

## A Proposal for the Content of the Chapter "Poverty Measures" in a Master's Subject in Economy and Development

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Accepted 13<sup>th</sup> November, 2018

### Abstract

The objective of this work is teaching. We ask ourselves, what should students be taught about statistical measures of poverty? In a master's course, Measures of Inequality and Poverty, given at the University of Seville, Spain, a chapter dedicated to Measures of Poverty is included. In this work we want to provide this chapter with content. We include the different conceptions of this phenomenon together with the most commonly used measures or indicators. Objective and subjective poverty (with the measures of Kaptein or Leiden), absolute and relative, one-dimensional or multidimensional, are ideas that are shelled in the development of this work. We also include a set of possible axioms or properties required for a good indicator of poverty.

**Keywords:** Poverty Line, Equivalence Scales, Absolute and Relative Poverty, Objective and Subjective Poverty, Kaptein Line.

### 1.0 INTRODUCTION

The Master in Economics and Development taught at the University of Seville includes a subject called "Measures of inequality and poverty." Well, the part dedicated to poverty measures is the one we present in this work, with a content proposal that seeks to convey to the student the most global vision possible on this issue of "measuring poverty" that is so socially worrying. The content proposal that we present is the one that we are currently developing in the aforementioned subject.

With the term "poverty" we identify the most disadvantaged group in a society, the one that enjoys the lowest level of life. Of course, it is a concept that is as intuitive as it is so difficult to profile at the technical-statistical level. In such a way that, at the time of carrying out the quantification of this concept, we find a great variety of measures depending on the different approaches and the possible plurality of data that support the quantification attempt. From defining poverty as "the most unfavorable part of a society" (intuitive definition but difficult to quantify) to using expressions such as "situation that means that we do not have or hardly have enough to stay alive" (definition rather applicable to societies in development pathways) or "situation in which we do not have enough to live a life that is considered normal in society" (applicable to developed societies, Hagenars, 1986), we find such a quantity of definitions in the literature, in which many of them imply the procedure to measure it.

Of course, the objective of a chapter of a subject taught in a master's degree, chapter entitled "poverty measures" is, as has already been said, to show and capture, in its greater generality, the broad casuistry associated with this concept, with special emphasis on the measures used in official statistics. The interest in measuring poverty in a society is justified because it will depend on it to be able to give solutions to this serious problem: it is necessary to know how many poor people there are, in what degree they are, why they are and where they are, if we want to design policies that allow these poor people to stop being poor.

We all know that there are certain goods and services whose access are essential to enjoy a decent life: housing with a

minimum of comfort, basic food, health, educational, cultural services... The lack of access to these goods, the stay away from the They can be a way of defining a person as poor. Therefore, these different goods, translated into quantifiable variables, can be used to define the concept. We would speak in this case of "multidimensional poverty", a definition closely linked to the concept of social exclusion. Its measurement is done through non-monetary variables and indicators of deprivation, with which aggregates are built that serve to classify or not people in this poverty status. It is also known as "poverty deficiency". The Council of Europe, bearing in mind the different options in the definition of poverty, recommends that those persons, families or groups of persons who are limited by the limitations of cultural, material and social resources, be considered poor if they exclude them from the minimum type of life considered acceptable. In the State in which they reside (OECD, 1984).

However, the inability to access these goods and services is usually due to the shortage of income of the person or family of which they are a part. Therefore, another way of measuring poverty in a more simplistic (one-dimensional) format is based on a variable that defines personal or family income (or expenses), which allow us to establish border levels or values such that, below , people are considered poor.

In literature we find different ways to classify poverty. Thus, we speak of "objective poverty" when the classification of the individual as poor has been made through economic data associated with the individual, while we speak of "subjective poverty" when for that classification the opinion of the individual has been taken into account.

Depending on the scale or reference used to establish the poverty threshold, we distinguish between "**absolute poverty**" and "**relative poverty**".

**Absolute poverty:** Situation in which the basic needs of the individual are not covered. That is, lack of basic goods, food, housing and clothing. This concept is closely linked to that of "misery", and should be applied equally in all countries and societies. That is, a person classified as poor according to this criterion should be classified in the same way anywhere in the world. However, it is very difficult to build pure measures of absolute poverty. Of course, absolute poverty is understood as a condition characterized by severe deprivation of basic human needs, including food, access to potable water, health benefits, protection, education and information. Therefore, it does not depend solely on income, also on access to social services.

**Relative poverty:** It is situated in the society under study. A person is classified as poor when they are in a situation of clear disadvantage, economically and socially, with respect to the rest of the people in their environment. So, this concept is closely linked to inequality. The classification between poor and non-poor will depend on the degree of development of the society studied in particular. For example, a country may consider anyone who receives an annual income of less than € 3,000 as poor, while another

country may consider it this way if its annual income is less than € 8,000. Yes, it may be the case that a person considered as poor in the second country is not so in the first.

On the other hand, poverty is not a static phenomenon over time. The economic situation of people can change so that an individual can go from being poor to not being poor and vice versa. Therefore, it is necessary that longitudinal studies are made, in time that take into account changes or transitions over more or less long periods. This is how the term "persistent poverty" or long-term appears. According to EUROSTAT, a person from the European Union is persistently immersed in poverty if it is classified as such in the last annual survey and in at least 2 of the previous 3.

For a better understanding of the poverty phenomenon, the joint use of absolute and relative measures of this concept is recommended.

An important detail to keep in mind is that most poverty studies are based on household surveys. Therefore, these surveys do not collect information on homeless people or residents of charitable institutions, so that individuals who are part of these groups, who are often affected by poverty with greater intensity and frequency, are not usually part of the group in the measurements that are usually made of it.

## 2.0 CONSTRUCTION OF THE POVERTY LINE

The first problem with which we find ourselves in a research on poverty is the identification of those who are poor. This is solved by introducing a poverty value or threshold, which is a level of income that separates the poor from those that are not. Thus, it is an imaginary line that is used to classify the people of a society into two groups: poor and not poor, the first being those that fall below that line. According to Kakwani (1986, p.239), *the poverty line is the level of income necessary to obtain the so-called minimum needs of life*. Authors such as Van Praag, Hagenaaars and Van Weeren (1982) define poverty in terms of well-being and obtain the poverty line based on the relationship between welfare and income.

As Domínguez and Martín (2006) point out, *there is no scientific basis on which one can unequivocally accept or reject a poverty line based on purely relative or purely subjective assumptions. Each has its merits and its limitations, or as Atkinson (1974: 48) says: any line of poverty will be influenced by the usual life models and would be only defined in relation to the life pattern of a particular society, or as Sen suggests (1983): the poverty line is such that it presents justification by itself and is one under which one cannot adequately participate in common activities, or be free from public shame because it does not satisfy needs. And returning to Kakwani (1986, p.273) he defines it as: the level of income sufficiently low that it is considered to create misfortune, in terms of the everyday life patterns of society.*

A first classification of the poverty lines is that of **objective** and **subjective**, depending on whether the income levels available to society or on the perception that the households themselves have of their social and economic situation, as we have already done. Defined objective and subjective poverty. In turn, the objective poverty lines are classified as **absolute** and **relative**. The first are those that establish the need to cover basic needs and are not related to the standard of living of a society. They are suitable for developing societies. The proportional growth of a society's income can reduce the number of individuals below this line, so poverty measured by this line could be eradicated through the economic growth of the society where it is measured. It is often said that the elasticity "economic growth / absolute poverty line" is zero.

The **relative poverty line** is plotted according to the distribution of wealth of society itself, so that each society has its own poverty line (it is, therefore, related to the idea of deprivation compared to its environment). In general, a change of scale in the variable used to construct the poverty line will involve a change of scale in the line itself, so that the group of poor will remain the same after the change of scale. Thus, if in a given society there is an increase in wealth of 5% for each and every one of its members, the poverty line of that society (relative poverty line) will also grow by 5% and, therefore, the poor before the increase will remain poor after the increase. We would speak in this case of an elasticity "economic growth / line of relative poverty" equal to one. In order to produce a reduction in the number of poor people calculated through this type of line, changes would have to be made in the distribution of wealth, that is, some kind of redistribution would have to be produced that favored the poor. In fact, most of the relative poverty lines that are built have a positive "economic growth / relative poverty line" elasticity, between zero and one (Kilpatrick, 1973).

Usually, the relative poverty lines use indicators based on monetary variables such as **income** or **expenditure**. In both cases, a minimum level of the chosen variable is set in such a way that a person below that level is classified as poor. Both variables present their advantages and disadvantages.

In the beginning, the best option is the annual income, since it reflects the economic capacity of a household. But the income only provides a partial image, because the homes besides income have assets, assets ..., which are part of the wealth of the household and influence their standard of living. On the other hand, the expenditure variable is more stable, since households do not modify their consumption habits with ease. The expense depends more on the concept of permanent income (income that allows families to live in the same conditions without modifying their wealth) than on current income. In addition, the concept of poverty is intimately linked to the so-called permanent income. Therefore, spending would also be a good variable for measuring poverty. However, spending also has the disadvantage of being largely conditioned by the environment in which one lives and by the customs acquired over time.

Also, both variables, income and expenses, are subject to measurement errors. It has sometimes been proven that the income figures that are collected in the surveys underestimate real income in the case of self-employment or capital income, while income from employment is usually more accurate. This, in addition to other conditioning factors, generates bias in the final information that is used in the analysis of poverty. In the measurement of expenditure there are also measurement problems that are linked to the methodology of the surveys that collect household consumption. No matter how well designed the surveys are, measurement errors in spending are inevitable in many cases, given the difficulty of obtaining this type of information, due to the effort that households must make to record the detailed expenses during the required period .

Therefore, the choice of the monetary variable used to measure poverty is not a banal issue, and affects the measures that are provided. In Europe, in recent years, income has been used as an official variable for the production of statistics on poverty and social exclusion. Thus, most of the poverty measures that we present use income or income as a support for construction, thus defining poverty as an economic deprivation. Foster (1984) talks about economic poverty, linked to the lack of resources of people, resources that are needed for the consumption of goods and services. We can also talk about *material poverty*.

### 3.0 INCOME PER UNIT OF CONSUMPTION

The relative poverty lines that are based on income (similarly would be done if they are based on the expense) are constructed in the following way:

- The total income of each household is calculated: income from work (self-employed or from others), capital income, social benefits, payments or returns of income tax (Income Tax), imputed rent, income from social assistance, transfers between households, mortgage interest paid, property taxes and property income.
- A fundamental decision is, what unit of analysis to use?, the home or the person? Although the income is per household, the person is usually chosen as the unit of analysis, since it is the one that is really affected by poverty. In any case, there is an intimate relationship between income per household and income per individual.
- Once the total household income is calculated, each person is awarded the same amount of "income". This per capita income could be calculated by dividing the total household income by the number of people who are part of that household. However, in the official statistics of most countries it is preferred to use another income to allocate to each person in the household. It is known as "income per consumption unit" or "equivalent income" which is the result of dividing the total household income by the "number of household consumption units", which is calculated taking into account factors such as the economy of scale.
- Equivalence scales. These scales try to reflect the reality of households based on the theory of the existence of economies of scale and equivalent consumption units. Thus, the increase in the number of members of a household does not have to be accompanied by the same proportional increase in income to maintain the level of welfare, given that many household expenses, housing, household appliances, etc. are shared. Children's consumption is usually different from that of adults, and those differences should be reflected in the number of household consumption units. The consumption units are calculated using what are called equivalence scales, for which there are several proposals, the most commonly used being the ones we quote below:
  - Scale of the OECD or Oxford scale. The number of consumption units in a household is calculated by adding the weights assigned to each member of the household, and these weights are: 1 for the first adult in the household, 0'7 for each of the rest of adults, and 0 '5 for each child under 14 years old. Thus, for a household in which adults and children live together, the number of consumption units of said household would be  $1+(a-1)\cdot 0'7+n\cdot 0'5$ . For example, a family with 2 adults and 2 children would have  $1+0'7+2\times 0'5=2'7$  equivalence units. In this way, the total annual income of the household would be divided by 2'7 and the result of that quotient would be the annual per capita income, assigned to each of the 4 members of the household. Proceeding in this way with all households, we can construct the distribution of frequencies for individualized income. Thus we will proceed, whatever the scale of equivalence used, the one quoted here from the OECD or those cited below.
  - Scale of the OECD modified. Similar to the previous one, where the first adult continues to weigh 1 point, but the rest

of adults 0'5 and children under 14 years 0'3. Thus, for a household in which adults and children live together, the number of consumption units of said household would be  $1+(a-1)\cdot 0'5+n\cdot 0'3$  in the same example above, a family with 2 adults and 2 children would have  $1+0'5+2\times 0'3=2'1$  equivalence units. This scale is what Eurostat usually uses, and with it the so-called Laeken indicators are built<sup>1</sup>.

- Parametric scales (Buhmann *et al.*, 1988). Recommended by some experts in the study of the distribution of income, and used in the international field to make comparisons between countries. It proposes to calculate the number of consumption units of a household through, where  $n$  is the number of household members and  $m$  is a parameter that we call "elasticity of equivalence" and that takes a value  $m\leq 1$ . If  $m=1$  we consider that there is no economy of scale in the household, and if there is economy of scale, the more accentuated the closer to 0 this parameter is. In some OECD studies it has been used, that is, number of consumption units  $=\sqrt[n]{n}$  In the case of the family of our example,  $\sqrt[4]{4}=2$  consumption units.
- Scale with two parameters (used in the USA). The number of consumption units in a household is calculated with the expression  $(a+k\cdot n)^m$ , being the number of adults,  $n$  the number of children under 14, and  $k$  and  $m$  two parameters of elasticity. For the family of our example, 2 adults and 2 children, and  $k=m=0'5$  for the number of consumption units of this household would be  $(2+0'5\times 2)^{0'5}=\sqrt{3}=1'732$ .

### 4.0 FIXING THE POVERTY LINE

Once the equivalence scale has been established and assigned to each member of each household, their per capita income, according to the units of consumption of the same, the distribution of the income of all the individuals of a society is constructed. Said variable is ordered from least to highest according to the level of income (income per consumption unit). We calculate the median of said variable. Until a few years ago the average was calculated, but this measure is influenced by extreme data, which does not reflect the reality of the majority of the population.

The poverty line or poverty threshold is established as a percentage of that median, which can be 40, 50, 60 or 70%, or even 20 or 25% if you want to study severe poverty. Currently EUROSTAT sets the poverty threshold at 60% of the median distribution of income per consumption unit.

The poverty line divides the population into poor and not poor. All those whose income per consumption unit is below the poverty line are classified as poor.

### 5.0 INCIDENCE, DISTRIBUTION AND INTENSITY OF POVERTY

To study poverty in a society, three types of measures are usually used: incidence, distribution and intensity measures.

**Incidence measures.** They inform about the extent of the problem, that is, they provide data on the number of people or households that are affected. They are usually expressed as a percentage of the population. They can be calculated for the entire population or for different subgroups of the same and, thus, detect which groups are most vulnerable within society.

**Distribution measures.** They tell us how the poor are distributed and what characteristics they have. That is, they provide a descriptive information of the poor group.

**Intensity measurements.** They focus on the degree of poverty suffered by people considered as poor.

By means of the set of measures related to the incidence, distribution and intensity of poverty, one can describe in a very detailed way what happens in a certain society in relation to this matter. We can meet with a great variety of situations: from a society with a high percentage of poor people, but where most of them are close to the poverty line, to another where there is a very small number of them, but very far away of said threshold.

We quote below some of the most commonly used measures. As an indicator of the incidence of poverty, the poverty rate, or  $H$  (Headcount ratio), is used, which is defined  $H = \frac{P}{n}$  as, with  $p$  = the number of the poor in society,  $y$   $n$  = and the total number of people, poor and non-poor, in society. Which is calculated this indicator. Poverty rates can be calculated for different population groups, according to demographic or socioeconomic variables: by sex, by age, by level of studies, by employment situation, etc.

The  $H$  index has serious drawbacks. Its greatest limitation is that it is insensitive to the changes experienced by the poor as long as they do not exceed the poverty line. Thus, it is an index insensitive to the intensity of poverty, since, although all the poor individuals were more and more, the index would not be altered by not depending on their expression of how poor they are.

In addition, it is also insensitive to inequality among the poor population, not verifying the principle of Pigou-Dalton transfers among the poor, since any monetary transfer between two individuals below the poverty line would not alter the index to Although this transfer is made in favor of an individual close to the line and to the detriment of one who is at the lowest levels of the distribution.

Example. Let us suppose a society formed by 60 people, of which we know their annual income of the previous year (calculated as income per consumption unit according to some equivalence scale), their age, and their sex (0 = woman, 1 = man). We have them sorted from least to greatest according to their annual income.

**Table 1.** Fictitious Data for Illustrative Example

Person	Annual income, in thousands of €	Age	Sex	Person	Annual income, in thousands of €	Age	Sex
1	2	15	0	31	11,2	33	1
2	2,3	24	1	32	11,3	42	1
3	3	51	0	33	12	57	0
4	3,4	24	0	34	12,3	8	1
5	4	22	0	35	12,7	14	1
6	4	33	1	36	12,9	33	0
7	4	55	1	37	13,2	45	0
8	5	47	1	38	13,8	74	0
9	5,5	22	0	39	13,9	82	1
10	5,8	78	1	40	14	67	1
11	5,9	64	0	41	14,3	12	0
12	6	33	0	42	14,7	35	0
13	6,2	25	0	43	14,9	48	1
14	6,7	12	1	44	15,4	17	1
15	7	8	1	45	15,9	27	1
16	7,2	32	1	46	16,2	25	0
17	7,5	50	0	47	16,5	33	1
18	8	32	1	48	16,8	15	0
19	8,3	33	1	49	17,4	84	0
20	8,5	42	0	50	17,6	56	0
21	9	57	0	51	17,9	24	1
22	9,2	61	0	52	18,5	33	1
23	9,3	33	1	53	18,6	18	1
24	9,7	42	1	54	19	24	0
25	10,1	21	1	55	19,3	33	1
26	10,7	12	0	56	20	12	0
27	10,8	18	0	57	20,3	54	1
28	10,8	69	1	58	20,4	56	0
29	10,9	58	1	59	21	69	1
30	11,2	21	0	60	22	48	0

Source: self-made

The foundations of the so-called Lisbon Strategy were set at the Nice European Council first (December 2000) and then at Laeken (December 2001). This aimed at a greater coordination of policies to reduce inequality and social exclusion -with the obligation of the different Member States to develop national plans for social inclusion-, for which a set of common basic indicators was defined, which would allow the progress of each country in reducing poverty and social exclusion. The selection of said set of indicators should allow Member States to use a "same language" in the evaluation of social reality. (Ayala, 2006, p.43)

The median of the variable annual income is 11'2 thousand euros. And 60% of the median is 6'72 thousand euros. Therefore, we set that amount as the poverty line, 6'72 thousand euros. We observe that there are 14 individuals with an income lower than 6.72. So,

$$H = \frac{14}{60} = 0'2333$$

That is, a 23'3% incidence of poverty. We can make a distribution by age of the incidence of poverty:

**Table 2.** Poverty Calculations for Table Data 1

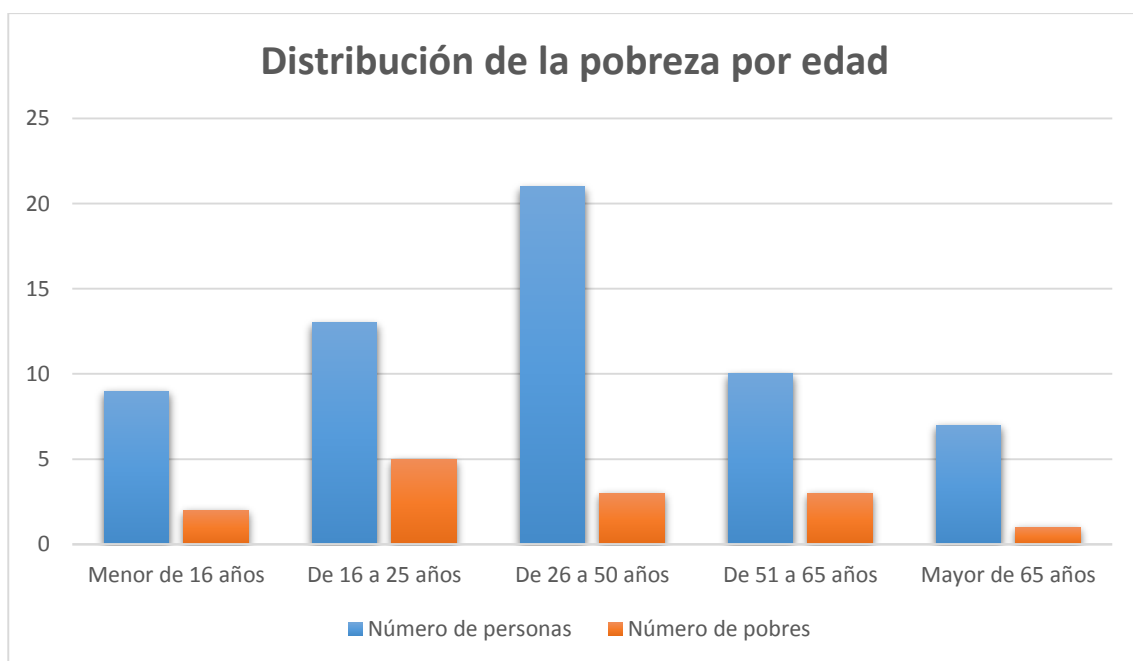
Age	Number of people	Number of poor	H index, by age bracket	% of poor people, over the total poor
<16 years	9	2	0,22	0,14
16-25	13	5	0,38	0,36
26-50	21	3	0,14	0,21
51-65	10	3	0,30	0,21
>65	7	1	0,14	0,07
	60	14		1,00

Source: self-made

We can say that the incidence of poverty in the 16 to 25 year stretch is the highest, with 38%, which in turn represents 36% of the total poor.

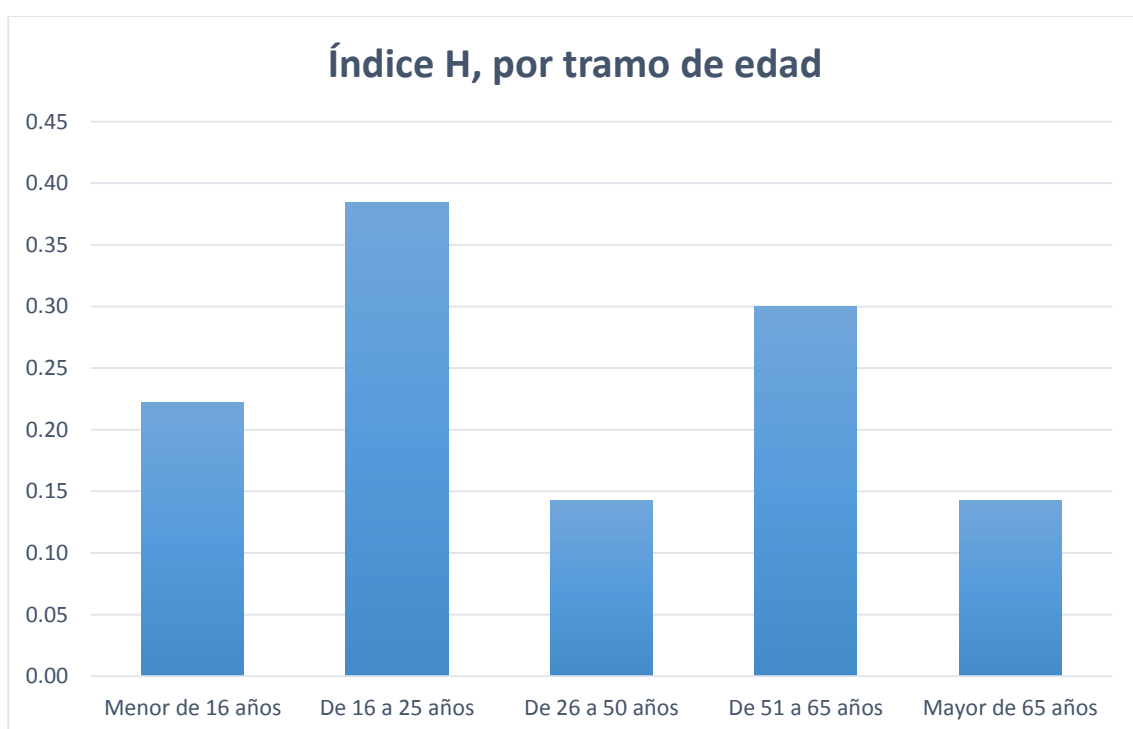
The weighted average of the H indices by age brackets (with weights number of people) is the global H index.

**Figure 1.** Distribution of Poverty by Age According to Data in Table 1



Source: Own Elaboration Based on Data from Table 1.

**Figure 2.** H Index by Age According to Data from Table 1



Source: Own Elaboration Based on Data from Table 1.

The distribution by gender of the poverty of this society is as follows:

**Table 3.** Distribution of Poverty by Sex for the Table Data 1

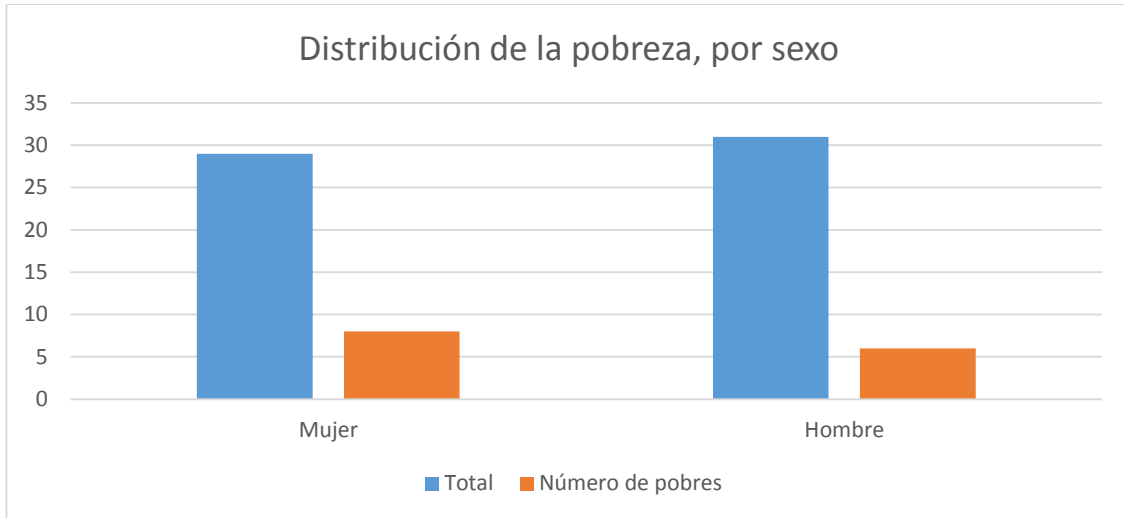
Sex	Total	Number of poor	H Index, by sex	Proportion of the poor, over the total
Woman	29	8	0,28	0,57
Man	31	6	0,19	0,43
	60	14		1,00

Source: self-made.

The incidence of poverty among women, 28%, is higher than among men, 19%. In addition, of the total of the poor in this society, 57% are women and 43% are men.

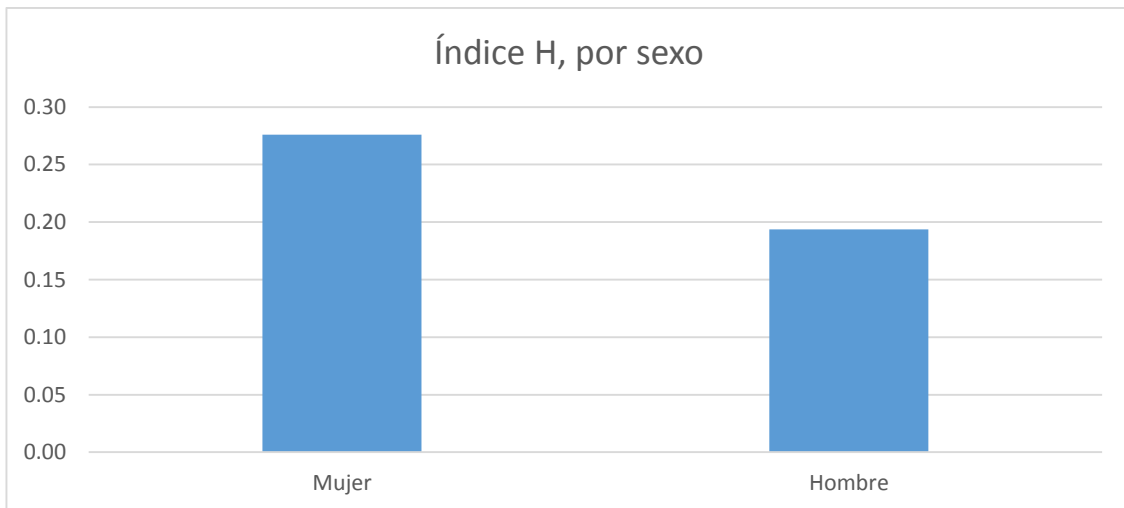
We could evaluate the level of income that would be necessary to transfer from rich to poor in order to eliminate poverty.

**Figure 3.** H Index by Age According to Data from Table 1



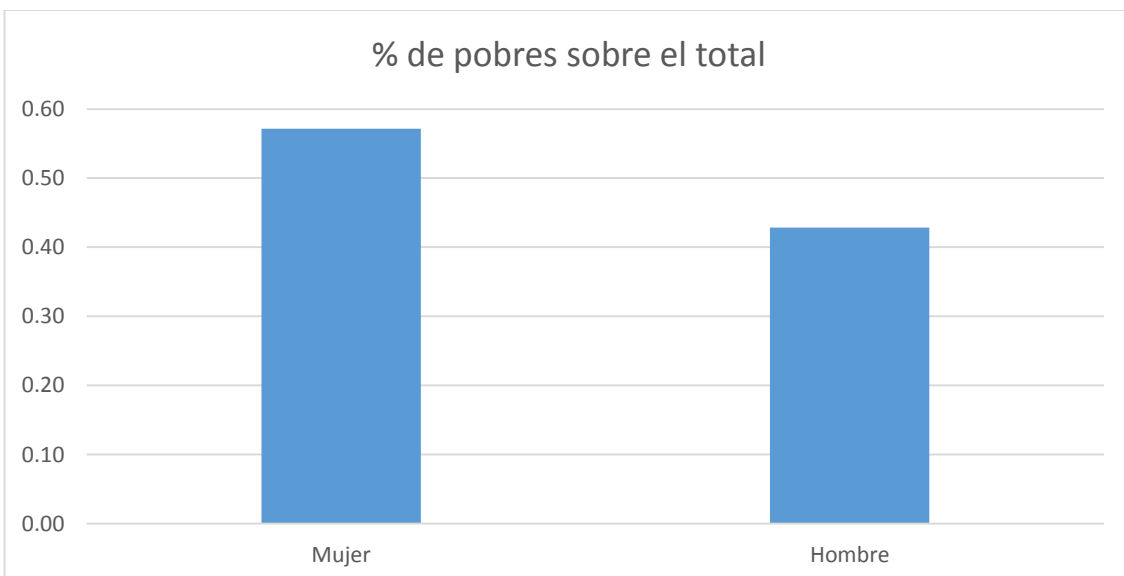
Source: Own Elaboration Based on Data from Table 1.

**Figure 4.** H Index by Sex According to Data from Table 1



Source: Own Elaboration Based on Data from Table 1.

**Figure 5.** Proportion of Poor by Sex According to Data from Table 1



Source: Own Elaboration Based on Data from Table 1.

This would be a way of measuring the intensity of poverty: the monetary amount necessary for the poor to stop being poor, and it is known as the Poverty Gap, and it is the distance, in terms of income, that separates the poor individuals of the poverty line. It is calculated in the following way:

$$PG = \sum_{i=1}^p (u - x_i) \quad (1)$$

The number of the poor, the poverty line, the level of income of the individual, which is considered to be poor (below the poverty line).

We can write,

Being  $p$  = number of poor,  $u$  = poverty line,  $x_i$  = and income level of individual  $i$ , which is considered poor (below the poverty line).

We can write,  $PG = p \cdot u - \sum_{i=1}^p x_i$ . That is,  $PG$  is the difference between  $p \cdot u$  = "total income that those poor people should have to stop being them", and  $y \sum_{i=1}^p x_i$  = "total

income that those poor people really have". In the example that concerns us,  $GP = 30'28$ . In other words, a transfer of € 30,280 would be necessary, from the most favored part of society to the poor, so that they would no longer be so.

We can relativize the previous amount taking as a reference the total amount necessary for the poor to stop being it  $p \cdot u$ ,  $y$  and in this way the (Income Gap Ratio) is constructed, which is defined as:

$$IG = \frac{PG}{p \cdot u} = \frac{p \cdot u - \sum_{i=1}^p x_i}{p \cdot u} = 1 - \frac{\sum_{i=1}^p x_i}{p \cdot u} = 1 - \frac{\bar{X}_p}{u}$$

where  $\bar{X}_p = \frac{\sum_{i=1}^p x_i}{p}$  is the average income of the poor.

In our example,  $IG = 0'3218$  and how,  $p \cdot u = 94'08$  that is, the poor should add an income of € 94,080 to stop being it. Then,  $IG$  is interpreted as that 32'18% of that amount is necessary, so that the poor cease to be so.

The problem of the  $GI$  index is that it is indifferent to the proportion represented by the poor in the total population, just the opposite of what happens with the  $H$  index, so a new index was later proposed, which combines the information of the previous two. This index is known as  $HIG$  or Poverty gap ratio and measures the quotient between the amount of income that would be necessary to place all the poor over the poverty line and the expression, where "total of individuals in society":

$$HIG = \frac{p \cdot u - \sum_{i=1}^p x_i}{n \cdot u} = \frac{p}{n} \cdot \frac{p \cdot u - \sum_{i=1}^p x_i}{p \cdot u} = H \left( 1 - \frac{\sum_{i=1}^p x_i}{p \cdot u} \right) = H \times IG \quad (2)$$

In the example that concerns us,  $HIG = H \times IG = 0'2333 \times 0'3218 = 0'075$ .

Another relative measure of poverty is what Eurostat uses to define the poverty gap. It is as follows:

$$Gap - Poverty - EUROSTAT = BPE = \frac{Poverty\ threshold - Medium\ of\ the\ poor}{2aPoverty\ threshold}$$

In the example this measure would take the value  $BPE = \frac{6'72 - 4'5}{6'72} = 0'3304$ . According to this, we would say that the poverty gap is almost 33% of the threshold.

## 6.0 OTHER POVERTY MEASURES

The previous indexes still do not take into account the third fundamental aspect of poverty, inequality among the poor,

so we must approach other indices that, even at the cost of losing simplicity, manage to correct this problem. There are, then, other forms of poverty measurement that are more complex and difficult to interpret, of which we try to introduce some of them in this section.

In its basic article, Sen (1976) incorporates for the first time in a single index the three elements of poverty, extension or incidence ( $H$ ), intensity ( $IG$ ), and inequality, the latter being collected by  $G_p$ , the Gini index calculated only on the poor population. Its index,  $S$ , can be interpreted as the weighted sum of the individual income gaps of the poor, where the weights depend on the relative position that each individual occupies among the poor. If the number of poor grows, the index approaches the following expression:

$$S = H (IG + (1 - IG) \cdot G_p) = \frac{p}{n} \left( 1 - \frac{\bar{X}_p (1 - G_p)}{u} \right) \quad (3)$$

If we calculate the Sen Index for the collective of 60 individuals we obtain  $S = 0'1062$ .

A variant of the Sen Index was proposed by Thon (1979), modifying the construction of the weights. Thus, in this new index,  $T$ , the relative position of the poor individual is calculated using the whole population as a reference, instead of using only the poor group. This simple change allows the fulfillment of several axioms that the Sen Index does not verify. Mathematically this index approaches for a sufficiently large  $p$  by the following expression:

$$T = H \cdot (S + 2(1 - H) \cdot IG) \quad (4)$$

Subsequently, Foster, Green and Thorbecke (1984) proposed a family of measures of poverty in which each of its components is characterized by having a different sensitivity to the deficit of individuals to reach the poverty line depending on how far from poverty. The same this is achieved through the parameter  $\alpha$  in the expression:

$$FGT(\alpha) = \frac{1}{n} \sum_{i=1}^p \left[ \frac{u - x_i}{u} \right]^\alpha, \text{ with } \alpha > 0 \quad (5)$$

Where  $\alpha$  can be interpreted as a parameter of aversion to poverty. The greater **the**  $\alpha$ , the greater the weight that the index gives to the larger income deficits (which reflect the situation of the most disadvantaged characterized by a greater distance between their equivalent income and the poverty line). Therefore, at higher values of  $\alpha$  greater importance is recognized by the index to progressive income transfers directed towards extreme poor. An infinite  $\alpha$  would only give value to the gap of the poorest individual, and only the situation of this individual would be reflected in the index.

It is easy to verify that this index is a generalization of some of the previous ones that are not more than particular cases. So, we have to:

$$FGT(0) = H$$

$$FGT(1) = HIG$$

For  $\alpha = 2$  the index it also adopts the following form:

$$FGT(2) = H \left[ IG^2 + (1 - IG)^2 CV_p^2 \right] \quad (6)$$

Where  $CV_p$  is the coefficient of variation among the poor.

It can be seen that this index is very similar to  $S$ , incorporating the three elements of poverty, with the difference that it uses the coefficient of variation instead of the Gini index.

The advantage of  $FGT(2)$  over the Sen index is that, like the other members of the family for integer and positive values of  $\alpha$ , it is additively decomposable.

For the example data,  $FGT(2) = 0'0361$ .

Another index commonly used in the empirical literature is the measure of Hagenaars (1986). It is a transformation of the *HIG* index in which *GI* is replaced by the percentage difference between the logarithm of the poverty line and the logarithm of the average economic position of the poor. For a given welfare function, the index tries to capture lost or squandered social welfare due to the existence of poverty:

$$HAG = \frac{p}{n} \left[ \frac{\log u - \log \bar{X}_p^g}{\log u} \right] \quad (7)$$

Where  $\bar{X}_p^g$  is the geometric mean of the income levels of the poor.

We calculate this index for the example data:  $HAG = 0'0551$ .

## 7.0 PROPERTIES REQUIRED BY POVERTY INDEXES

There is a wide literature on the problems related to the measurement of poverty, and throughout the twentieth century many indicators have been proposed to make this measurement in its different aspects. The previous sections of this work are proof of this. For that reason, the theoretical literature has been introducing axioms or basic properties that would be required to an indicator of poverty and, thus, bear in mind the advantages and disadvantages of it when using it. Amartya Sen (1983) formulated two axioms that, according to him, every measure of poverty should comply: monotonicity and transference. The first one tells us that a reduction in the income of a person who is below the poverty line, keeping all other individuals with the same level of income, should increase the value of the poverty index (that is, an increase in the intensity of poverty in an individual should be reflected in the index). The axiom of transfer tells us that an economic transfer from a person who is below the poverty line to one that has a higher income level, although this other one was also below the poverty line, keeping the level constant of income of the rest of individuals, must increase the value of the poverty index. Interestingly, one of the most used indexes, which we have represented as H, does not meet either of the two axioms.

Our objective in this section is, without intending to be exhaustive, to compile a series of properties or axioms (including the two already mentioned of Sen) required to the poverty indices that usually appear in the literature. Thus, the students of this subject will also be aware of the rigor that is present in all the theory associated with the measurement of poverty.

Let  $X$  be a vector of income, that is,  $X = (x_1, x_2, \dots, x_n)$  representing the income of the  $n$  individuals of a society. Suppose that the poverty line has been set at  $z$ . Be  $Y = (y_1, y_2, \dots, y_n)$  the income vector resulting from making some transformation in vector  $X$ . Be  $I(X, z)$  an indicator of poverty of a society whose income vector is  $X$  and with poverty line  $z$ . In the same way  $I(Y, z)$ , it is the same index but now measured in a society with income vector  $Y$ . We enunciate the following axioms required for this poverty index:

1. **Domain.** If the vector  $Y$  is obtained from the vector  $X$  through transfers between the incomes of those that are above the poverty line, then the poverty index does not change:  $I(X, z) = I(Y, z)$ .

2. **Symmetry (or anonymity).** If the vector  $Y$  is obtained from the vector  $X$  by means of a simple permutation (we change the position of the incomes within the vector, but we do not alter the values of them) then the poverty index does not change:  $I(X, z) = I(Y, z)$ .

3. **Invariance before population replicas.** If the vector  $Y$  is obtained from the vector  $X$  by replicating it  $m$  times then the poverty index does not change:  $I(X, z) = I(Y, z)$ .

4. **Monotonicity.** If the vector  $Y$  is obtained from the vector  $X$  by reducing the income of a poor individual, below the poverty line, then  $I(X, z) < I(Y, z)$ .

5. **Principle of transfers.** If the vector  $Y$  is obtained from the vector  $X$  through a transfer between two poor individuals so that the one that receives the transfer remains below the poverty line after it, then  $I(X, z) < I(Y, z)$ .

6. **Principle of sensitivity to transfers.** If from vector  $X$  two  $Y$  and  $Y'$  vectors are obtained by respective regressive transfers (from poorer to less poor), so that in both cases the individuals involved are at the same distance in terms of income level, then the poverty index will grow more the poorer the person from whom the transfer comes out.

7. **Subgroup consistency.** If we have an exhaustive partition of the vector  $X$  in  $k$  subpopulations, where are the sizes of these subpopulations, so that, with  $n$  the size of the population. Then, if the vector  $Y$  is obtained from the vector  $X$  increasing the poverty in a subpopulation without altering the rest,  $I(X, z) < I(Y, z)$ .

8. **Decomposition.** If we have an exhaustive partition of the vector  $X$  in  $k$  subpopulations, where are the sizes of these subpopulations, so that,  $n_1 + n_2 + \dots + n_k = n$  with  $n$  the size of the population, with  $x^1, x^2, \dots, x^k$  representing the different subvectors resulting from the partition, then:

$$I(X, z) = \sum_{i=1}^k \frac{n_i}{n} \cdot I(x^i, z)$$

9. **Growth of the non-poor.** If the vector  $Y$  is obtained from the vector  $X$  by adding a non-poor individual, above the poverty line, then  $I(X, z) > I(Y, z)$ .

We observe that the domain axiom, axiom 1, tells us that poverty rates should only be sensitive to what happens to the poor, to those who are below the poverty line, showing indifference with possible transfers between poor.

The axioms 2 and 3 are basic axioms required in the construction of both indexes of inequality and poverty. The axiom 4, of monotonicity, has to do with the idea of intensity of poverty, so that, if the poverty of some poor person is even more acute, an adequate index must pick it up. The axiom 5, almost in the same line as the 4, if it increases the inequality among the poor themselves, that fact must be reflected in the index itself.

The axiom 6 in some way the *aversion to poverty* that should be in society and reflected in the index: a regressive transfer should be reflected in the poverty index if the transfer affects poor individuals (we could say that it is the version of the Principle of Pigou-Dalton of inequality measures for the case of poverty measures).

If we consider subpopulations within the population, and within them, individuals below the poverty line, we have two axioms, 7 and 8, required at the poverty index for the case of analyzing those subpopulations within the whole.

In axiom 7 we are told that if the poverty level of a subpopulation increases, this should be reflected in the poverty rate of the entire population. And axiom 8 indicates that a poverty index should be possible to construct it based on the poverty indices of the subpopulations, as a weighted average thereof, with weights proportional to the sizes of said subpopulations.



If that is possible, it will allow us to measure the weight that each subpopulation has in the value that the global poverty index takes.

These two axioms are not unanimously accepted. Authors as prominent as Sen say that the poverty of a subpopulation does not depend only on itself, but on society as a whole and,

therefore, that separation into subpopulations does not make much sense.

The axiom 9 is justified for the case of comparing levels of poverty of populations of different sizes. Table 4 shows a version of our table published by Zheng (1997) on the axioms that meet the poverty rates mentioned so far.

**Table 4.** Axioms That Satisfy Poverty Indices

Axioms	Poverty indices						
	<i>H</i>	<i>PG</i>	<i>HIG</i>	<i>S</i>	<i>T</i>	<i>FGT</i> ( $\alpha \geq 2$ )	<i>HAG</i>
<b>Domain</b>	v	v	v	v	v	v	v
<b>Anonymity</b>	v	v	v	v	v	v	v
<b>Population</b>	v	v	v	x	x	v	v
<b>Monotonicity</b>	x	v	v	v	v	v	v
<b>Transfers</b>	x	x	x	v	v	v	v
<b>Sensitivity to transfers</b>	x	x	x	x	x	v for $\alpha > 2$	v
<b>Subgroup consistency</b>	v	v	v	x	x	v	v
<b>Decomposibility</b>	v	x	v	x	x	v	v
<b>Growth not poor</b>	v	x	v	v	v	v	v

Source: Own Elaboration from the Table of Zheng (1997).

v: meets the axiom, x: does not meet the axiom.

To complete the information in the table, we add that the most complete indices fulfilling normative properties are HAG and FGT (for  $\alpha > 2$ ) since they fulfill all the aforementioned axioms. The fact of fulfilling axioms 7 and 8, related to the division into groups, allow us a more detailed analysis of the behavior of poverty. As has been said before, one of the most used indicators (perhaps because of its simplicity of calculation), the H index, does not meet the axioms of Monotonicity, Transfers and Sensitivity to transfers, that is, the index is not altered, although they produce changes in the distribution of the poor or the intensity of their poverty deepens. Also, the PG and HIG indices remain constant before changes in the distribution of poverty, and the S and T indices, although they comply with transfer axiom 5, are not sensitive to the increase in the intensity of poverty. In any case, measuring a phenomenon as complex as that of poverty is difficult through a one-dimensional indicator. The usual thing is to use several indicators and different poverty lines in order to get as close as possible to the reality of a society regarding this phenomenon. Thus the concept of extreme poverty or indigence appears for the case in which the poverty line is located in very low places of the distribution of wealth.

**8.0 LINES OF SUBJECTIVE POVERTY**

They are based on the opinion that individuals have about themselves, in relation to the whole of the society in which they live. In other words, in these cases, the concept of poverty that is used to divide society between the poor and the non-poor is based on the perception that the households themselves, and the people who form it, have what it is to be poor.

This approach assumes that "each individual is the best judge of their own situation" (van Praag et al, 1980). This avoids, in some way, the value judgments that are implicit in the measurement of relative poverty: the choice of the threshold, the use of equivalence scales...

The three most popular lines of "subjective poverty" are those of Kaptein, Leyden and Deleeck, although the latter has been losing interest over time. These lines construct the poverty threshold based on the responses that households give to certain questions that are formulated in the so-called "household surveys".

**8.1 Kaptein poverty line**

The proposal for the construction of this line is an investigation of households seeking information from them on the minimum income that each household believes necessary to reach the end of the month. Specifically, each household is asked a question of this style: "In your opinion,

what are the net monthly income that would at least be necessary for a household like yours to arrive at the end of the month?"

In addition to the answer to this question, information is collected on the monthly income that the household actually has and on the number of members that comprise it.

Under the hypothesis that the answer given by the household to the previous question depends fundamentally on the number of members of the household and the level of income it has, a linear model is estimated in a logarithmic scale that relates these three variables:

$$\log(y_{\min})_i = \alpha + \beta_1 \cdot \log(m)_i + \beta_2 \cdot \log y_i + u_i \quad i = 1, \dots, n$$

Where  $(y_{\min})_i$  = minimum income declared as response by household *i*,

- m* = Number of household members *i*,
- y<sub>i</sub>* = Real income declared by household *i*,
- n* = Number of families surveyed,
- u<sub>i</sub>* = Random disturbance term associated with the home *i*.

The natural observation is as follows:

- Households with high incomes say that they need, to reach the end of the month, a smaller amount than what they actually enter.
- Households with low incomes manifest the opposite, they need to arrive at the end of the month, an amount higher than what they actually enter.

Therefore, it is possible to think that those households in which the answer to the question is close to their real income are the ones that most precisely define these minimums.

Then proceed as follows: For each household size, the previous model is represented, which has been estimated by Ordinary Least Squares (OLS), so that the OX axis represents the real income of all households in the survey that have the size considered, and in the OY axis the minimum income declared by those households, necessary to reach the end of the month. Also, on the same graph, we represent the bisector of the first and third quadrants, line that represents the situation of coincidence between real income and minimum income. At the cut-off point between the two lines there is a coincidence between real income and minimum income, for the estimated model. Well, the value provided by

this cut-off point is set as a poverty line for households of that size.

Thus, for each household size we will have a different poverty threshold and this threshold has been built on the information base that the households themselves have provided on the monetary amount they believe necessary to reach the end of the month.

Kaptein's methodology has served to inspire other methodologies used in the construction of subjective poverty lines. They include other explanatory variables such as "age of the main breadwinner", "number of children in the household", etc.

**8.2 Leyden poverty line**

For the construction of this line, the income that households relate to 6 economic situations that are ordered from worst to best is used. Each household is asked a question of this style.

Given the current circumstances of your home, say approximately, what monthly net income will be associated

with each of the following economic situations or level of well-being:

- Very bad: \_\_\_\_\_ €
- Bad: \_\_\_\_\_ €
- Insufficient: \_\_\_\_\_ €
- Enough: \_\_\_\_\_ €
- Good: \_\_\_\_\_ €
- Very good: \_\_\_\_\_ €

For each household a kind of utility function could be constructed, U, that relates the manifested income with the level of well-being, where said level is on a scale between 0 and 1, and is represented in the OY axis, leaving the axis OX for those stated income. Within the range of the OY axis, we assign the sub-interval  $\left[0, \frac{1}{6}\right]$  to the category "Very bad"  $\frac{1}{12}$

. Your class mark is associated with the income stated in that category (suppose,  $x_1$ ). Therefore, we represent the point of coordinates  $\left(x_1, \frac{1}{12}\right)$ . Thus, we proceed with the other categories:

**Table 5.** Construction of Coordinates for the Graph of Utility Curve

Level of well-being	Interval associated with welfare level	Class mark of the interval	Level of income	Coordinates of the point to represent
Very bad	$\left(0, \frac{1}{6}\right)$	$\frac{1}{12}$	$x_1$	$\left(x_1, \frac{1}{12}\right)$
Bad	$\left(\frac{1}{6}, \frac{2}{6}\right)$	$\frac{3}{12}$	$x_2$	$\left(x_2, \frac{3}{12}\right)$
Insufficient	$\left(\frac{2}{6}, \frac{3}{6}\right)$	$\frac{5}{12}$	$x_3$	$\left(x_3, \frac{5}{12}\right)$
Enough	$\left(\frac{3}{6}, \frac{4}{6}\right)$	$\frac{7}{12}$	$x_4$	$\left(x_4, \frac{7}{12}\right)$
Good	$\left(\frac{4}{6}, \frac{5}{6}\right)$	$\frac{9}{12}$	$x_5$	$\left(x_5, \frac{9}{12}\right)$
Very good	$\left(\frac{5}{6}, 1\right)$	$\frac{11}{12}$	$x_6$	$\left(x_6, \frac{11}{12}\right)$

If we represent the points of the last column in the XY plane and join them with a smoothed curve, starting from the origin of coordinates, we will have a rising curve that reminds us of the distribution function of a continuous random variable. Well, there are studies that confirm that the resulting graph closely approximates the distribution of a normal log model. Based on this approximation, if for household  $t$ , we represent  $x_{it}$  by the income that would provide to said household a welfare or utility whose classmark is defined by  $\frac{i - \frac{1}{2}}{6}$ , then, based on the property of the normal log model, it can be written,  $\log x_{it} = \mu_t + \sigma_t \cdot u_i$ ,  $i = 1, 2, \dots, 6$  with  $F(u_i) = \frac{i - \frac{1}{2}}{6}$ , where  $F$  is the distribution function of a distribution.

The hypothesis is established that the average associated with the household  $t$  depends on the real income  $x_t$  of said household, and on the number of members thereof,  $n_t$ .

Then, we relate that mean with the two variables mentioned by the regression model  $\mu_t = \alpha + \beta_1 \cdot \ln x_t + \beta_2 \cdot n_t + \varepsilon_t$ . The standard deviation,  $\sigma_t$ , is estimated by calculating the standard deviation of the 6 responses of each household (from  $x_1$  up to  $x_6$ ) and averaging the standard deviations of all households,  $\bar{\sigma}$ .

For each size of household,  $n$ , a minimum level of well-being is fixed  $\alpha$  (in the OY axis of utility) and all households with a level of welfare lower than that are considered poor  $\alpha$ . Then, the  $x$  income that provides that level of well-being is sought on the horizontal axis. That income is what gives us the poverty line for homes of that size.

So, for a household size, given that  $\log x = \mu_t + \sigma_t \cdot \alpha$  and taking into account the regression that defines  $\mu_t$  we can write  $\log x = \hat{\alpha} + \hat{\beta}_1 \cdot \log x + \hat{\beta}_2 \cdot n_t + \bar{\sigma} \cdot \alpha$ . If we clear, we have:  $\log x = \frac{\hat{\alpha} + \hat{\beta}_2 \cdot n_t + \bar{\sigma} \cdot \alpha}{1 - \hat{\beta}_1}$ . Therefore, the poverty line for a family of size would be:  $x = e^{\frac{\hat{\alpha} + \hat{\beta}_2 \cdot n_t + \bar{\sigma} \cdot \alpha}{1 - \hat{\beta}_1}}$ .

**8.3 Deeleck poverty line**

Use the information provided by households in response to two questions:

- The first question is the same as the one formulated to fix the Kaptein line: Minimum income needed to reach the end of the month.
- The second question is: In relation to the total monthly income of your household, how do you usually get to the end of the month? Point to an X:

This methodology constructs the poverty line using only the information of those households that in the second question have indicated "with some difficulty", that is, households that are close to the poverty line.

Using those homes, those who arrive at the end of the month with some difficulty, a new variable is built which is called "minimum income", in the following way:

$$\text{Minimum income} = \min(x^*, x^{\min})$$

Where  $x^*$  = the real income of the household,  $x^{\min}$  = answer that said household has given to the first question (minimum income to reach the end of the month).

According to the study that you want to carry out, you can divide the households into groups, each one defined by certain characteristics, and on which you intend to establish the poverty line.

With respect to the variable "minimum income", for each group, the mean  $\mu$  (by means of the sample mean  $\bar{X}$ ) and the standard deviation  $\sigma$  (by means of the sample standard deviation) are estimated, and the interval  $(\mu - 2\sigma, \mu + 2\sigma)$  is estimated. Those values that fall outside this range are eliminated. The average of the "minimum income" values that remain inside is recalculated. That will be the poverty line of that group. Therefore, the poverty line of each group will be the average of those households that arrive at the end of the month with some difficulty, once the outliers have been eliminated.

## 9.0 MULTIDIMENSIONAL DEPRIVATION

We have already commented that poverty is a phenomenon that manifests itself in many different ways and is the result of multiple factors. Therefore, it is impossible to define in a unique and absolute way. Behind every analysis that is made underlies a definition and a concrete way of conceiving it. Therefore, it is important to carry out poverty analysis that takes into account the various aspects of poverty and its multidimensional dimension. So far, the approaches we have taken to measure poverty have been monetary and are based on real or subjective income, established by the households themselves. In all the measures of poverty that we have given so far, household income is considered a good variable "proxi" of the resources available to the household to achieve adequate welfare.

However, it is necessary to provide other measures of poverty, which are not based solely on monetary indicators, which base the conclusions on variables that reflect the deprivations suffered by the household, trying to extend the concept of poverty to that of social exclusion. This growing need to build other measures of poverty and that are non-monetary is due to several reasons:

- Monetary poverty only shows part of the phenomenon. It assumes that households with the same income enjoy similar living standards.
- Even if we consider income as a good indicator of well-being, sometimes the measurement is complicated and little adjusted, the truth of income in certain households is not always available.
- People have other types of resources, which are not reflected in monetary poverty measures, and which can be used to avoid falling into poverty: savings, wealth, family support, friendships, and access to credit...

On the other hand, studies have been carried out in which an attempt is made to analyze the relationship between "monetary poverty" and "material deprivation". These studies classify households as poor or non-poor according to the two proposals: there is a fairly large common part, households or people, classified as poor according to both

approaches, but we also find another part, in some cases important, that is classified as poor according to one form, but not according to the other. Those groups with inconsistencies have been studied and, indeed, appear households that have sufficient income, but with material deprivation and, on the contrary, households that do not suffer basic deprivation and, nevertheless, their income is below the established poverty line. Therefore, the relationship between "monetary poverty" and "material deprivation" is not perfect. Of course, in order to provide as complete a view as possible of the phenomenon of poverty, it is necessary to obtain and use other measures to analyze deprivation and social exclusion.

Finally, comment that, after the last enlargement of the European Union, it is necessary to provide measures that allow comparison between such heterogeneous countries. The measures of relative monetary poverty have many disadvantages in this aspect: sometimes they provide similar results for countries that enjoy a very different quality of life. A solution that allows comparison is that given by non-monetary poverty measures, which is developing this type of measures at European level.

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