde{F}:[0,+\infty)\to L_Q^0(Y,X)$ are appropriate continuous functions and will be specified later. The impulsive times t_i satisfy $0=t_0<t_1<t_2<...,<t_m\to+\infty$, which implies that in every interval $[0,b]$ there exists only a finite number of t_j. $I_i:X\to X$, $\Delta x(t_i)$ represents the jump in the state x at t_i determining the size of the jump, which is defined by $\Delta x(t_i)=x(t_i^+)-x(t_i^-)$, where $x(t_i^+)$ and $x(t_i^-)$ are respectively the right and left limits of $x(t)$ at t_i. For any continuous function x and any $t\in[0,b]$, we denote by x_t the element of $\mathscr{PC}$ defined by $x_t(\theta)=x(t+\theta)$, $-r\leq\theta\leq 0$.

The analysis of (1)-(3) driven by fractional Brownian motion when $\Upsilon(t)=0$, for all $t\geq 0$, was initiated in Arthi et al. [2], where the authors proved the existence and exponential stability of solutions by using an impulsive integral inequality and a strict contraction principle. Now, in our current paper, we will investigate the existence of solutions and stability problems for the previously mentioned neutral stochastic integro–differential system with impulses driven by a Rosenblatt process, since this problem still has not been considered in the literature. The Rosenblatt process is still of interest in practical applications because of its selfsimilarity, stationarity of increments and long-range dependence. Therefore, it is necessary to consider the impulsive effects for the stability of solutions of neutral stochastic integro–differential equations driven by such a Rosenblatt process. In particular, here we highlight that the results of [2] for fractional Brownian motion can be extended to the case $\Upsilon(t)\neq 0$. The main contributions of this paper are summarized as follows:
In this work, a general class of impulsive neutral stochastic integrodifferential equations driven by a Rosenblatt process is considered firstly. Then, using methods of functional analysis, a set of sufficient conditions are proposed ensuring exponential stability or solutions. The results are established with the use of the resolvent operator approach. Our

paper expands the usefulness of stochastic integro-differential equations, since the literature shows results for existence and exponential stability for such equations in the case of semigroup only.

The structure of this paper is organized as follows. In Section 2, we recall some basic definitions and preliminary facts which will be used throughout this work. In Section 3, some results on the existence and uniqueness of mild solutions are established. Section 4 is devoted to the proof of exponential stability of a mild solution in mean square, followed by an illustrative example in Section 5.

2. **Preliminaries.** Let us start with some basic facts about Rosenblatt process. Also we review some fundamentals about the resolvent operator theory which will be crucial for our study.

2.1. **Rosenblatt process.** We recall in this subsection, some basic concepts about Rosenblatt processes as well as the Wiener integral with respect to them. Consider $(\chi_n)_{n\in Z}$ a stationary Gaussian sequence with mean zero and variance 1 such that its correlation function satisfies that $R(n) := \mathbb{E}(\chi_0\chi_n) = n^{\frac{2H-2}{k}}L(n)$, with $H \in (\frac{1}{2},1)$ and L is a slowly varying function at infinity. Let h be a function of Hermite rank k, that is, if h admits the following expansion in Hermite polynomials

$$h(x) = \sum_{j\geq 0} c_j H_j(x), \quad c_j = \frac{1}{j!}\mathbb{E}(h(\chi_0 H_j(\chi_0))),$$

then $k = \min\{j|c_j \neq 0\} \geq 1$, where $H_j(x)$ is the Hermite polynomial of degree j given by $H_j(x) = (-1)^j e^{\frac{x^2}{2}} \frac{d^j}{dx^j} e^{-\frac{x^2}{2}}$. Then, the Non-Central Limit Theorem (see, for example, Dobrushin & Major [10]) says $\frac{1}{n^H}\sum_{j=1}^{[nt]} h(\xi_j)$ converges as $n \to \infty$, in the sense of finite dimensional distributions, to the process

$$Z_H^k(t) = c(H,k)\int_{\mathbb{R}^k}\int_0^t \left(\prod_{j=1}^{k}(s-y_j)_+^{\left(-\frac{1}{2}+\frac{1-H}{k}\right)}\right) ds dB(y_1)\cdots dB(y_k), \tag{4}$$

where the above integral is a Wiener-Itô multiple integral of order k with respect to the standard Brownian motion $(B(y))_{y\in\mathbb{R}}$ and $c(H,k)$ is a positive normalization constant depending only on H and k. The process $(Z_H^k(t))_{t\geq 0}$ is called as the Hermite process and it is H self-similar in the sense that for any $c > 0$, $(Z_H^k(ct)) \stackrel{d}{=} (c^H Z_H^k(t))$ and it has stationary increments.

The fractional Brownian motion (which is obtained from (4) when $k = 1$) is the most used Hermite process to study evolution equations due to its large range of applications. When $k = 2$ in (4), Taqqu [29] named the process as Rosenblatt process. The stationarity of increments, self-similarity and long range dependence (see Tindel et al. [30]) imply that Rosenblatt processes become very important in practical applications.

Let $\{Z_H(t), t \in [0,T], T > 0\}$ be a one-dimensional Rosenblatt process with parameter $H \in (\frac{1}{2},1)$. According to the work by Tudor [31], the Rosenblatt process with parameter $H > \frac{1}{2}$ can be written as

$$Z_H(t) = d(H)\int_0^t\int_0^t \left[\int_{y_1\vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u,y_1)\frac{\partial K^{H'}}{\partial u}(u,y_2)du\right] dB(y_1)dB(y_2), \tag{5}$$

where $K^H(t,s)$ is given by

$$K^H(t,s) = c_H s^{\frac{1}{2}-H} \int_s^t (u-s)^{H-3/2} u^{H-1/2} du \text{ for } t > s,$$

with

$$c_H = \sqrt{\frac{H(2H-1)}{\beta(2-2H, H-\frac{1}{2})}},$$

$\beta(.,.)$ denotes the Beta function, $K^H(t,s) = 0$ when $t \leq s$, $(B(t), t \in [0,T])$ is a Brownian motion, $H' = \frac{H+1}{2}$ and $d(H) = \frac{1}{H+1}\sqrt{\frac{H}{2(2H-1)}}$ is a normalizing constant. The covariance of the Rosenblatt process $\{Z_H(t), t \in [0,T]\}$ satisfy

$$\mathbb{E}(Z_H(t) Z_H(s)) = \frac{1}{2}\left(s^{2H} + t^{2H} - |s-t|^{2H}\right).$$

The covariance structure of the Rosenblatt process allows to construct the Wiener integral with respect to it. We refer to Maejima and Tudor [19] for the definition of Wiener integral with respect to general Hermite processes and to Kruk et al. [15] for a more general context (see also Tudor [31]).

Note that

$$Z_H(t) = \int_0^T \int_0^T I(1_{[0,t]})(y_1, y_2) dB(y_1) dB(y_2),$$

where the operator I is defined on the set of functions $f : [0,T] \to \mathbb{R}$, which takes its values in the set of functions $h : [0,T]^2 \to \mathbb{R}^2$ and is given by

$$I(f)(y_1, y_2) = d(H) \int_{y_1 \vee y_2}^T f(u) \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2) du.$$

Let φ be an element of the set $\mathscr{E}$ of step functions on $[0,T]$ of the form

$$\varphi = \sum_{i=0}^{n-1} a_i 1_{(t_i, t_{i+1}]},\ t_i \in [0,T].$$

Then, it is natural to define its Wiener integral with respect to Z_H as

$$\int_0^T \varphi(u) dZ_H(u) := \sum_{i=0}^{n-1} a_i (Z_H(t_{i+1}) - Z_H(t_i)) = \int_0^T \int_0^T \varphi(f)(y_1, y_2) dB(y_1) dB(y_2).$$

Let $\mathscr{H}$ be the set of functions φ such that

$$\|\varphi\|^2_{\mathscr{H}} := 2 \int_0^T \int_0^T (I(\varphi)(y_1, y_2))^2 dy_1 dy_2 < \infty.$$

It follows that (see Tudor[31])

$$\|\varphi\|^2_{\mathscr{H}} = H(2H-1) \int_0^T \int_0^T \varphi(u)\varphi(v)|u-v|^{2H-2} du dv.$$

It has been proved in Maejima and Tudor [19] that the mapping

$$\varphi \to \int_0^T \varphi(u) dZ_H(u)$$

defines an isometry from $\mathscr{E}$ to $L^2(\Omega)$ and it can be extended continuously to an isometry from $\mathscr{H}$ to $L^2(\Omega)$ because $\mathscr{E}$ is dense in $\mathscr{H}$. We call this extension as the Wiener integral of $\varphi \in \mathscr{H}$ with respect to Z_H. It is noted that the space $\mathscr{H}$ contains not only functions but its elements could be also distributions. Therefore it is suitable to know subspaces $|\mathscr{H}|$ of $\mathscr{H}$: $|\mathscr{H}| = \left\{\varphi : [0,T] \to \mathbb{R} | \int_0^T \int_0^T |\varphi(u)||\varphi(v)||u-v|^{2H-2} du dv < \infty\right\}$. The space $|\mathscr{H}|$ is not complete with respect to the norm $\|.\|_{\mathscr{H}}$ but it is a Banach space with respect to the norm

$$\|\varphi\|^2_{|\mathscr{H}|} = H(2H-1)\int_0^T\int_0^T |\varphi(u)||\varphi(v)||u-v|^{2H-2}dudv.$$

As a consequence, we have

$$L^2([0,T]) \subset L^{1/H}([0,T]) \subset |\mathscr{H}| \subset \mathscr{H}.$$

For any $\varphi \in L^2([0,T])$, we have

$$\|\varphi\|^2_{|\mathscr{H}|} \leq 2HT^{2H-1}\int_0^T |\varphi(s)|^2 ds$$

and

$$\|\varphi\|^2_{|\mathscr{H}|} \leq C(H)\|\varphi\|^2_{L^{1/H}([0,T])}, \tag{6}$$

for some constant $C(H) > 0$. Whenever we write $C(H) > 0$ we mean a positive constant depending only on H, and its value may be different in different places.

Define the linear operator K_H^* from $\mathscr{E}$ to $L^2([0,T])$ by

$$(K_H^*\varphi)(y_1,y_2) = \int_{y1\vee y_2}^T \varphi(t)\frac{\partial\mathscr{K}}{\partial t}(t,y_1,y_2)dt,$$

where $\mathscr{K}$ is the kernel of Rosenblatt process in representation (5)

$$\mathscr{K}(t,y_1,y_2) = 1_{[0,t]}(y_1)1_{[0,t]}(y_2)\int_{y_1\vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u,y_1)\frac{\partial K^{H'}}{\partial u}(u,y_2)du.$$

Note that $(K_H^* 1_{[0,t]})(y_1,y_2) = \mathscr{K}(t,y_1,y_2)1_{[0,t]}(y_1)1_{[0,t]}(y_2)$. The operator K_H^* is an isometry between $\mathscr{E}$ to $L^2([0,T])$, which can be extended to the Hilbert space $\mathscr{H}$. In fact, for any $s,t \in [0,T]$ we have

$$\begin{aligned}
\left\langle K_H^* 1_{[0,t]}, K_H^* 1_{[0,s]}\right\rangle_{L^2([0,T])} &= \left\langle \mathscr{K}(t,.,.)1_{[0,t]}, \mathscr{K}(s,.,.)1_{[0,s]}\right\rangle_{L^2([0,T])} \\
&= \int_0^{t\wedge s}\int_0^{t\wedge s} \mathscr{K}(t,y_1,y_2)\mathscr{K}(s,y_1,y_2)dy_1dy_2 \\
&= H(2H-1)\int_0^t\int_0^s |u-v|^{2H-2}dudv \\
&= \left\langle 1_{[0,t]}, 1_{[0,s]}\right\rangle_{\mathscr{H}}.
\end{aligned}$$

Moreover, for $\varphi \in \mathscr{H}$, we have

$$Z_H(\varphi) = \int_0^T\int_0^T (K_H^*\varphi)(y_1,y_2)dB(y_1)dB(y_2).$$

Let $\{Z_n(t)\}_{n\in\mathbb{N}}$ be a sequence of two-sided one dimensional Rosenblatt process mutually independent on $(\Omega,\mathscr{F},\mathbb{P})$. We consider a K-valued stochastic process $Z_Q(t)$ given by the following series:

$$Z_Q(t) = \sum_{n=1}^{\infty} z_n(t)Q^{1/2}e_n, \quad t \geq 0.$$

Moreover, if Q is a non-negative self-adjoint trace class operator, then this series converges in the space K, that is, it holds that $Z_Q(t) \in L^2(\Omega,K)$. Then, we say that the above $Z_Q(t)$ is a K-valued Q- Rosenblatt process with covariance operator Q. For instance, if $\{\sigma_n\}_{n\in\mathbb{N}}$ is a bounded sequence of non-negative real numbers such that $Qe_n = \sigma_n e_n$, by assuming that Q is a nuclear operator in K, then the stochastic process

$$Z_Q(t) = \sum_{n=1}^{\infty} z_n(t)Q^{1/2}e_n = \sum_{n=1}^{\infty}\sqrt{\sigma_n}z_n(t)e_n, \quad t \geq 0,$$

is well-defined as a K-valued Q- Rosenblatt process.

Definition 2.1. *(Tudor [31]). Let $\varphi : [0,T] \to L_Q^0(Y,X)$ such that*

$$\sum_{n=1}^{\infty} \|K_H^*(\varphi Q^{1/2} e_n)\|_{L^2([0,T];\mathscr{H})} < \infty.$$

Then, its stochastic integral with respect to the Rosenblatt process $Z_Q(t)$ is defined, for $t \geq 0$, as follows :

$$\begin{aligned} \int_0^t \varphi(s) dZ_Q(s) &:= \sum_{n=1}^{\infty} \int_0^t \varphi(s) Q^{1/2} e_n dz_n(s) \\ &= \sum_{n=1}^{\infty} \int_0^t \int_0^t (K_H^*(\varphi Q^{1/2} e_n))(y_1, y_2) dB(y_1) dB(y_2). \end{aligned} \tag{7}$$

Lemma 1. *For $\psi : [0,T] \to L_Q^0(Y,X)$ such that $\sum_{n=1}^{\infty} \|\psi Q^{1/2} e_n\|_{L^{1/H}([0,T];U)} < \infty$ holds, and for any $a, b \in [0,T]$ with $b > a$, we have*

$$\mathbb{E}\left\| \int_a^b \psi(s) dZ_Q(s) \right\|^2 \leq c(H)(b-a)^{2H-1} \sum_{n=1}^{\infty} \int_a^b \|\psi(s) Q^{1/2} e_n\|^2 ds.$$

If, in addition,

$$\sum_{n=1}^{\infty} \|\psi(t) Q^{1/2} e_n\| \text{ is uniformly convergent for } t \in [0,T],$$

then, it holds that

$$\mathbb{E}\left\| \int_a^b \psi(s) dZ_Q(s) \right\|^2 \leq C(H)(b-a)^{2H-1} \int_a^b \|\psi(s)\|_{L_Q^0(K,\mathrm{H})}^2 ds.$$

Proof. Let $\{e_n\}_{n\in\mathbb{N}}$ be the complete orthonormal basis of K introduced above. Applying (7) and Hölder inequality, we have

$$\begin{aligned} &\mathbb{E}\left\| \int_a^b \psi(s) dZ_Q(s) \right\|^2 \\ &\quad = \mathbb{E}\left\| \sum_{n=1}^{\infty} \int_a^b \psi(s) Q^{1/2} e_n dz_n(s) \right\|^2 \\ &\quad = \sum_{n=1}^{\infty} \mathbb{E}\left\| \int_a^b \psi(s) Q^{1/2} e_n dz_n(s) \right\|^2 \\ &\quad = \sum_{n=1}^{\infty} H(2H-1) \int_a^b \int_a^b \|\psi(s) Q^{1/2} e_n\| \|\psi(t) Q^{1/2} e_n\| |t-s|^{2H-2} ds dt \\ &\quad \leq C(H) \sum_{n=1}^{\infty} \left(\int_a^b \|\psi(s) Q^{1/2} e_n\|^{1/H} ds \right)^{2H} \\ &\quad \leq C(H)(b-a)^{2H-1} \sum_{n=1}^{\infty} \int_a^b \|\psi(s) Q^{1/2} e_n\|^2 ds. \end{aligned} \tag{8}$$

□

2.2. **Partial integro-differential equations in Banach spaces.** In the current section, we recall some definitions, notations and properties needed in the sequel. In what follows, X will denote a Banach space, A and $\Upsilon(t)$ are closed linear operators on X. Y represents the Banach space $D(A)$, the domain of operator A, equipped with the graph norm

$$|y|_Y := |Ay| + |y| \quad \text{for } \ y \in Y.$$

The notation $C([0,+\infty);Y)$ stands for the space of all continuous functions from $[0,+\infty)$ into Y. We then consider the following Cauchy problem

$$\begin{cases} \xi'(t) &= A\xi(t) + \displaystyle\int_0^t \Upsilon(t-s)\xi(s)ds \ \text{ for } \ t \geq 0, \\ \xi(0) &= \xi_0 \in X. \end{cases} \tag{9}$$

Definition 2.2. *(*[12]*) A resolvent operator for Eq.* (9) *is a bounded linear operator valued function* $\Psi(t) \in L(X)$ *for* $t \geq 0$*, satisfying the following properties :*

(i) $\Psi(0) = I$ *and* $\|\Psi(t)\| \leq Ne^{\beta t}$ *for some constants* N *and* β*.*
(ii) For each $x \in X$, $\Psi(t)x$ *is strongly continuous for* $t \geq 0$*.*
(iii) For $x \in Y$, $\Psi(\cdot)x \in C^1([0,+\infty);X) \cap C([0,+\infty);Y)$ *and*

$$\begin{aligned} \Psi'(t)x &= A\Psi(t)x + \int_0^t \Upsilon(t-s)\Psi(s)xds \\ &= \Psi(t)Ax + \int_0^t \Psi(t-s)\Upsilon(s)xds \qquad \textit{for } \ t \geq 0. \end{aligned}$$

In the sequel, the assumptions below are important:

(H1) A is the infinitesimal generator of a C_0-semigroup $(T(t))_{t\geq 0}$ on X.

(H2) For all $t \geq 0$, $\Upsilon(t)$ is a continuous linear operator from $(Y, |\cdot|_Y)$ into $(X, |\cdot|_X)$. Moreover, there exists an integrable function $c : [0,+\infty) \to \mathbb{R}^+$ such that for any $y \in Y$, $y \mapsto \Upsilon(t)y$ belongs to $W^{1,1}([0,+\infty),X)$ and

$$\left| \frac{d}{dt}\Upsilon(t)y \right|_X \leq c(t)|y|_Y \ \text{ for } \ y \in Y \ \text{ and } \ t \geq 0.$$

We recall that $W^{k,p}(\Omega) = \{u \in L^p(\Omega) \ : \ D^\alpha u \in L^p(\Omega), \ \forall |\alpha| \leq k\}$, where $D^\alpha u$ is the weak α-th partial derivative of u.

Theorem 1. *(*[12]*) Assume that hypotheses* **(H1)** *and* **(H2)** *hold. Then Eq.* (9) *admits a resolvent operator* $(R(t))_{t\geq 0}$*.*

Theorem 2. *(*[18]*) Assume that hypotheses* **(H1)** *and* **(H2)** *hold. Let* $T(t)$ *be a compact operator for* $t > 0$*. Then, the corresponding resolvent operator* $\Psi(t)$ *of Eq.* (9) *is continuous for* $t > 0$ *in the operator norm, namely, for all* $t_0 > 0$*, it holds that* $\lim_{h\to 0} \|\Psi(t_0+h) - \Psi(t_0)\| = 0$.

We recall now some results on the existence of solutions for the following integro–differential equation

$$\begin{cases} \xi'(t) &= A\xi(t) + \displaystyle\int_0^t \Upsilon(t-s)\xi(s)ds + \pi(t) \ \text{ for } \ t \geq 0, \\ \xi(0) &= \xi_0 \in X, \end{cases} \tag{10}$$

where $\pi : [0,+\infty[\to X$ is a continuous function.

Definition 2.3. *([12]) A continuous function $\xi : [0,+\infty) \to X$ is said to be a strict solution of Eq.* (10) *if*

(i) $\xi \in C^1([0,+\infty);X) \cap C([0,+\infty);Y)$,

(ii) ξ *satisfies Eq.* (10) *for* $t \geq 0$.

Remark 1. *From this definition we deduce that $\xi(t) \in D(A)$, and the function $\Upsilon(t-s)\xi(s)$ is integrable, for all $t > 0$ and $s \in [0,+\infty)$.*

Theorem 3. *([12]) Assume that **(H1)-(H2)** hold. If v is a strict solution of Eq.* (10)*, then the following variation of constants formula holds*

$$\xi(t) = \Psi(t)v_0 + \int_0^t \Psi(t-s)\pi(s)ds \quad for \quad t \geq 0. \tag{11}$$

Accordingly, we can establish the following definition.

Definition 2.4. *([12]) A function $\xi : [0,+\infty) \to X$ is called a mild solution of* (10)*, for $\xi_0 \in X$, if v satisfies the variation of constants formula* (11).

Next theorem provides sufficient conditions ensuring the regularity of solutions of Eq. (10).

Theorem 4. *([12]) Let $\pi \in C^1([0,+\infty);X)$ and let ξ be defined by* (11)*. If $\xi_0 \in D(A)$, then ξ is a strict solution of Eq.* (10).

The following integral inequality is a key tool in proving the exponential stability of mild solution of the neutral system with impulsive moments, which is stated by Lemma 3.1 in [33].

Lemma 2. *For any $\omega > 0$, assume that there exist some positive constants $\alpha_j (j = 1,2,3)$ and $\beta_i (i = 1,2,\cdots,m)$ and a function $\Lambda : [-r,+\infty) \to [0,+\infty)$ such that*

$$\Lambda(t) \leq \alpha_1 e^{-\omega t}, \ for \ t \in [-r,0],$$

and

$$\begin{aligned}\Lambda(t) \quad &\leq \quad \alpha_1 e^{-\omega t} + \alpha_2 \sup_{\theta \in [-r,0]} \Lambda(t+\theta) + \alpha_3 \int_0^t e^{-\omega(t-s)} \sup_{\theta \in [-r,0]} \Lambda(s+\theta)ds \\ &\quad + \sum_{t_i < t} \beta_i e^{-\omega(t-t_i)} \Lambda(t_i^-),\end{aligned}$$

for each $t \geq 0$. If $\alpha_2 + \frac{\alpha_3}{\omega} + \sum_{i=1}^m \beta_i < 1$, then $\Lambda(t) \leq Me^{-\mu t}$ for $t \geq -r$, where $\mu > 0$ is the unique solution to the equation: $\alpha_2 e^{\mu r} + \frac{\alpha_3}{(\omega-\mu)} e^{\mu r} + \sum_{i=1}^m \beta_i = 1$ and $M_0 = \max\{\alpha_1, \frac{\alpha_1(\omega-\mu)}{\alpha_3 e^{\mu r}}\} > 0$.

Remark 2. *If $\beta_i = 0$ $(i = 1,2,...)$ in Lemma 2, then we obtain the results proved in Lemma 3.1 in* [5]*. Furthermore, if $\alpha_2 = 0$, then Lemma 2 turns out to be Lemma 3.1 in* [6].

3. **Existence and Uniqueness.**

Before stating and proving the existence and uniqueness of solutions, we will describe now the following conditions on the data of system (1)-(3) which will be imposed in our subsequent results.

(H3) There exists a positive constant M, such that for all $t \geq 0$, $\|\Psi(t)\| \leq M$.

(H4) There exists a positive constant M_{a_1}, such that for all $t \in J, \kappa_1, \kappa_2 \in \mathscr{PC}$,

$$\left\| \int_0^t [a_1(t,s,\kappa_1) - a_1(t,s,\kappa_2)]ds \right\| \leq M_{a_1} \|\kappa_1 - \kappa_2\|.$$

Also $\tilde{M}_{a_1} = b \sup_{0 \leq s \leq t \leq b} \|a_1(t,s,0)\|$.

(H5) There exists a positive constant M_g such that g is $X-$valued and for all $t \in J$, $\kappa_j, \phi_j \in \mathscr{PC}, j = 1,2,$

$$\|g(t,\kappa_1,\phi_1) - g(t,\kappa_2,\phi_2)\| \le M_g[\|\kappa_1 - \kappa_2\| + \|\phi_1 - \phi_2\|].$$

Also $\tilde{M}_g = \sup_{t\in J} \|g(t,0,0)\|$ and $k = M_g(1+M_{a_1}) < 1$.

(H6) There exists a positive constant M_{a_2}, such that for all $t \in J, \kappa_1, \kappa_2 \in \mathscr{PC}$,

$$\left\| \int_0^t [a_2(t,s,\kappa_1) - a_2(t,s,\kappa_2)]ds \right\| \le M_{a_2}\|\kappa_1 - \kappa_2\|.$$

Also $\tilde{M}_{a_2} = b\sup_{0\le s\le t\le b} \|a_2(t,s,0)\|$.

(H7) There exists a positive constant M_f such that f is $X-$valued and for all $t \in J$, $\kappa_j, \phi_j \in \mathscr{PC}, j = 1,2,$

$$\|f(t,\kappa_1,\phi_1) - f(t,\kappa_2,\phi_2)\| \le M_f[\|\kappa_1 - \kappa_2\| + \|\phi_1 - \phi_2\|].$$

Also $\tilde{M}_f = \sup_{t\in J} \|f(t,0,0)\|$ and $\tilde{M} = M_f(1+M_{a_2})$.

(H8) The function g is continuous on its time variable, i.e., for all $\kappa, \phi \in \mathscr{PC}$,

$$\lim_{t\to s} \|g(t,\kappa,\phi) - g(s,\kappa,\phi)\|^2 = 0.$$

(H9) The function $\tilde{F} : [0,+\infty) \to L_Q^0(Y,X)$ satisfies

$$\int_0^t \|\tilde{F}(s)\|^2_{L_Q^0} ds < \infty, \forall t \in [0,b].$$

For a complete orthonormal basis $\{a_n\}_{n\in\mathbb{N}}$ in Y , we have

- **(C.1)** $\sum_{n=1}^{\infty} \|\tilde{F}Q^{1/2}a_n\|_{L^2([0,b];X)} < \infty$.
- **(C.2)** $\sum_{n=1}^{\infty} |\tilde{F}(t)Q^{1/2}a_n|_X$ is uniformly convergent for all $t \in [0,b]$.

(H10) The impulsive functions $I_i(i = 1,2,...)$ satisfy the following condition: there exist some positive numbers $d_i(i = 1,2,...)$ such that

$$\|I_i(\kappa_1) - I_i(\kappa_2)\| \le d_i\|\kappa_1 - \kappa_2\| \text{ and } \|I_i(0)\| = 0,$$

for all $\kappa_1, \kappa_2 \in \mathscr{PC}$ and $\sum_{i=1}^{+\infty} d_i < \infty$.

We present now the definition of mild solution for the stochastic system (1)-(3).

Definition 3.1. *An X-valued stochastic process $\{x(t), t \in [-r,b]\}$ is called a mild solution of the abstract Cauchy problem (1)-(3) if*

(1) $x(\cdot) \in \mathscr{PC}([-r,b], L^2(\Omega,X))$;
(2) For $t \in [-r,0]$, $x(t) = \phi(t)$;
(3) For $t \in J$, $x(t)$ *satisfies the following integral equation:*

$$\begin{aligned} x(t) &= \Psi(t)[\phi(0) - g(0,\phi,0)] + g(t,x_t,\int_0^t a_1(t,s,x_s)ds) \\ &\quad + \int_0^t \Psi(t-s)f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)ds \\ &\quad + \sum_{0<t_i<t} \Psi(t-t_i)I_i(x(t_i^-)) + \int_0^t \Psi(t-s)\tilde{F}(s)dZ_Q^H(s) \;\; \mathbb{P}-a.s. \end{aligned}$$

Theorem 5. *If hypotheses **(H1)-(H10)** hold for every* $\phi \in \mathscr{PC}, b > 0$*, then system (1)-(3) has a unique mild solution on* $[-r,b]$ *provided that*

$$\theta = 3M^2\Big(\sum_{i=1}^{+\infty} d_i\Big)^2 < (1-k)^2 \tag{12}$$

where $k = M_g(1+M_{a_1}) < 1$.

Proof. First, let us introduce the set $\mathscr{PC}_b := \mathscr{PC}([-r,b],L^2(\Omega,X))$, which is the Banach space of all piecewise continuous functions from $[-r,b]$ into $L^2(\Omega,X)$, equipped with the norm $\|\zeta\|^2_{\mathscr{PC}_b} = \sup_{s\in[-r,b]}(\mathbb{E}\|\zeta(s)\|^2)$.

Let $\widehat{\mathscr{PC}_b}$ be the closed subset of $\mathscr{PC}_b$ defined as $\widehat{\mathscr{PC}_b} = \{x \in \mathscr{PC}_b : x(\tau) = \phi(\tau)$, for $\tau \in [-r,0]\}$ with the norm $\|\cdot\|_{\mathscr{PC}_b}$. We consider the operator $\mathbb{L} : \widehat{\mathscr{PC}_b} \to \widehat{\mathscr{PC}_b}$ defined by

$$\begin{aligned}(\mathbb{L}x)(t) \quad &= \quad \Psi(t)[\phi(0) - g(0,\phi,0)] + g(t,x_t,\int_0^t a_1(t,s,x_s)ds) \\ &\quad + \int_0^t \Psi(t-s)f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)ds \\ &\quad + \int_0^t \Psi(t-s)\tilde{F}(s)dZ_Q^H(s) + \sum_{0<t_i<t} \Psi(t-t_i)I_i(x(t_i^-)),\ t \in\ J, \qquad (13)\end{aligned}$$

and $(\mathbb{L}x)(t) = \varphi(t)$ for $t \in [-r,0]$.

In the following parts, we show that the operator $\mathbb{L}$ has a fixed point in $\widehat{\mathscr{PC}_b}$. For convenience, we split the proof into two steps.

Step 1: The map $\mathbb{L}$ is well defined, i.e., for $x \in \widehat{\mathscr{PC}_b}$, the function $(\mathbb{L}x)(\cdot)$ is continuous on the interval J.

Let $x \in \widehat{\mathscr{PC}_b},\ t \in J$ and $|\rho|$ be enough small, then

$$\begin{aligned}\mathbb{E}\|(\mathbb{L}x)(t+\rho) - (\mathbb{L}x)(t)\|^2 \quad &\leq \quad 5\{\mathbb{E}\|(\Psi(t+\rho) - \Psi(t)[\varphi(0) - g(0,\varphi,0)]\|^2\} \\ &\quad +5\sum_{j=1}^{4} \mathbb{E}\|F_j(t+\rho) - F_j(t)\|^2. \qquad (14)\end{aligned}$$

From hypothesis **(H3)**, we derive

$$\|(\Psi(t+\rho) - \Psi(t))[\varphi(0) - g(0,\varphi,0)]\|^2 \leq 2M^2\|\varphi(0) - g(0,\varphi,0)\|^2.$$

Then by the norm continuity of $\Psi(t)$ combined with Lebesgue dominated theorem it follows immediately that

$$\lim_{\rho\to 0} \mathbb{E}\|(\Psi(t+\rho) - \Psi(t))[\varphi(0) - g(0,\varphi,0)]\|^2 = 0.$$

By condition (**H8**), we conclude that

$$\mathbb{E}\|F_1(t+\rho) - F_1(t)\|^2 \to 0 \text{ as } |\rho| \to 0.$$

Furthermore, by virtue of **(H3)**,**(H4)**,**(H6)** and **(H7)**, we have

$$\begin{aligned}&\mathbb{E}\|F_2(t+\rho) - F_2(t)\|^2 \\ &\leq 2\mathbb{E}\left\|\int_0^t [\Psi(t+\rho-s) - \Psi(t-s)]f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)ds\right\|^2 \\ &\quad +2\mathbb{E}\left\|\int_t^{t+\rho} \Psi(t+\rho-s)f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)ds\right\|^2 \qquad (15)\end{aligned}$$

By estimating the terms on the right side of the above inequality, we obtain

$$\begin{aligned}&\left\|(\Psi(t+\rho-s) - \Psi(t-s))f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)\right\|^2 \\ &\qquad \leq 2M^2[M_f(1+M_{a_2})\|x_s\|^2 + M_f\tilde{M}_{a_2} + \tilde{M}_{a_2}].\end{aligned}$$

and

$$\left\|\Psi(t+\rho-s)f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)\right\|^2 \leq M^2[M_f(1+M_{a_2})\|x_s\|^2+M_f\tilde{M}_{a_2}+\tilde{M}_{a_2}],$$

Thanks to Lebesgue's dominated theorem and the above inequalities along with the norm continuity of $\Psi(t)$ we deduce

$$\mathbb{E}\|F_2(t+\rho)-F_2(t)\|^2 \to 0 \text{ as } |\rho| \to 0.$$

Now, using **(H3)** and **(H10)**,

$$\begin{aligned}\mathbb{E}\|F_4(t+\rho)-F_4(t)\|^2 &\leq 2\mathbb{E}\left\|\sum_{0<t_i<t}(\Psi(t+\rho-t_i)-\Psi(t-t_i))I_i(x(t_i^-))\right\|^2 \\ &+2\mathbb{E}\left\|\sum_{t\leq t_i<t+\rho}\Psi(t+\rho-t_i)I_i(x(t_i^-))\right\|^2,\end{aligned}$$

and evaluating the terms on the right hand side,

$$\begin{aligned}\|(\Psi(t+\rho-t_i)-\Psi(t-t_i))I_i(x(t_i^-))\|^2 &\leq \|\Psi(t+\rho-t_i)-\Psi(t-t_i)\|^2[d_i\|x(t_i^-)\|^2] \\ &\leq 2M^2[d_i\|x(t_i^-)\|^2]\end{aligned}$$

and

$$\|\Psi(t+\rho-t_i)I_i(x(t_i^-))\|^2 \leq M^2[d_i\|x(t_i^-)\|^2].$$

Thus we obtain

$$\mathbb{E}\|F_4(t+\rho)-F_4(t)\|^2 \to 0 \text{ as } |\rho| \to 0.$$

Furthermore

$$\begin{aligned}\mathbb{E}\|F_3(t+\rho)-F_3(t)\|^2 &\leq 2\mathbb{E}\left\|\int_0^t[(\Psi(t+\rho-s)-\Psi(t-s))]\tilde{F}(s)dZ_Q^H(s)\right\|^2 \\ &+2\mathbb{E}\left\|\int_t^{t+\rho}\Psi(t+\rho-s)\tilde{F}(s)dZ_Q^H(s)\right\|^2 \\ &:= N_1+N_2.\end{aligned}$$

In view of Lemma 1 and **(H3)**,

$$\begin{aligned}N_1 &\leq 2c(H)t^{2H-1}\int_0^t\|[(\Psi(t+\rho-s)-\Psi(t-s))]\tilde{F}(s)\|_{L_Q^0}^2 ds \\ &\leq 2c(H)t^{2H-1}M^2\int_0^t\|\tilde{F}(s)\|_{L_Q^0}^2 ds \to 0 \,\text{as}\, |\rho| \to 0,\end{aligned}$$

since, for every fixed s,

$$\|(\Psi(t+\rho-s)-\Psi(t-s))\tilde{F}(s)\|_{L_Q^0}^2 \leq 2M^2\|\tilde{F}(s)\|_{L_Q^0}^2.$$

Next, using again Lemma 1, we have

$$N_2 \leq 2c(H)\rho^{2H-1}M^2\int_t^{t+\rho}\|\tilde{F}(s)\|_{L_Q^0}^2 ds \to 0 \text{ as } |\rho| \to 0.$$

Further, we have

$$\lim_{\rho\to 0}\mathbb{E}\|F_5(t+\rho)-F_5(t)\|^2 = 0.$$

Therefore, we can conclude that

$$\lim_{\rho\to 0}\mathbb{E}\|(\mathbb{L}x)(t+\rho)-(\mathbb{L}x)(t)\|^2 = 0.$$

That is to say, the function $t \to (\mathbb{L}x)(t)$ is continuous on the interval J and, consequently, $\mathbb{L}$ is well defined.

Step 2: In this part we show that $\mathbb{L}$ is a contraction mapping.

For $x, y \in \widehat{\mathscr{PC}_b}$, we have

$$
\begin{aligned}
&\|(\mathbb{L}x)(t)-(\mathbb{L}y)(t)\|^2 \\
&\leq \frac{1}{k}\left\|\left(g(t,x_t,\int_0^t a_1(t,s,x_s)ds)-g(t,y_t,\int_0^t a_1(t,s,y_s)ds)\right)\right\|^2 \qquad (16)\\
&\quad+\frac{3}{1-k}\left\{\left\|\int_0^t \Psi(t-s)\left[f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)-f(s,y_s,\int_0^s a_2(s,\tau,y_\tau)d\tau)\right]ds\right\|^2\right.\\
&\quad\left.+\left\|\sum_{0<t_i<t}\Psi(t-t_i)[I_i(x(t_i^-))-I_i(y(t_i^-))]\right\|^2\right\}\\
&\leq \frac{1}{k}\left\|\left(g(t,x_t,\int_0^t a_1(t,s,x_s)ds)-g(t,y_t,\int_0^t a_1(t,s,y_s)ds)\right)\right\|^2\\
&\quad+\frac{3}{1-k}\left\|\int_0^t \Psi(t-s)\left[f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)-f(s,y_s,\int_0^s a_2(s,\tau,y_\tau)d\tau)\right]ds\right\|^2\\
&\quad+\frac{3}{1-k}\left\|\sum_{0<t_i<t}\Psi(t-t_i)[I_i(x(t_i^-))-I_i(y(t_i^-))]\right\|^2.
\end{aligned}
$$

Applying Hölder's inequality, and using the Lipschitz property of g, f and $I_i, i=1,2,\cdots,m$, we find

$$
\begin{aligned}
\mathbb{E}\|(\mathbb{L}x)(t)-(\mathbb{L}y)(t)\|^2 \quad \leq \quad & k\mathbb{E}\|x_t-y_t\|^2+\frac{3}{1-k}tM^2\tilde{M}^2\int_0^t \mathbb{E}\|x_s-y_s\|^2ds\\
&+\frac{3}{1-k}M^2\left(\sum_{i=1}^{+\infty}d_i\right)^2.
\end{aligned}
$$

Therefore,

$$\sup_{s\in[-r,t]}\mathbb{E}\|(\mathbb{L}x)(s)-(\mathbb{L}y)(s)\|^2 \leq \omega(t)\sup_{s\in[-r,t]}\mathbb{E}\|x(s)-y(s)\|^2,$$

where

$$\omega(t)=k+\frac{3M^2\tilde{M}^2}{1-k}t^2+\frac{3M^2}{1-k}\left(\sum_{i=1}^{+\infty}d_i\right)^2.$$

Then, using inequality (12), we obtain

$$\omega(0)=k+\frac{3M^2}{1-k}\left(\sum_{i=1}^{+\infty}d_i\right)^2<1.$$

Therefore it follows that there exists a sufficiently small $b_1>0$ such that $0<b_1\leq b$ and $0<\omega(b_1)<1$. This implies that $\mathbb{L}$ is a contraction mapping. Then, the fixed point theorem implies that system (1)-(3) possesses a unique solution in $\widehat{\mathscr{PC}_{b_1}}$. This procedure can be repeated in order to extend the solution to the entire interval $[-r,b]$ in finitely many steps. This completes the proof. □

Remark 3. *Notice that we can extend the solution for $t \geq b$. Indeed, if we assume that the constants M_{a_1}, M_{a_2}, M_f and M_g which appear in assumptions **(H4)–(H7)** are independent of $b > 0$, then the mild solution is defined for all $t \in [-r, b]$, for each $b > 0$. This will play a crucial role in our analysis of stability. Therefore, in the next section we will assume that the solutions are defined globally in time (for instance, under the previous assumptions).*

4. **Mean square exponential stability.** Now we will analyze the exponential stability in the mean square moment for the mild solution to system (1)-(3). We need to impose some additional assumptions:

(H11) The resolvent operator $(\Psi(t))_{t\geq 0}$ satisfies the further condition:
There exist a constant $M > 0$ and a real number $\mu > 0$ such that $\|\Psi(t)\| \leq Me^{-\mu t}, \forall t \geq 0$. In other words, the resolvent operator $(\Psi(t))_{t\geq 0}$ is exponentially stable.

(H12) There exist non-negative real numbers $Q_1, Q_2 \geq 0$ and a continuous function $\zeta_1 : [0, +\infty) \to \mathbb{R}_+$ such that

$$\left\| \int_0^t a_1(t,s,\kappa)ds \right\|^2 \leq Q_1\|\kappa\|^2, \; \|g(t,\kappa,\phi)\|^2 \leq Q_2[\|\kappa\|^2 + \|\phi\|^2] + \zeta_1(t),$$

for all $t \geq 0$ and $\kappa, \phi \in \mathscr{PC}$.

(H13) There exist non-negative real numbers $R_1, R_2 \geq 0$ and a continuous function $\zeta_2 : [0, +\infty) \to \mathbb{R}_+$ such that

$$\left\| \int_0^t a_2(t,s,\kappa)ds \right\|^2 \leq R_1\|\kappa\|^2, \; \|f(t,\kappa,\varphi)\|^2 = R_2[\|\kappa\|^2 + \|\varphi\|^2] + \zeta^2(t),$$

for all $t \geq 0$ and $\kappa, \varphi \in \mathscr{PC}$.

(H14) There exist non-negative real numbers $S_1, S_2 \geq 0$ such that

$$\zeta_j(t) \leq S_j e^{-\mu t}, \forall t \geq 0, j = 1, 2.$$

(H15) In addition to assumptions **(C.1)** and **(C.2)**, the function $\tilde{F} : [0, +\infty) \to L_Q^0(Y, X)$ satisfies

$$\int_0^{+\infty} e^{\mu s}\|\tilde{F}(s)\|^2_{L_Q^0} ds < \infty.$$

To facilitate the computations we set $\tilde{Q} = Q_2[1 + Q_1]$ and $\tilde{R} = R_2[1 + R_1]$. Now we can establish the exponential stability of system (1)-(3).

Theorem 6. *Assume that conditions **(H10)-(H15)** and the following inequality*

$$\frac{4M^2\tilde{R}/\mu^2 + 4M^2\left(\sum_{i=1}^{+\infty} d_i\right)^2}{(1-k)^2} < 1 (17)$$

hold, where $k := \tilde{Q}^{\frac{1}{2}} < 1$. Then the mild solution of system (1)-(3) is exponentially stable in mean square.

Proof. Thanks to inequality (17) we can find a suitable number $\varepsilon > 0$, small enough, such that

$$k + \frac{4M^2\tilde{R}}{\mu(\mu-\varepsilon)(1-k)} + \frac{4M^2\left(\sum_{i=1}^{+\infty} d_i\right)^2}{1-k} < 1.$$

Let $x(t)$ be the solution of the impulsive stochastic system (1)-(3) and $\eta = \mu - \varepsilon$. Therefore, from Eq. (12),

$$\begin{aligned}
\mathbb{E}\|x(t)\|^2 &\leq \frac{1}{k}\mathbb{E}\left\|g(t,x_t,\int_0^t a_1(t,s,x_s)ds)\right\|^2 + \frac{4}{1-k}\mathbb{E}\Big\{\|\Psi(t)[\phi(0)-g(0,\phi,0)]\|^2 \\
&+ \left\|\int_0^t \Psi(t-s)\tilde{F}(s)dZ_Q^H(s)\right\|^2 + \left\|\int_0^t \Psi(t-s)f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)ds\right\|^2 \\
&+ \left\|\sum_{0<t_i<t}\Psi(t-t_i)I_i(x(t_i^-))\right\|^2\Big\} \\
&\leq \sum_{j=1}^5 G_j(t). \qquad (18)
\end{aligned}$$

Now, we compute the terms on the right-hand side of the above inequality. From hypotheses **(H12)** and **(H14)**, we have

$$\begin{aligned}
G_1(t) &= \frac{1}{k}\mathbb{E}\|g(t,x_t,\int_0^t a_1(t,s,x_s)ds)\|^2 \\
&\leq \frac{1}{k}\left\{\tilde{Q}\mathbb{E}\|x_t\|^2 + \zeta_1(t)\right\} \\
&\leq k\mathbb{E}\|x_t\|^2 + E_1 e^{-\eta t},
\end{aligned}$$

where $E_1 = \frac{S_1}{k}$.

From the hypotheses **(H11)**, **(H12)** and **(H13)**, we obtain that

$$\begin{aligned}
G_2(t) &\leq \frac{8}{1-k}\mathbb{E}\|\Psi(t)\phi(0)\|^2 + \frac{8M^2}{1-k}e^{-2\mu t}\mathbb{E}\left\{\tilde{Q}\mathbb{E}\|\phi\|^2 + \zeta_1(t)\right\} \\
&\leq E_2 e^{-\eta t}, \qquad (19)
\end{aligned}$$

where $E_2 = \dfrac{8}{1-k}\left[\mathbb{E}\|\phi(0)\|^2 + \left\{\tilde{Q}\mathbb{E}\|\phi\|^2 + S_1\right\}\right]$.

From hypotheses **(H11)**, **(H12)**, **(H14)** and Hölder inequality, we have the following estimate

$$\begin{aligned}
G_3 &\leq \frac{4}{1-k}\mathbb{E}\left(\int_0^t Me^{-\mu(t-s)}\left\|f(s,x_s,\int_0^s a_2(s,\tau,x_\tau)d\tau)\right\|ds\right)^2 \\
&\leq \frac{4M^2\tilde{R}}{\mu(1-k)}\int_0^t e^{-\mu(t-s)}\mathbb{E}\|x_s\|^2 ds + E_4 e^{-\eta t}, \qquad (20)
\end{aligned}$$

where $E_4 = \dfrac{4M^2}{\mu(1-k)}\dfrac{S_2}{\mu-\eta}$.

Using Lemma 1 and hypotheses **(H11)**, we derive

$$\begin{aligned}
G_4(t) &\leq \frac{4}{1-k}M^2 c(H)t^{2H-1}\int_0^t e^{-2\mu(t-s)}\|\tilde{F}(s)\|^2_{L_Q^0}ds \\
&\leq e^{-\eta t}\frac{4M^2}{(1-k)}c(H)t^{2H-1}e^{-\varepsilon t}\int_0^t e^{\mu s}\|\tilde{F}(s)\|^2_{L_Q^0}ds. \qquad (21)
\end{aligned}$$

Noting that condition **(H15)** guarantees the existence of a constant $E_5 > 0$ such that, for all $t \geq 0$,

$$\frac{4M^2}{(1-k)}c(H)t^{2H-1}e^{-\varepsilon t}\int_0^t e^{\mu s}\|\tilde{F}(s)\|^2_{L_Q^0}ds \leq E_5,$$

we deduce

$$G_4(t) \leq E_5 e^{-\eta t}. \tag{22}$$

As for the last term, hypothesis **(H10)** implies

$$\begin{aligned} G_5(t) &\leq \frac{4M^2}{(1-k)}\left(\sum_{0<t_i<t} d_i\right)^2 e^{-2\mu(t-t_i)}\mathbb{E}\|x(t_i^-)\|^2 \\ &\leq \frac{4M^2}{(1-k)}\left(\sum_{0<t_i<t} d_i\right)\sum_{0<t_i<t} d_i e^{-\mu(t-t_i)}\mathbb{E}\|x(t_i^-)\|^2. \end{aligned} \tag{23}$$

Connecting the inequalities (19) – (23) along with (18) and using Lemma 2 we conclude

$$\mathbb{E}\|x(t)\|^2 \leq \omega e^{-\eta t}, \quad \text{for } t \in [-r,0]$$

and

$$\begin{aligned} \mathbb{E}\|x(t)\|^2 &\leq \omega e^{-\eta t} + k \sup_{-r\leq\theta\leq 0}\mathbb{E}\|x(t+\theta)\|^2 + \hat{k}\int_0^t e^{-\eta(t-s)}\sup_{-r\leq\theta\leq 0}\mathbb{E}\|x(s+\theta)\|^2 ds \\ &+ \sum_{i=1}^{+\infty} d_i e^{-\eta(t-t_i)}\mathbb{E}\|x(t_i^-)\|^2, \text{ for each } t \geq 0. \end{aligned}$$

Here

$$\omega = \max\left(\sum_{j=1}^{4} E_j, \sup_{-r\leq\theta\leq 0}\mathbb{E}\|\phi(\theta)\|^2\right)$$

and

$$\hat{k} = \frac{4M^2\tilde{R}}{\mu(1-k)}$$

We observe that $k + \frac{\hat{k}}{\eta} + \sum_{i=1}^{+\infty} d_i < 1$. Using Lemma 2.6, we have the existence of positive constants E and θ such that $\mathbb{E}\|x(t)\|^2 \leq E e^{-\theta t}$, for any $t \geq -r$. Hence we conclude that the mild solution of system (1)-(3) is exponentially stable in mean square and the proof is completed. □

Remark 4. *In the absence of impulsive effects, system (1)-(3) becomes the following one*

$$d\left[x(t) - g(t, x_t, \int_0^t a_1(t,s,x_s)ds)\right] \tag{24}$$

$$= A\left[x(t) - g(t, x_t, \int_0^t a_1(t,s,x_s)ds)\right] dt \tag{25}$$

$$+ \int_0^t B(t-s)\left[x(s) - g(t, x_s, \int_0^s a_1(t,\tau,x_\tau)d\tau)\right] dsdt \tag{26}$$

$$+ f(t, x_t, \int_0^t a_2(t,s,x_s)ds)dt + \tilde{F}(t)dZ_Q^H(t), t \in [0,b], t \neq t_i, \tag{27}$$

$$x_0(t) = \phi(t) \in \mathscr{C}, -r \leq t \leq 0, \tag{28}$$

and the mild solution of system (24)-(28) is exponentially stable in mean square provided

$$\frac{4M^2\tilde{R}/\mu^2}{(1-k)^2} < 1, \tag{29}$$

which can be obtained by using the same technique used in Theorem 6.

5. **Example.** In this section we make use of our previous results to study the existence, uniqueness and exponential stability of mild solution to a concrete neutral stochastic partial integro–differential equations with finite delays.
Consider the neutral stochastic partial integro–differential equations with impulses of the form

$$\begin{aligned}
&d\left[z(t,\tau)-\sigma_1\left(t,z(t-\zeta,\tau),\int_0^t \sigma_2(t,s,z(s-\zeta,\tau))ds\right)\right]\\
&=\frac{\partial^2}{\partial\xi^2}\left[z(t,\tau)-\sigma_1\left(t,z(t-\zeta,\tau),\int_0^t \sigma_2(t,s,z(s-\zeta,\tau))ds\right)\right]dt\\
&+\int_0^t \gamma(t-s)\left[z(s,\tau)-\sigma_1\left(s,z(s-\zeta,\tau),\int_0^s \sigma_2(t,l,z(l-\zeta,\tau))dl\right)\right]dt\\
&+\left[\sigma_3\left(t,z(t-\zeta,\tau),\int_0^t \sigma_4(t,s,z(s-\zeta,\tau))ds\right)\right]dt+\tilde{F}(t)dZ_Q^H(t),
\end{aligned}\tag{30}$$

$0\le\tau\le\pi, t\neq t_i, t\in[0,+\infty)$, subject to the initial conditions

$$\begin{aligned}
z(t,0) &= z(t,\pi)=0,\ 0\le t<+\infty,\\
\Delta z(t_i,.)(\tau) &= \frac{l}{i^2}z(t_i^-,\tau),\ t=t_i,\ i=1,2,...,\\
z(t,\cdot) &= \phi(t,\cdot), -r\le t\le 0,
\end{aligned}$$

where $\phi(\cdot,\cdot)\in\mathscr{PC}([-r,0],L^2[0,\pi])$, $\sigma_j(\cdot), j=1,2,3,4$, are functions defined below, and l is a positive constant, and $\tilde{F}:\mathbb{R}_+\to\mathbb{R}$ is a continuous function such that $\tilde{F}$ satisfies assumption **(H13)**, Z^H denotes a Rosenblatt process. Let $Y=L^2([0,\pi])$ and $e_n:=\sqrt{\frac{2}{\pi}}\sin(nx),\ (n=1,2,3,\cdots)$. Then $(e_n)_{n\in\mathbb{N}}$ is a complete orthonormal basis in Y. In order to define the operator $Q:Y\to Y$, we choose a sequence $\{\lambda_n\}_{n\ge1}\subset\mathbb{R}^+$ and set $Qe_n=\lambda_n e_n$, and assume that $tr(Q)=\sum_{n=1}^\infty\sqrt{\lambda_n}<\infty$. Define the process Z^H by

$$Z^H(t)=\sum_{n=1}^\infty\sqrt{\lambda_n}\gamma_n^H(t)e_n,$$

where $H\in(\frac12,1)$ and $\{\gamma_n^H\}_{n\in\mathbb{N}}$ is a sequence of two-sided one-dimensional Rosenblatt process mutually independent. Define $A:D(A)\subset\mathbb{H}\to\mathbb{H}$ by $A=\frac{\partial^2}{\partial z^2}$, with domain $D(A)=H^2([0,\pi])\cap H_0^1([0,\pi])$. Then, A is the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t\ge0}$ on X, which is given by
$T(t)\phi=\sum_{n=1}^\infty e^{-n^2t}<\phi,e_n>e_n,\ \ \phi\in D(A)$.
Let $\Gamma:D(A)\subset X\to X$ be the operator defined by

$$\Gamma(t)(z)=\gamma(t)Az\ \text{ for }\ t\ge0\ \text{ and }\ z\in D(A),$$

and let the functions $\sigma_1(t,x,y),\ \sigma_2(t,z,z'),\ \sigma_3(t,x,y),\ \sigma_4(t,z,z')$ be defined as follows:

$$\begin{aligned}
\sigma_1 &: \mathbb{R}_+\times\mathbb{R}\times\mathbb{R}\longrightarrow\mathbb{R}\\
\sigma_2 &: \mathbb{R}_+\times\mathbb{R}_+\times\mathbb{R}\longrightarrow\mathbb{R}\\
\sigma_3 &: \mathbb{R}_+\times\mathbb{R}\times\mathbb{R}\longrightarrow\mathbb{R}\\
\sigma_4 &: \mathbb{R}_+\times\mathbb{R}_+\times\mathbb{R}\longrightarrow\mathbb{R}.
\end{aligned}$$

Moreover, let the following assumptions hold a.s.:

(A1) The functions $\sigma_i, i=1,2$, are continuous in the sense that

$$\lim_{t\to s}|\ \sigma_i(t,x,y)-\sigma_i(s,x,y)|^2=0,\ \text{for } x,y\in\mathbb{R}.$$

(A2) There exist non-negative real number q_1 such that

$$|\int_0^t \sigma_2(t,x,y)ds|^2 \leq q_1|y|^2, \text{ for } x,y \in \mathbb{R}.$$

(A3) There exist non-negative real number r_1 such that

$$|\int_0^t \sigma_4(t,x,y)ds|^2 \leq r_1|y|^2, \text{ for } x,y \in \mathbb{R}.$$

(A4) There exist non-negative real numbers $q_1, q_2 \geq 0$ and a continuous function $\zeta_1 : [0,+\infty) \to \mathbb{R}_+$ such that

$$|\sigma_1(t,x,y)|^2 \leq q_2[|x|^2 + |y|^2] + \zeta_1(t),$$

for all $t \geq 0$ and $x,y \in \mathbb{R}$.

(A5) There exist non-negative real numbers $r_1, r_2 \geq 0$ and a continuous function $\zeta_2 : [0,+\infty) \to \mathbb{R}_+$ such that

$$|\sigma_3(t,x,y)|^2 = r_2[|x|^2 + |y|^2] + \zeta_2(t,)$$

for all $t \geq 0$ and $x,y \in \mathbb{R}$.

(A6) There exist non-negative real numbers $S_1, S_2 \geq 0$ such that

$$\zeta_j(t) \leq S_j e^{-\mu t}, \forall t \geq 0, j = 1,2.$$

Let

$$\begin{aligned}
a_1(t,s,\phi)(\tau) &= \sigma_2(t,s,\phi(\theta,\tau)),\\
g(t,\phi,\int_0^t a_1(t,s,\phi)ds)(\tau) &= \sigma_1(t,\phi(\theta,\tau),\int_0^t \sigma_2(t,s,\phi(\theta,\tau))ds),\\
&= \int_{-r}^0 \nu_1(\theta)\phi(\theta)(\xi)d\theta + \int_0^t\int_{-r}^0 b_2(t)b_3(l)\phi(l,\xi)dlds,\\
a_2(t,s,\phi)(\tau) &= \sigma_4(t,s,\phi(\theta,\tau)),\\
f(t,\phi,\int_0^t a_2(t,s,\phi)ds)(\tau) &= \sigma_3(t,\phi(\theta,\tau),\int_0^t \sigma_4(t,s,\phi(\theta,\tau))ds),\\
&= \int_{-r}^0 \tilde{b}_1(t,s,\tau,\phi(s,\tau))ds \qquad (31)\\
&\quad + \int_0^t\int_{-r}^0 \tilde{b}_2(s)\tilde{b}_3(s,l,\tau,\phi(l,\tau))dlds,
\end{aligned}$$

where,

(1) the function $\nu_1(\theta) \geq 0$ is continuous in $(-r,0]$ satisfying

$$\int_{-r}^0 \nu_1^2(\theta)d\theta < \infty, \quad \gamma_g^1\left(\int_{-r}^0 \nu_1^2(\theta)d\theta\right)^{\frac{1}{2}} < \infty.$$

(2) $b_2, b_3 : \mathbb{R} \to \mathbb{R}$ are continuous, and

$$\gamma_g^2\left(\int_{-r}^0 (b_3(s))^2 ds\right)^{\frac{1}{2}} < \infty.$$

(3) The function $\tilde{b}_2 : \mathbb{R} \to \mathbb{R}$ is continuous and $\tilde{b}_i : \mathbb{R} \to \mathbb{R}$, $i = 1,3$ are continuous and there exist continuous functions $r_j : \mathbb{R} \to \mathbb{R}, j = 1,2,3,4$ such that

$$\begin{aligned}
|\tilde{b}_1(t,s,x,y)| &\leq r_1(t)r_2(s)|y|; \quad (t,s,x,y) \in \mathbb{R}^4,\\
|\tilde{b}_3(t,s,x,y)| &\leq r_3(t)r_4(s)|y|; \quad (t,s,x,y) \in \mathbb{R}^4,
\end{aligned}$$

with $\tilde{L}_1^b = \left(\int_{-r}^0 (r_2(s))^2 ds\right)^{\frac{1}{2}} < \infty, \quad \tilde{L}_2^b = \left(\int_{-r}^0 (r_4(s))^2 ds\right)^{\frac{1}{2}} < \infty.$

Notice that the function g is continuous with respect to variable t.
If we put

$$\begin{cases} x(t) := x(t)(\tau) & = & z(t,\tau) \;\text{ for }\; t \geq 0 \;\text{ and }\; \tau \in [0,\pi] \\ \phi(t)(\tau) & = & z_0(t,\tau) \;\text{ for }\; t \in [-r,0] \;\text{ and }\; \tau \in [0,\pi], \end{cases}$$

then equation (30) takes the following abstract form

$$\begin{aligned} & d\left[x(t) - g(t,x_t,\int_0^t a_1(t,s,x_s)ds)\right] \\ =& A\left[x(t) - g(t,x_t,\int_0^t a_1(t,s,x_s)ds)\right]dt \\ & \quad + \int_0^t B(t-s)\left[x(s) - g(s,x_s,\int_0^s a_1(t,\tau,x_\tau)d\tau)\right]dsdt \\ & \quad + f(t,x_t,\int_0^t a_2(t,s,x_s)ds)dt + \tilde{F}(t)dZ_Q^H(t), t \in [0,+\infty), t \neq t_i, \qquad (32) \\ & \Delta x(t_i) = I_i(x(t_i^-)), t = t_i,\; i = 1,2,..., \\ & x_0(t) = \phi(t),\; -r \leq t \leq 0, \end{aligned}$$

Moreover, if γ is bounded and a C^1 function such that is derivative γ' is bounded and uniformly continuous, then **(H1)** and **(H2)** are satisfied and hence, by Theorem 2.2, Eq. (9) has a resolvent operator $(\Psi(t))_{t\geq 0}$ on X.

Moreover for $\phi_j \in \mathscr{PC}, j = 1,2$

$$\begin{aligned}
&\left\| g\left(t,\phi_1,\int_0^t a_1(t,s,\phi_1)\,ds\right) - g\left(t,\phi_2,\int_0^t a_2(t,s,\phi_2)\,ds\right)\right\|_{L^2[0,\pi]} \\
&= \Bigg[\int_0^\pi \Bigg(\int_{-r}^0 v_1(\theta)\left[\phi_1(\theta)(\rho) - \phi_2(\theta)(\rho)\right]d\theta \\
&\qquad + \int_0^t\int_{-r}^0 b_2(s)b_3(l)\left[\phi_1(l,\rho) - \phi_2(l,\rho)\right]dlds\Bigg)^2 d\rho\Bigg]^{1/2} \\
&\le \sqrt{2}\left[\int_0^\pi\left(\int_{-r}^0 v_1(\theta)\left[\phi_1(\theta)(\rho) - \phi_2(\theta)(\rho)\right]d\theta\right)^2 d\rho\right]^{1/2} \\
&\qquad + \sqrt{2}\left[\int_0^\pi\left(\int_0^t\int_{-r}^0 b_2(s)b_3(l)\left[\phi_1(l,\rho) - \phi_2(l,\rho)\right]dlds\right)^2 d\rho\right]^{1/2} \\
&\le \sqrt{2}\left[\int_0^\pi\left(\int_{-r}^0 v_1(\theta)\,d\theta\right)^2 d\rho\right]^{1/2}\|\phi_1 - \phi_2\| \\
&\qquad + b\sqrt{2}\left[\int_0^\pi\left(\int_0^t b_2^2(s)ds\right)\left(\int_{-r}^0 b_3(l)\left[\phi_1(l,\rho) - \phi_1(l,\rho)\right]dl\right)^2 d\rho\right]^{1/2} \\
&\le \sqrt{2\pi}\left[\left(\int_{-r}^0 v_1(\theta)\,d\theta\right)^2\right]^{1/2}\|\phi_1 - \phi_2\| \\
&\qquad + b\sqrt{2}\sqrt{\pi}\left(\int_0^t b_2^2(s)ds\right)^{1/2}\times\left(\int_{-r}^0 b_3^2(l)dl\right)^{1/2}\|\phi_1 - \phi_2\| \\
&\qquad \le k_2\|\phi_1 - \phi_2\|
\end{aligned}$$

where $k_2 = \gamma_g^1 + b\|b_2\|_\infty\gamma_g^2$. In the same way we obtain

$$\begin{aligned}
&\left\| f\left(t,\phi_1,\int_0^t a_2(t,s,\phi_1)\,ds\right) - f\left(t,\phi_2,\int_0^t a_2(t,s,\phi_2)\,ds\right)\right\|_{L^2[0,\pi]} \\
&\qquad \le l_2\|\phi_1 - \phi_2\|
\end{aligned}$$

where $l_2 = \|b_2\|_\infty\tilde{L}_1^b + \|\tilde{b}_2\|_\infty\|r_3\|_{L^1}\tilde{L}_2^b$. Therefore, we may easily verify all the assumptions of Theorem 5 and hence, there exists a mild solution for (30).
We assume moreover that there exists $\beta > a > 1$ and $b(t) < \frac{1}{a}e^{-\beta t}$ for all $t \ge 0$. Thanks to Lemma 5.2 in [9], we have the following estimates $\|\Psi(t)\| \le e^{-\lambda t}$ where $\lambda = 1 - \frac{1}{a}$.
Consequently, all the hypotheses of Theorem 6 are fulfilled. Therefore, Eq. (30) possesses a unique mild solution which is exponentially stable provided that

$$\sqrt{q_2(1+q_1))} + \frac{\frac{4(r_2(1+r_1))}{\lambda^2} + 4(\sum_{i=1}^\infty \frac{l}{i^2})^2}{(1-\sqrt{q_2(1+q_1)})^2} < 1.$$

6. **Conclusion.** The interest of this work is the study of neutral impulsive stochastic integro–differential equations driven by a Rosenblatt process in a real separable Hilbert space. The existence, uniqueness and exponential stability were obtained by using the fixed point theorem and the resolvent operators theory. Finally, an example is analyzed to illustrate the effectiveness of the main results. It should be emphasized that system (1)-(3)

considered in this paper is more general than those in the existing literature. There are two direct issues that require further study. First, we will investigate the existence and exponential stability of mild solutions to functional non autonomous stochastic integro-differential equations driven by Rosenblatt process with index $H \in (\frac{1}{2}, 1)$. Second, we will study the solvability and stability for neutral stochastic integro-differential equations driven by Rosenblatt process with impulses.

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