Predictive control of a solar air conditioning plant with simultaneous identification

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Abstract—This paper presents the application of a predictive controller with simultaneous identification to a solar air conditioning plant. The time varying nature of the process makes necessary an adjustment of the controller parameters to the varying operational conditions. The main novelty with respect to classic adaptive MPC scheme is to penalize the identification error in the cost function used for control. The behaviour of the controller is illustrated by simulations and experimental results. The integration of identification and control avoids the tedious identification procedure that is necessary before the start-up of any predictive controller. This new adaptive MPC scheme shows its effectiveness in controlling the outlet temperature in the solar thermal plant.

I. INTRODUCTION

Model Predictive Control (MPC) has developed considerably over the last years, both within the research control community and in industry [1]. This success can be attributed to the fact that MPC is, perhaps, the most general way of posing the process control problem in the time domain. MPC formulation integrates optimal control, stochastic control, control of processes with dead time, multivariable control and future references when available. Another advantage of MPC is that because of the finite control horizon used, constraints and, in general nonlinear processes which are frequently found in industry, can be handled.

However, one of the major drawbacks of this type of control strategy is the need to obtain a dynamic model of the plant. Most of the success of commercial predictive controllers such as Dynamic Matrix Control DMC [2] comes from its ability to use a step response model of the plant, which can be easily identified with experimental tests. However, the identification phase needs a lot of expertise and time to perform the experiments and this is usually done only once, at the process start-up. The model is not updated frequently even if process dynamics changes along time.

Model updating is particularly important in processes with changing operating conditions, where the process parameters are continually evolving. In processes involving mass transportation (as the one controlled in this work), the characteristic time constant and delay are affected by flow changes, giving rise to a process dynamics that changes during variable operating regimes.

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Model uncertainty and disturbances are important concerns in MPC and have been thoroughly studied in recent years. The main approaches to the subject appear in the fields of adaptive control and robust control. There is a lot of work done in robust MPC, with significant contributions in the min-max environment which, in spite of their theoretical importance, are difficult to be implemented in practice [3].

Adaptive MPC has also been widely studied by a number of authors, for example [4] [5]. The application of a selftuning controller with a recursive least squares identification algorithm gives rise to a solution that is easily implementable but shows numerical problems when the excitation vanishes [6]. A supervisory level is needed which makes the procedure more complex. This can be solved with the methodology proposed by Shouche et al. [7], Model Predictive Control and Identification MPCI, which is an adaptive MPC scheme that employs the persistent excitation condition [8] to guarantee identifiability. The main drawback of this method is that the use of the persistent excitation condition deteriorates the control performance. Although the identification capabilities of the method are very good, the fact that the control signals are calculated in order to guarantee excitation gives poor control features. In addition, there are no reports of applications to real plants. The method presented here tries to overcome this problem.

Solar thermal plants are usually difficult to control because the energy source (solar radiation) is not manipulable [5] and is continually changing. This makes constant flow changes necessary for reference tracking, provoking sudden process dynamics changes. Lots of control strategies have been applied to these plants, ranging from classical PIDs to MPC [9]. In this paper, a method that uses predictive control and identification simultaneously has been tested on a solar air conditioning plant.

The proposed method makes use of a cost function that includes tracking error and control effort (as any predictive controller) as well as the identification error in a past receding horizon. This is a combination of the control problem and the identification problem in just one cost function. This method is in the framework of the dual control [10]. The problem is not too costly and has been implemented on an industrial low-cost SCADA.

The paper is organized as follows. In section II a description of the solar plant is presented. Section III describes the proposed control strategy, which is tested under simulation and compared to a self-tuning MPC in section IV. The results of applying the proposed controller to the real plant are shown in section V and finally the conclusions are drawn.

II. PLANT DESCRIPTION

The solar air conditioning plant is located in Seville (Spain). It is used to cool the Laboratories of the System Engineering and Automatic Control Department of the University of Seville. It consists of a solar field that produces hot water which feeds an absorption machine generating chilled water and injects it into the air conditioning system, achieving a cooling power of 35 kW.

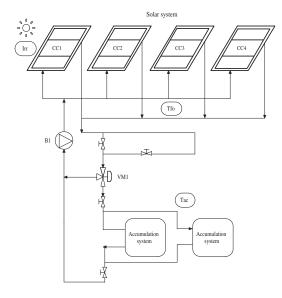


Fig. 1. Plant description

The solar plant can be analyzed as an air conditioning installation that uses thermal energy to produce cold air. A complete description of the plant can be found in [9].

The overall control objective is to supply chilled water to the air distribution system at the required temperature. This is accomplished by controlling the temperature of the hot water supplied by the solar field. Since the primary energy (solar radiation) is not manipulable, the desired temperature is achieved by acting on the circulating flow. The solar contribution, in addition to radiation seasonal and daily cyclic variations, is also dependent on atmospheric conditions such as cloud cover, humidity, and air transparency. It is important to maintain a constant outlet temperature as the solar conditions change, and the only means available for achieving this is via adjustment of the fluid flow.

The control problem addressed in this paper is the regulation of the solar field outlet temperature (T_{fo}) . Figure 1 shows the main components of the plant, which are the following:

- a) Solar system, composed of a set of flat solar collectors. The primary source of energy is solar radiation which is used by the solar collectors to increase the temperature of the circulating water. The solar field is composed of 151 m² of flat collectors which work within the range of 60 to 100 $^{\circ}C$ and supply a nominal power of 50 kW.
- Accumulation system, composed of two 2500-liter tanks working in parallel. This system acts as a buffer, storing

hot water to be used in transient situations where the solar radiation does not allow the desired temperature to be obtained at the end of the hot water circuit.

The objective of the control system is to maintain the outlet oil temperature T_{fo} at a desired level in spite of disturbances such as changes in the solar irradiance level (caused by clouds), mirror reflectivity or inlet water temperature. This is accomplished by varying the flow of the fluid through the field manipulating the three-way valve (VM1). The field exhibits a variable delay time that depends on the control variable (flow). The transfer function of the process varies with factors such as irradiance level or water inlet temperature. The maintenance of a constant outlet temperature throughout the day as the solar conditions change requires a wide variation in the operational flow level. This leads to substantial variations in the general dynamic performance and in particular, from the control viewpoint, gives rise to a system time delay which varies significantly. The controller parameters need to be adjusted to suit the operating conditions, and the proposed method offers one approach which can accommodate such a requirement.

The proposed control strategy is implemented on a small-size Distributed Control System (DCS) as a routine that communicates through the standard interface OLE for Process Control (OPC). OPC facilitates the interoperability between automation and control applications.

III. CONTROL STRATEGY

This section is dedicated to describing the proposed control strategy. The predictive controller with simultaneous identification on-line is based on Generalized Predictive Control (GPC), that consists of applying a control sequence that minimizes a multistage cost function that considers both tracking error and control effort.

For control purposes a simple, linear model is required which relates changes in fluid flow to changes in outlet temperature. In this section the theoretical development for n-order systems is shown and the use of first-order systems is justified.

A. n-order systems

The proposed controller extends the cost function of the original GPC ([11]) with an identification error term added in the following way:

$$\min_{x} J = \sum_{j=N_{1}}^{N_{2}} \delta(j) [\hat{y}(t+j \mid t) - w(t+j)]^{2} + (1)$$

$$+ \sum_{j=1}^{N_{u}} \lambda(j) [\Delta u(t+j-1)]^{2} + (1)$$

$$+ \sum_{j=1}^{N_{3}} \gamma(j) [y(t-j+1 \mid t) - \phi \theta]^{2}$$

s.t
$$\Delta u_{max} \leq \Delta u \leq \Delta u_{min}$$
, $u_{max} \leq u \leq u_{min}$
 $y_{max} \leq y \leq y_{min}$, $a_{i_{max}} \leq a_i \leq a_{i_{min}}$
 $b_{k_{max}} \leq b_k \leq b_{k_{min}}$, $d_{max} \leq d \leq d_{min}$

 $\forall i=1\dots na$ and $\forall k=1\dots nb$, where N_1 and N_2 are the minimum and maximum predictions horizons (taken as $N_1=d+1$ and $N_2=d+N$), N_u is the control horizon and N_3 is the identification horizon, d is the delay of the input-output process model and $\delta(j)$, $\lambda(j)$ and $\gamma(j)$ are weighting sequences. w(t+j) is a future set-point or reference sequence, $\Delta u(t)$ is the incremental control action $(\Delta u(t)=u(t)-u(t-1))$, $\hat{y}(t+j\mid t)$ is the j-step ahead prediction of the system output on data up to time t and $y(t-j+1\mid t)$ is the j-step backwards of the system real output on data up to time t. ϕ is the regression matrix, θ is the parameter vector to be identified and finally a_i,b_i,d , are the transfer function parameters of the discrete polynomials of degree na and nb as shown below.

If a CARIMA model is used to model the random disturbances in the system and the noise polynomial is chosen to be 1, the following equations are obtained¹:

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t) + \frac{\epsilon(t)}{\Lambda}$$
 (2)

Where A and B are the following polynomials in the backward shift operator z^{-1} :

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_{na} z^{-na}$$
(3)

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{nb} z^{-nb}$$

then the best expected value for the output prediction $\hat{y}(t+d+j\mid t)$ is given by,

$$\hat{y}(t+d+j|t) = (1-a_1)\hat{y}(t+d+j-1|t) + (4)$$

$$(a_1 - a_2)\hat{y}(t+d+j-2|t) + \dots +$$

$$a_{na}\hat{y}(t+d+j-na-1|t) + b_0\Delta u(t+j-1) +$$

$$b_1\Delta u(t+j-2) + \dots + b_{nb}\Delta u(t+j-1-nb)$$

If equation (4) is applied recursively for $j=1,2,\ldots,N$, the prediction vector is given by the following equation expressed in condensed form as:

$$\hat{y} = Gu^{+} + S\hat{y}^{-} + Hu^{-} \tag{5}$$

Where \hat{y} , u^+ , \hat{y}^- and u^- are vectors of sizes $N \times 1$, $N_u \times 1$, $(na+1) \times 1$ and $nb \times 1$ respectively.

$$\hat{y} = \begin{bmatrix} \hat{y}(t+d+1 \mid t) \\ \hat{y}(t+d+2 \mid t) \\ \dots \\ \hat{y}(t+d+N \mid t) \end{bmatrix} u^{+} = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \dots \\ \Delta u(t+N_{u}-1) \end{bmatrix}$$

$$\hat{y}^{-} = \begin{bmatrix} \hat{y}(t+d\mid t) \\ \hat{y}(t+d-1\mid t) \\ \dots \\ \hat{y}(t+d-na\mid t) \end{bmatrix} u^{-} = \begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \dots \\ \Delta u(t-nb) \end{bmatrix}$$

And G, S and H are matrices of dimensions $N \times N_u$, $N \times (na+1)$ and $N \times nb$, respectively. The following equations show how the matrices G and S can be obtained for n-order systems in a standard form.

G is a lower triangular matrix which takes the form:

$$G = \begin{bmatrix} g_0 & 0 & \dots & 0 \\ g_1 & g_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ g_N & g_{N-1} & \dots & g_0 \end{bmatrix}$$

and their elements are given by

$$g_{0} = b_{0}$$

$$g_{j} = \sum_{i=1}^{j} a_{i} g_{j-i} + \sum_{i=0}^{j-1} b_{i} \quad j = 1, \dots, N$$
If $j < 0 \Rightarrow g_{j} = 0$ (6)

S is given by

$$s_{1,j} = -\tilde{a}_{j+1}, \quad j = 1, \dots, n\tilde{a}$$

$$s_{i,j} = \sum_{k=1}^{i-1} s_{1,k} s_{i-k,j}$$

$$i = 2, \dots, N; \quad j = 1, \dots, n\tilde{a}$$
(7)

Where \tilde{a} and $n\tilde{a}$ are the elements and degree respectively of the polynomial $\tilde{A}(z^{-1})$, that is, $\tilde{A}(z^{-1}) = \Delta A(z^{-1}) = (1-z^{-1})A(z^{-1})$.

H is given by

$$h_{1,j} = b_j, \quad j = 1, \dots, nb$$

$$h_{i,j} = \sum_{k=1}^{i-1} (\tilde{a}_{k+1} h_{i-k,j}) + h_{1,i+j-1}$$

$$i = 2, \dots, N; \quad j = 1, \dots, nb$$
(8)

And finally ϕ is the regression matrix of dimension $N_3 \times (na+nb+1)$ and θ is the parameter vector to be identified of dimension $(na+nb+1)\times 1$, which is calculated at every sampling time using the receding horizon identification.

$$\phi = [y(t-j \mid t) \quad y(t-j-1 \mid t) \dots y(t-j-na \mid t)$$

$$\Delta u(t-d-j \mid t) \quad \dots \quad \Delta u(t-d-j-nb \mid t)]$$
 (9)

$$\theta = [1 - a_1(t) \quad a_1(t) - a_2(t) \quad \dots \quad a_{na}(t)$$
 (10)
$$b_0(t) \quad \dots \quad b_{nb}(t)]^T$$

The decision variables of the problem proposed are the following:

$$x = \begin{bmatrix} a_1(t) & \dots & a_{na}(t) & b_0(t) & \dots & b_{nb}(t) & d(t) & (11) \\ \Delta u(t) & \Delta u(t+1) & \dots & \Delta u(t+N_u-1) & \end{bmatrix}^T$$

The algorithm complexity grows with regard to order system and control horizon.

B. First-order systems

Most processes in industry, when considering small changes around an operating point can be described by a linear model of, normally, very high order. This is because most industrial processes are composed of many dynamic elements, usually first order, so the full model is of an order equal to the number of elements. In fact, each mass or energy storage element in the process provides a first-order element

¹Parameters a_i , b_i and d are time-dependant.

in the model. Consider, for instance, a long pipe used for heat exchanging purposes, as the case of solar collector. The pipe can be modelled by breaking it into a set of small pieces, each of which can be considered a first-order system. The resulting model will have an order equal to the number of pieces used to model the pipe, that is, a very high-order model. These very high-order models would be difficult to use for control purposes but, fortunately, as shown in [12], it is possible to approximate the behaviour of such high-order processes by a system with one time constant and a dead time.

The plant to be controlled can be described by this kind of model. If the sampling time is an integer multiple of the delay, the discrete transfer function is given by:

$$G(z^{-1}) = \frac{bz^{-1}}{1 - az^{-1}}z^{-d}$$

In this case na=1 and nb=1 and the methodology shown above is reduced considerably. Therefore ϕ and θ are reduced to dimensions $N_3 \times 3$ and 3×1 respectively:

$$\phi = \begin{bmatrix} y(t-j-1 \mid t) & y(t-j-2 \mid t) & \Delta u(t-d-j \mid t) \end{bmatrix}$$
 (12)

$$\theta = \begin{bmatrix} 1 - a(t) & a(t) & b(t) \end{bmatrix}^T$$
 (13)

And the decision variables number becomes $3 + N_u$:

$$x = \begin{bmatrix} a(t) & b(t) & d(t) & \Delta u(t) & \Delta u(t+1) \text{ (14)} \\ \dots & \Delta u(t+N_u-1) \end{bmatrix}^T$$

Consequently the optimization problem is also reduced and now the algorithm complexity grows linearly with the control horizon. The algorithm complexity is independent on the system parameters.

C. Optimization problem

The optimization problem is composed of a bilinear objective function (control and estimation cannot be designed separately, the estimation is affected by the control) subject to inequality constraints (the ones that can be handled by any MPC plus those imposed on model parameters) in the presence of continuous and integer variables (dead time d). Therefore it is a non-convex Mixed Integer Non-Linear Programming (MINLP) problem.

This kind of problem has a high computational burden, mainly if the global minimum want to be found. There are Branch&Bound algorithms available in the market that solve this optimization problem with the help of the user, who can influence the choice of branching variable by providing priorities for the integer variables. Anyway, this is not an easy problem to be solved on-line.

In order to simplify the method so that can be used in real time, two approximation have been used:

a) A simpler optimization algorithm has been used. The Matlab Optimization Toolbox function (*fmincon*) has been used to solve the problem. *fmincon* uses derivativebased search algorithm and do not guarantee a global minimum. All the parameters of the optimization

- function can be modified in order to reach an acceptable compromise between execution time and the suboptimal solution of the algorithm.
- b) On the other hand, the problem has been relaxed treating the integer variable as real, that is, it is converted into Non-Linear Programming with real variables. The value given by the algorithm is truncated in order to satisfy the requirements of the system.

The only tuning parameters of the controller are:

- 1) Control horizon: N_u .
- 2) Prediction horizon: $N_1 = d + 1$, $N_2 = d + N$.
- 3) Identification horizon: N_3 .
- 4) Output weighting factor: δ_i .
- 5) Input weighting factor: λ_i .
- 6) Identification weighting factor: γ_i .

IV. SIMULATIONS RESULTS

In order to test the proposed method before the final implementation and to compare it with others controllers, a simulation study was made. This section shows simulation of the proposed controller compared to a standard approach using an self-tunig GPC with RLS identification and to the MPCI proposed by Shouche *et al.* [7].

The nominal model used for the design is the following first order linear system with a dead time of three sampling periods:

$$G_m(z^{-1}) = \frac{-0.009546z^{-1}}{1 - 0.89654z^{-1}}z^{-4}$$
 (15)

The following figures show the behaviour of the process output, which is the solar field outlet temperature (T_{fo}) and the manipulated variable, which is valve opening (VM1)as well as model parameters a,b and d. The tuning values for the predictive controllers are: $N_u=10,\ N=60,\ N_3=60,\ \lambda=1,\ \delta=1,\ \gamma=1000,$ being the sampling time $T_s=40s$. All the decision variables have an initial value equal to zero, that is, the controller does not know the process model. And the maximum and minimum values of the variables are 100 and -100.

Fig 2 shows a comparison of the evolution of the process output responses under a self-tuning GPC with a RLS identification procedure (thin solid line) and the proposed method GPC with simultaneous identification (bold line). This simulation was performed in order to illustrate the behaviour of both controllers under changes in the operating point and set-point. Initially, the model parameters are $a=-0.89654,\ b=-0.009546$ and d=3. They are changed from their nominal values at t=159, taking the new values $a=-0.627,\ b=-1$. The dead time was not changed so that both controllers could work in the same conditions, since the self-tuning controller does not estimate this value.

As can be seen, both controllers behave well in the nominal case, but the proposed controller is able con go on controlling with the new values of the parameters while the self-tuning GPC behaves worst. The RLS finds new values of the parameters that make the self-tuning controller behave well, although they are not the true ones. At t=340, the

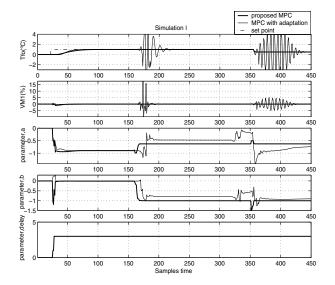


Fig. 2. Simulation I

MPC with adaptation modifies the values parameters. This effect is based on the absence of Persistent Excitation in closed loop identification [13]. At t=351, where a step change in the reference is performed, the proposed strategy gives a good closed loop response while the MPC with adaptation is fluctuating during 100 samples.

The following simulation (figure 3) presents the results of a test performed to show how the proposed strategy is able to identify the plant dead time, apart from the other model parameters.

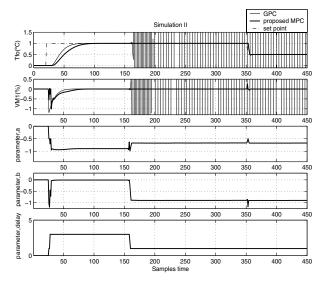


Fig. 3. Simulation II

In this case the comparison has been carried out with a fixed GPC. The GPC logically behaves worst, since it is not able to adapt to this change. The model used in the simulation has been changed from the one in equation (15) to

$$G_m(z^{-1}) = \frac{-0.9z^{-1}}{1 - 0.6723z^{-1}}z^{-1}$$

Note how, in spite of the great variation of the parameters (even dead time), the strategy described in this paper is able to meet the new model parameters without the need of a Persistent Excitation. It is able to track the set-point in steady state and when it is changed. The standard GPC is not able to meet these changes.

The last simulation compares the proposed method with the MPCI proposed by Shouche *et al.*. The model is taken from case study (B) of [7] and is given by:

$$y(t) = ay(t-1) + bu(t-1) + e(t)$$

Where the initial model parameters are a=0.4, b=0.4 and e=-0.05. The true parameters are a=0.6, b=0.2 and e=0, which perfectly identified by both controllers.

Table I shows the tracking capabilities of both controllers quantified as IAE (Integral of Absolute Error) and ISE (Integral of Square Error). Both controllers find good estimates of the true values, but MPCI imposes constraints on the input in order to have Persistent Excitation, deteriorating the tracking capabilities of the controller. The difference in controllers performance is clearly shown.

TABLE I
MPCI VS PROPOSED MPC

Controller	IAE	ISE
MPCI	0.0817	2.5017×10^{-5}
Proposed MPC	1.3279 ×10 ⁻⁴	6.6304×10^{-11}

V. EXPERIMENTAL RESULTS

Several experiments have been performed on the solar plant to show the behaviour of the proposed controller. The tuning values used are: $N_u=10,\ N=60,\ N_3=150,$ $\lambda=1,\ \delta=10,\ \gamma=100.$ All the decision variables start with a initial value equal to zero, that is, there is no previous knowledge of the plant dynamics. The bounds of all the variables are 100 and -100 except $\Delta u_{max}=20$ and $\Delta u_{min}=-20$. The following graphics show the actual solar field outlet temperature (T_{fo}) together with its reference, valve opening, solar radiation, field inlet temperature, which is the accumulators output temperature (T_{ac}) as well as model parameters a, b and d.

Figure 4 presents the result of the experiment carried out to show reference tracking capabilities. The experiment takes over three hours, and corresponds to a clear day (see solar radiation).

There exists a slow variation in the solar radiation and inlet temperature T_{ac} during all day which gives rise to changes in process dynamics. In this case the delay remains unchanged (d=5), but the other parameters (a and b), are modified by the controller in order to obtain a good closed loop behaviour.

One of the most appealing features of this method, as is it capability of starting to control without prior knowledge of the plant, is shown in the next experiment.

Figure 5 shows the evolution of the plant at the beginning of the day. During the start-up phase, the control strategy is

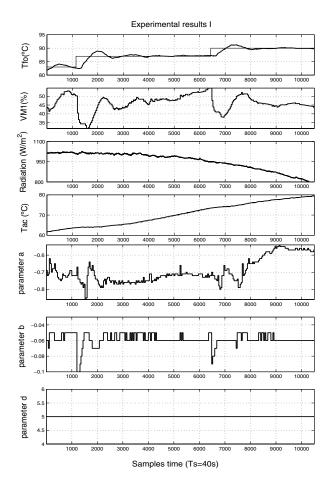


Fig. 4. Experimental results I

able to drive the plant towards the desired operating regime and track the setpoint, even with changes in radiation. In this case the parameters constraints are: $a_{max} = -0.6$, $a_{min} = -1$, $b_{max} = 1$, $b_{min} = -0.9$, $d_{max} = 10$, $d_{min} = 1$. Notice that some of these constraints are active during the experiment, showing that a constrained MPC is solved on line.

The results obtained in both experiments are good in spite of the varying conditions, showing that the proposed method is a good candidate to control this kind of plants.

VI. CONCLUSIONS

The paper has shown the application of a predictive controller with simultaneous identification to a solar plant. The control strategy allows the start-up of the plant without a tedious identification procedure and has shown good performance in changing operating conditions. The use of a sub-optimal solution of the MINLP problem allows its use in real time with low computational requirements. While the applicability of the method has been illustrated, future investigation is needed relating the optimization procedure and stability issues.

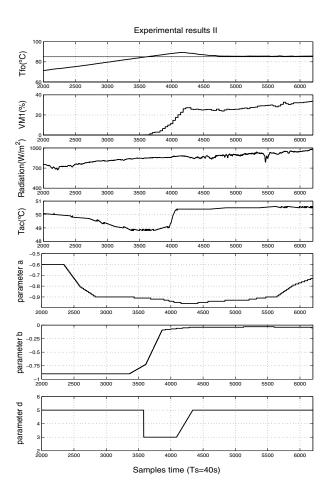


Fig. 5. Experimental results II

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