

# Research Article Increasing Sustainability of Logistic Networks by Reducing Product Losses: A Network DEA Approach

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This paper considers a multiproduct supply network, in which losses (e.g., spoilage of perishable products) can occur at either the nodes or the arcs. Using observed data, a Network Data Envelopment Analysis (NDEA) approach is proposed to assess the efficiency of the product flows in varying periods. Losses occur in each process as the observed output flows are lower than the observed input flows. The proposed NDEA model computes, within the NDEA technology, input and output targets for each process. The target operating points correspond to the minimum losses attainable using the best observed practice. The efficiency scores are computed comparing the observed losses with the minimum feasible losses. In addition to computing relative efficiency scores, an overall loss factor for each product and each node and link can be determined, both for the observed data and for the computed targets. A detailed illustration and an experimental design are used to study and validate the proposed approach. The results indicate that the proposed approach can identify and remove the inefficiencies in the observed data and that the potential spoilage reduction increases with the variability in the losses observed in the different periods.

## 1. Introduction

Economic growth and globalisation represent major challenges for the logistics sector. As larger quantities of products are transported over greater distances, more complex logistics networks have to be designed to guarantee that all material will arrive safely and on time [1, 2]. Designing such networks involves many different decisions. Altiparmak et al. [3] include in this process the definition of manufacturing plants' characteristics (i.e., their capacity and types of production), the number of warehouses and distribution centres used to store and forward the products, their location, defining the distribution channels and retailers to serve, and determining the quantity of products flowing through each edge in the network.

Normally, for mathematical modelling purposes, the logistics network is represented using a directed graph. The nodes correspond to the suppliers, manufacturing plants, warehouses, wholesalers, and retailers, while the arcs represent the product flows among nodes. Additionally, when there are restrictions on the corresponding product flows, capacity limits are attached to nodes and arcs. In this way, all decisions previously listed can be translated into the model and defined using a graph.

The complexity of logistics network design has attracted a great deal of research, mainly looking to optimise the material flows. From some review reports (e.g., [4, 5]) it can be derived that existing papers have analysed different design objectives, but almost always these are endeavouring to reduce costs (or, alternatively, maximize profits). However, the costs aspect, although always important, is not the only factor to be taken into consideration when deciding where to source and how to deliver and move products. Other factors are also relevant, such as providing a high service level and sustainability. Issues such as the environmental impact of logistics operations have come under the scrutiny of the logistics community, opening a new research field usually referred to as green logistics [6]. Events such as terrorist acts, natural disasters (floods, hurricanes, etc.), nuclear accidents, or labour strikes have also created interest in the design of more resilient logistics networks. Finally, designing logistics networks able to guarantee a high service level is also a main goal of many companies, for which their competitive strategy is aligned with that goal. Therefore, considering these types of alternative objectives is becoming more common when studying logistics networks, with some researchers considering more than one of these objectives and proposing a multiobjective approach [7, 8].

This paper will focus on the efficiency of the product flows along a logistics network in which product losses can occur along the nodes and links. Note that a network can be costeffective, it can even be resilient too, but it may still have some service level problems, due to product losses, inherent with product handling and spoilage. For that reason, the ability of a supply network to minimize such losses becomes an important objective of many companies.

In this paper, a new approach to assess the efficiency of logistics networks and ascertain the minimum loss product flows is proposed. The methodology used is based on Network Data Envelopment Analysis (NDEA), which, as will be seen in the next section, has been extensively used to assess the efficiency of supply chains. The NDEA approach proposed in this paper is, however, different from existing approaches as it is focused specifically on product losses. To the best of our knowledge, there are no similar approaches in the literature. Moreover, the proposed approach uses an innovative NDEA modelling of the supply chain so that each processing node and transportation link is mapped to an NDEA process whose inputs and outputs are the corresponding products flows entering and leaving the node or link. The observed inputs and outputs implicitly convey the product loss information in each node and link. Based on this, NDEA is able to capture the best practice and compute minimum loss product flows. Thus, the contribution of the paper is twofold. On the one hand, it is the first to propose a method to assess the product losses efficiency of a supply chain, thus helping managers to identify and enforce the best practices, setting efficient product flow targets along the different nodes and links of the supply chain. On the other hand, an innovative NDEA modelling approach has been developed.

The structure of the paper is the following. In Section 2, the relevant literature is reviewed. The proposed mathematical models are presented in Section 3. Section 4 presents an illustration of the proposed approach. In Section 5 an experimental study on the influence of the observed losses variability on the performance of the proposed approach is carried out. Finally, Section 6 summarises and concludes.

## 2. Relevant Literature

As indicated in the introduction, logistics network design is a very wide field, including different goals and constraints. Some reviews have been published so far, including the influential general work of Beamon [4] and the review of Geunes and Pardalos [9]. However, other authors have proposed specific ways of solving the design problem. As an example, great effort has been made in the field of defining sustainable supply chains [10].

Regarding our goal to consider product losses along the network, there are few published works appertaining to this and they usually deal with the problem using different metrics: minimizing defective material [11], maximizing the system responsiveness (i.e., the ratio between the quantity of products shipped and the total amount of products demanded) (e.g., [12]) or maximizing the coverage percentage of customer demand delivered within the preferred delivery lead time [13]. Environments in which the material moves under strict risk or tight timeframes are especially interesting in this context. An example of this situation is the problem of improving the performance in the management of healthcare products such as blood [14] or the design of military networks [15] and disaster response supply chains [16].

In this paper the focus is on the product losses that occur along the logistics network. These losses, especially in perishable food logistics networks, can be significant and therefore cannot be ignored. Given the customer demand, the flow along the logistics network must be planned so that the required quantities are available at the retailers. However, due to product losses, the product flows along the logistics network must be greater than the customer demand. The goal is that the products flow efficiently along the logistics network, i.e., minimizing the losses. We can assess the efficiency of the logistics network in a given period if we have data about observed losses in the different processing nodes and transportation links and we use those data to identify the best practices along each route. Those best practices are used to compute a target efficient pattern of product flows against which we can benchmark the observed losses.

To assess the efficiency of logistics networks, Data Envelopment Analysis (DEA), a well-known nonparametric methodology [37], is proposed. With the observed input and output data, DEA constructs a Production Possibility Set (PPS) that contains all the feasible operating points. The nondominated set of feasible operating points is labelled the efficient frontier (EF). DEA models generally project each observation (usually called a Decision Making Unit, DMU) onto the EF, measuring efficiency as the distance to the EF.

Conventional DEA considers the DMU as a black box with inputs and outputs. However, to properly model logistics networks with DEA, it is necessary to use NDEA, which distinguishes the different processes within the DMU and considers intermediate flows between the processes [38, 39]. NDEA represents a more fine-grained level of analysis that requires more detailed data. A basic feature of NDEA is that it constructs a specific PPS for each process, computing target values for all inputs, outputs, and intermediate flows to enable the whole system to operate efficiently. Note that, in this regard, efficiency in NDEA implies efficient operation in all the processes of a DMU.

There are different NDEA approaches, for both technical efficiency (e.g., [40]) and cost efficiency (e.g., [41]). The main difference between technical efficiency and cost efficiency is that the former does not use information about unit prices for the inputs and hence always projects the DMU onto an efficient operating point that dominates it. A cost efficiency model projects onto the minimum cost operating point which may not dominate the observed DMU. With regard to technical efficiency NDEA models, there are also variants,

depending on the method used to measure the distance to the EF. Thus, the relational NDEA approach [42] uses a radial efficiency metric while the additive efficiency decomposition [43] computes the overall system efficiency as a weighted average of the efficiency of the different processes and the directional distance function NDEA approach (e.g., [44]) uses a direction vector to guide the input and output improvements. There are also slacks-based NDEA approaches such as the Network Slacks-Based Measure of efficiency (NSBM [45– 47]) or Network Slacks-Based Inefficiency (NSBI [48, 49]). The former computes the efficiency, in the nonoriented case, as the ratio of the average normalized input reduction to the average normalized output increase, while the latter computes the efficiency as the sum of those average normalized input and output improvements instead of using a ratio.

Both conventional DEA and NDEA have been applied to assess the efficiency and benchmark supply chains (e.g., [50, 51]). In particular, as regards NDEA applications to supply chains, Table 1 shows the main features of existing approaches. Most approaches consider either two stages in series or multiple layers in series (typically four: supplier, manufacturers, distributors, and retailers) with parallel processes within each layer. Apart from the variety in the NDEA models used (with some approaches using a multiplier formulation and others using an envelopment formulation) note the type of inputs, outputs, and intermediate products considered. This paper approaches the efficiency of logistics networks from a different angle. Each supply chain node and transportation link is mapped to an NDEA process and the input and output variables of each process are just the product flows entering and leaving, respectively, that process. Thus, instead of assessing the efficiency of the supply chain from a global perspective that includes operations and economic and sustainability variables as most other approaches do, the proposed NDEA approach focuses on the losses that occur in the different nodes and links of the network. The authors are not aware of any similar approach.

This paper approaches the efficiency of logistics networks from a different angle. Thus, each supply chain node and transportation link is mapped to an NDEA process and the input and output variables of each process are just the product flows entering and leaving, respectively, that process. In addition, the proposed NDEA model focuses on the losses that occur in the different nodes and links of the network. Lozano and Adenso-Díaz [52] have used this new NDEA approach for planning the operation of supply chains using a biobjective optimisation approach. In this paper, more than in planning the future operation of the logistic network (using forecast demand data) we aim at assessing the past performance of the network using an input-oriented NSBM approach.

## 3. NDEA Computation of Efficient Flows

This section presents an NDEA model to assess the spoilage efficiency of a logistics network in different periods. It is assumed that the data involving the amounts of the various products that entered and left each node and link in the logistics network in different time periods are available. The proposed NDEA model has a number of innovative NDEA modelling features. Thus, each DMU corresponds, therefore, to the observed product flows along the logistics network in each period. The processes in the DEA network are the different nodes and transportation links in the logistics network and, for each process, the inputs are the material flows that enter the process upstream, and the outputs are the material flows that leave the process downstream. In this problem, conservation of flow in each process does not generally hold. On the contrary, the outputs are generally lower than the corresponding inputs, because losses (e.g., spoilage) can occur within each process.

Therefore, each node and link of the logistics network is mapped onto a process of the NDEA model. The NDEA network of processes has a feedforward topology that mimics the actual structure of the logistics network so that the first layer corresponds to the suppliers. The inputs to the suppliers are the quantities they produce and the outputs are the amounts they deliver using each of the transportation links between the supplier and the different plants. These transportation links are also NDEA processes and the outputs of the supplier process are intermediate products that enter these transportation link processes. The outputs of the transportation link processes are the amounts that reach the plant at the end of the link. These amounts are considered as intermediate products generated within the transportation link process.

Analogously, the outputs of the plant processes are the amounts they deliver using each of the transportation links between the plant and the different wholesalers. These are intermediate products that enter the transportation link processes. The outputs of each of those transportation links are the amounts that reach the wholesaler at the end of the link. These are again intermediate products that enter the wholesaler processes. In turn, the outputs of the wholesaler processes are the amounts they deliver using each of the transportation links between the wholesaler and the different retailers. These are intermediate products that enter the corresponding transportation link processes while the amount that reaches the end of the link corresponds to the intermediate products consumed by the retailers. The retailer processes represent the final layer of the network and their outputs are the amounts demanded by (i.e., sold to) customers. Figure 1 shows an example of the mapping between a very simple logistics network (with two suppliers, one plant, one wholesaler, and two retailers) and its corresponding NDEA network.

Summarizing the above, the NDEA processes correspond to the different processing nodes and transportation links of the actual logistics network. For each process, the inputs (which, except in the case of the suppliers, are intermediate products generated by a previous process) correspond to the amount of products that enter the process while the outputs (which, except in the case of retailers, are intermediate products to be consumed by the next process) are the amounts that leave. Therefore, for each process the number of inputs/incoming intermediate products is the same as the number of outputs/outgoing intermediate products and, for

|                                     | •  |  |        |                   |  |
|-------------------------------------|--|--|--------|-------------------|--|
| Reference                           | NDEA topology  | NDEA model   | # DMUs | SC sector         | Variables (inputs, outputs and intermediate)   |
| Liang et al. [17]                   | two stages in series                                 | multiplier formulation,<br>average individual stages<br>eff. | 10     | not specified     | Labour; Operating cost, Shipping cost, Number # units of<br>product A shipped, # units of product B shipped, Sales,<br>Profit  |
| Alfonso et al. [18]                 | four stages in series                                | envelopment formulation,<br>input-oriented NSBM eff.         | ю      | leather           | Stock of equipment, Equipment's average years of service,<br>Number of employees, Capital stock, Operational costs,<br>Sales, Return On Investment, Requisitions rate, Average<br>lead time, Orders arrival rate, Average time of the agent's<br>payment, etc.   |
| Khalili-Damghani and Tavana<br>[19] | convergent multiple stages<br>in series              | fuzzy DEA formulation,<br>radial efficiency                  | 40     | dairy product     | Providers of agility in sourcing, Capabilities of agility in<br>sourcing, Performance of sourcing, Providers of agility in<br>making, Capabilities of agility in making, Performance of<br>making, Providers of agility in delivery, Capabilities of<br>agility in delivery, Performance of delivery, Final goals of<br>SC   |
| Tàvana et al. [20]                  | multiple layers in series<br>with parallel processes | envelopment formulation,<br>network epsilon-based<br>model   | 10     | semiconductor     | On-time delivery standard deviation, supplier's distance<br>from the manufacturer, Supplier price, Supplier quality,<br>Number of employees, Number of machines, Reciprocal of<br>Work-in-Process, Reciprocal of Flow Time, Flexibility,<br>Cost per dollar revenue, Service level, Customer tenure,<br>Material flow, etc.  |
| Shafiee et al. [21]                 | four-process complex<br>network topology             | multiplier formulation,<br>radial efficiency                 | 22     | food industry     | Production capacity utilization, Average number of on<br>time delivery per year, Total inventory cost, Customer<br>response time, Volume of new goods sold, Volume of<br>qualified delivery goods per year, Return on investment,<br>Gross revenue, Profit before tax, Cost of information<br>sharing, etc.  |
| Mirhedayatian et al. [22]           | multiple stages in series                            | envelopment formulation,<br>output-oriented NSBM<br>eff.     | 10     | soft drinks       | Material purchase cost, Transportation cost, Staff cost,<br>Cost of quality, Advertising cost, Reliability cost, R&D<br>cost, Facility technology level, Supplier flexibility,<br>Capability of suppliers, Services, Parts per million,<br>Eco-design cost, CO <sub>2</sub> emission, Producer reputation,<br>On-time delivery to customers, Customer satisfaction, etc. |
| Khodakarami et al. [23]             | two stages in series                                 | envelopment formulation,<br>SBM and Russell-type eff.        | 27     | chemical industry | Annual cost, Annual personnel turnover, Environmental<br>cost, Number of products from supplier to manufacturer,<br>Partnership cost in green production plans, Number of<br>trained personnel in the fields of job, safety, and health,<br>Number of green products, Revenue  |

TABLE 1: Main features of existing supply chain NDEA applications.

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|                                      |  | TABLE   | 1: Continued. |  |   |
|--------------------------------------|--|---|---------------|--|---|
| Reference                            | NDEA topology  | NDEA model  | # DMUs        | SC sector                                      | Variables (inputs, outputs and intermediate)  |
| Tavassoli et al. [24]                | multiple layers in series<br>with parallel processes | envelopment formulation,<br>radial efficiency                             | П             | air transport                                  | Labour, Passenger-plane-km, Cargo-plane-km,<br>Passenger-km, Ton-km   |
| Momeni et al. [25]                   | complex reverse logistics<br>topology                | envelopment formulation,<br>multi-objective additive<br>eff.              | 10            | automotive                                     | Material purchase cost, Transportation cost, Labour cost,<br>No. of delivered parts for recycling, Transportation cost<br>from supplier to facilities, Investment on required<br>equipment, No. of on-time delivered parts, Percent of<br>intact parts, Rate of customer relationship, etc.                     |
| Yousefi et al. [26]                  | multiple stages in series                            | envelopment formulation,<br>Tchebycheff eff.                              | 12            | conferences<br>organization                    | Service staff, Host staff, Partly-prepared halls and booths,<br>Hosts, Fully-prepared halls and booths, Experts, Extra<br>services, Readiness for holding conference, Guests and<br>inventors, Satisfaction of guests and inventors,<br>Encouraging inventors, Introducing inventions to industry<br>and market |
| Tajbakhsh and Hassini [27]           | multiple layers in series<br>with parallel processes | envelopment formulation,<br>input-oriented radial eff.                    | 27<br>10      | banking<br>beverage                            | Fixed assets, IT budget, No. of employees, Deposits, Profit,<br>Fraction of loans recovered<br>Raw material cost, Transportation cost, Supplier<br>defect-free parts per million, Advertising cost, CO <sub>2</sub><br>emission, Average reputation factor, No. of green<br>products, Personnel cost, etc.      |
| Fu et al. [28]                       | multiple layers in series<br>with parallel processes | envelopment formulation,<br>NSBM eff.                                     | 10            | not specified                                  | Material purchase cost, Fixed asset maintenance cost,<br>Transportation cost, Staff cost, Advertising cost, Gross<br>profit, Pollutant emissions, R&D costs, Service level, No. of<br>on-time delivered products, etc.  |
| Izadikhah and Farzipoor Saen<br>[29] | two stages in series                                 | envelopment formulation,<br>weighted average of<br>individual stages eff. | 29            | Equipment for<br>expendable medical<br>devices | Cost of work safety and labour health, Annual cost,<br>Environmental cost, # units from supplier to<br>manufacturer, No. of ISO certificates, Revenue growth<br>rate, etc.  |
| Omrani and Keshavarz [30]            | multiple layers in series<br>with parallel processes | multiplier formulation,<br>ratio efficiency                               | 4             | maritime transport                             | Ship purchase cost, Crew cost, Costs of spare<br>parts+provisions+insurance, Costs of repairs, Commercial<br>container operation cost, Commercial passenger<br>operation cost, Lease purchasing by instalments, # of<br>containers carried per year, # of passenger+cars carried<br>per year, Net income, etc.  |

| Reference                     | NDEA topology                           | NDEA model  | # DMUs          | SC sector          | Variables (inputs, outputs and intermediate)  |
|-------------------------------|---|---|-----------------|--------------------|---|
| Tavana et al. [31]            | multiple stages in series               | multiplier formulation,<br>weighted average of<br>individual stages eff.  | 7               | cement             | Capital, Cooperation experience, Transport cost, Timely<br>delivery, Technology level, Inventory, Order amount,<br>Order profit   |
| Motevali Haghighi et al. [32] | four stages in series                   | envelopment formulation,<br>weighted average of<br>individual stages eff. | 40              | plastic recycling  | Delivery cost, Supplier rejection rate, Hazardous materials,<br>Flexibility, Health and Safety Staff, No. of green products,<br>Delivery cost, Service quality, on-time delivery, etc.  |
| Yousefi et al. [33]           | series, parallel and<br>series-parallel | envelopment formulation,<br>robust fuzzy GP-NDEA                          | 18              | insulating windows | Eco-design costs, Workplace safety costs, Raw materials<br>costs, Kg glass ready for installation, Profile production,<br>Glass received, Profiles received, Cost of glasses, Cost of<br>profiles, Number of shipped windows, Cost of windows,<br>Transportation cost, Quality of installation, Revenue |
| Shokri Kahi et al. [34]       | series                                  | multiplier formulation,<br>dynamic NDEA                                   | 17 (x3 periods) | tomato products    | Environmental costs, Cost of labor safety, Other costs,<br>Revenue from sales of raw tomatoes, Cost of purchasing<br>tomato from farm, Volume of tomato paste, Shipping<br>costs, Uncollected revenue, Unpaid costs, Green R&D<br>costs, Annual profit, Degree of customer satisfaction                 |
| Moslemi and Mirzazadeh [35]   | four-stage series                       | multiplier formulation,<br>crossefficiency, interval<br>data              | ∞               | blood supply chain | Space and facilities, Costs, Intellectual capital, Budget,<br>Waste disposal, Waste management, Loss rate, Revenue,<br>Quality, No. of donors, Patient satisfaction, Inventory<br>management, Social actions, Recording and archiving,<br>Information sharing   |
| Badiezadeh et al. [36]        | multi-stage series                      | multiplier formulation,<br>double frontier                                | 6               | tomato paste       | Material purchasing cost, Environmental cost,<br>Staff welfare cost, CO2 emission, Number of delivered<br>products, Number of products from supplier to<br>manufacturer, Number of green products, Revenue  |

TABLE 1: Continued.



FIGURE 1: Example of mapping a logistics network to its corresponding NDEA network.

each of them, the difference between what leaves and what enters is the losses. This logical NDEA network of processes, derived from the physical supply network, is completely original and differs from the NDEA topology used in any previous supply chain NDEA applications. It is also a novel feature of the proposed approach that the inputs and outputs of the processes are similar; i.e., they correspond to flows of the same products. Normally, in all other NDEA applications, the inputs and outputs of a process correspond to variables that are different (e.g., inputs may be costs or lead time while outputs may be revenue or service level). Since the proposed approach is focused on assessing product losses efficiency the inputs and outputs of a process are simply the product flows entering and leaving a node or transportation link so that the inefficiencies of a process correspond to the output product flows being smaller than the input product flows.

The proposed NDEA model formulated below computes a complete product flow target solution based on the PPS of each of the processes. The efficiency metric used is the inputoriented variant of the NSBM DEA proposed in Lozano [47] which fits perfectly the objective function of minimizing the amount of products input at the suppliers. Recall that the inputs at the suppliers are equal to the amount that reaches the customers plus the amount corresponding to the product losses that occur along the logistics network.

Although the resulting NDEA model may seem complicated because of its many constraints, in fact these constraints simply stipulate that the target operating point in each process is computed as a linear combination of the observed DMUs (constant returns to scale are assumed) and that the amount of intermediate products consumed by a process must be equal to the amount produced by the previous process that provides those intermediate products.

In order to formulate the problem mathematically, note the following.

Data

 $s \in S$ : index and set of suppliers

 $p \in P$ : index and set of plants

 $w \in W$ : index and set of wholesalers

 $r{\in}R{:}$  index and set of retailers

SP = SxP: set of links between suppliers and plants

 $PW \subseteq PxW$ : set of links between plants and wholesalers

 $WR \subseteq WxR$ : set of links between wholesalers and retailers

j: index on DMUs (periods)

i=1,2,...,m: index on products

 $x_{ij}^s$ : amount of product i that entered node s of DMU j

 $x_{ij} = \sum_{s} x_{ij}^{s}$ : total amount of product i supplied to DMU j

 $z_{ij}^{sp}$ : amount of product i sent by node s to node p of DMU j

 $u_{ij}^{sp}$ : amount of product i received by node p from node s of DMU j

 $z_{ij}^{pw}$  : amount of product i sent by node p to node w of DMU j

 $u_{ij}^{pw}$ : amount of product i received by node w from node p of DMU j

 $z_{ij}^{wr}$ : amount of product i sent by node w to node r of DMU j

 $u_{ij}^{wr}$ : amount of product i received by node r from node w of DMU j

 $y_{ii}^r$ : amount of product i sold by retailer r in DMU j

Note that, for the observed data, it is possible to compute, for each product, the losses in each node/link of the logistics network for a given DMU 0. Thus, we have the following:

Overall loss factor of product i:

$$\zeta_{i0} = 1 - \frac{\sum_{r} y_{i0}^{r}}{\sum_{s} x_{i0}^{s}} = 1 - \frac{\sum_{r} y_{i0}^{r}}{x_{i0}}$$
(1)

Average overall loss factor:

$$\zeta_0 = \frac{1}{m} \sum_i \zeta_{i0} \tag{2}$$

Loss factor of product i in process s:

$$\zeta_{i0}^{s} = 1 - \frac{\sum_{p} z_{i0}^{sp}}{x_{i0}^{s}}$$
(3)

Loss factor of product i in process (s,p):

$$\zeta_{i0}^{sp} = 1 - \frac{u_{i0}^{sp}}{z_{i0}^{sp}} \tag{4}$$

Loss factor of product i in process p:

$$\zeta_{i0}^{p} = 1 - \frac{\sum_{w} z_{i0}^{pw}}{\sum_{s} u_{i0}^{sp}}$$
(5)

Loss factor of product i in process (p,w):

$$\zeta_{i0}^{pw} = 1 - \frac{u_{i0}^{pw}}{z_{i0}^{pw}} \tag{6}$$

Loss factor of product i in process w:

$$\zeta_{i0}^{w} = 1 - \frac{\sum_{r} z_{i0}^{wr}}{\sum_{p} u_{i0}^{pw}}$$
(7)

Loss factor of product i in process (w,r):

$$\zeta_{i0}^{wr} = 1 - \frac{u_{i0}^{wr}}{z_{i0}^{wr}} \tag{8}$$

Loss factor of product i in process r:

$$S_{i0}^{r} = 1 - \frac{y_{i0}'}{\sum_{w} u_{i0}^{wr}}$$
(9)

In what follows, an NDEA model is formulated to assess the material flow efficiency of a given DMU 0. The model uses an input orientation; i.e., it seeks to minimize the number of products injected into the network by the suppliers guaranteeing that the amounts of each product that can be sold by the retailers are at least equal to the observed values for the given DMU, i.e.,  $y_{i0}^r$ , which are assumed to correspond to the demand of the products at each retailer. The model computes target values for the material flow that enters and leaves each node and link and also determines an NSBM efficiency score. Thus, the proposed NDEA model below determines the minimum feasible losses along the supply chain (i.e., the target operating point within the PPS of each node and link of the chain so that the average loss ratio is minimized) and computes the efficiency scores of the DMUs by comparing those minimum feasible losses with the observed ones. Hence, as always in DEA, the role played by the PPS inferred from the observations in determining the reductions in the product losses that can be achieved is essential.

Note the following.

#### Variables

 $\hat{x}_i^s$ : target amount of product i entering node s

 $\hat{z}_i^{sp}$  : target amount of product i sent from node s to node p

 $\hat{u}_i^{sp}$ : target amount of product i received by node p from node s

 $\widehat{z}_i^{pw}$  : target amount of product i sent from node p to node w

 $\hat{u}_i^{pw}$ : target amount of product i received by node w from node p

 $\widehat{\boldsymbol{z}}_i^{wr}$  : target amount of product i sent from node w to node r

 $\widehat{u}_i^{wr}$ : target amount of product i received by node r from node w

 $\widehat{h}_{i}^{s}$ : target loss of product i in process s

 $\hat{h}_{i}^{sp}$ : target loss of product i in process (s,p)

 $\hat{h}_{i}^{p}$ : target loss of product i in process p

$$\hat{h}_{i}^{pw}$$
: target loss of product i in process (p,w)

 $\widehat{h}_{i}^{w}$ : target loss of product i in process w

 $\hat{h}_i^{wr}$ : target loss of product i in process (w,r)

 $\hat{h}_i^r$ : target loss of product i in process r

 $\hat{h}_i^-$ : target loss of product i along the whole supply chain

 $\hat{h}_i$ : loss of product i reduction along the whole supply chain in the target solution (w.r.t. DMU 0)

 $\lambda_j^s$ : intensity variables to compute process s target values

 $\lambda_{j}^{sp}$  : intensity variables to compute process (s,p) target values

 $\boldsymbol{\lambda}_{j}^{p}:$  intensity variables to compute process p target values

 $\lambda_j^{pw}$ : intensity variables to compute process (p,w) target values

 $\lambda_j^w$ : intensity variables to compute process w target values

 $\lambda_j^{wr}$ : intensity variables to compute process (w,r) target values

 $\lambda^r_{j}$  intensity variables to compute process r target values

Assuming that all processes exhibit constant returns to scale (CRS), the proposed NSBM DEA model for spoilage efficiency assessment is the following:

$$\theta_0 = \max \frac{1}{m} \sum_i \frac{\sum_s \hat{x}_i^s}{\sum_s x_{i0}^s} = 1 - \frac{1}{m} \sum_i \frac{\hat{h}_i^-}{x_{i0}}$$
(10)

s.t.

$$\sum_{j} \lambda_{j}^{s} x_{ij}^{s} = \hat{x}_{i}^{s} \quad \forall i \; \forall s \tag{11}$$

$$\sum_{s} \widehat{x}_{i}^{s} = x_{i0} - \widehat{\widehat{h}}_{i}^{-} \quad \forall i$$
(12)

$$\sum_{j} \lambda_{j}^{s} z_{ij}^{sp} = \widehat{z}_{i}^{sp} = \sum_{j} \lambda_{j}^{sp} z_{ij}^{sp} \quad \forall i \forall (s, p) \in SP$$
(13)

$$\sum_{p:(s,p)\in SP} \widehat{z}_i^{sp} = \widehat{x}_i^s - \widehat{h}_i^s \quad \forall i \forall s$$
(14)

$$\sum_{j} \lambda_{j}^{sp} u_{ij}^{sp} = \widehat{u}_{i}^{sp} = \sum_{j} \lambda_{j}^{p} u_{ij}^{sp} \quad \forall i \; \forall \, (s, p) \in SP$$
(15)

$$\widehat{u}_{i}^{sp} = \widehat{z}_{i}^{sp} - \widehat{h}_{i}^{sp} \quad \forall i \; \forall \left(s, p\right) \in SP$$
(16)

$$\sum_{j} \lambda_{j}^{p} z_{ij}^{pw} = \hat{z}_{i}^{pw} = \sum_{j} \lambda_{j}^{pw} z_{ij}^{pw}$$

$$\forall i \ \forall (p, w) \in PW$$
(17)

$$\sum_{w:(p,w)\in PW} \widehat{z}_i^{pw} = \sum_{s:(s,p)\in SP} \widehat{u}_i^{sp} - \widehat{h}_i^p \quad \forall i \; \forall p \tag{18}$$

$$\sum_{j} \lambda_{j}^{pw} u_{ij}^{pw} = \hat{u}_{i}^{pw} = \sum_{j} \lambda_{j}^{w} \ddot{u}_{ij}^{pw} \quad \forall i \; \forall (p, w) \in PW$$
(19)

$$\widehat{u}_{i}^{pw} = \widehat{z}_{i}^{pw} - \widehat{h}_{i}^{pw} \quad \forall i \; \forall (p, w) \in PW$$
(20)

$$\sum_{j} \lambda_{j}^{w} z_{ij}^{wr} = \widehat{z}_{i}^{wr} = \sum_{j} \lambda_{j}^{wr} z_{ij}^{wr} \quad \forall i \; \forall \; (w,r) \in WR$$
(21)

$$\sum_{r:(w,r)\in WR} \widehat{z}_i^{wr} = \sum_{p:(p,w)\in PW} \widehat{u}_i^{pw} - \widehat{h}_i^w \quad \forall i \ \forall w$$
(22)

$$\sum_{j} \lambda_{j}^{wr} u_{ij}^{wr} = \widehat{u}_{i}^{wr} = \sum_{j} \lambda_{j}^{r} u_{ij}^{wr} \quad \forall i \; \forall (w, r) \in WR$$
(23)

$$\widehat{u}_{i}^{wr} = \widehat{z}_{i}^{wr} - \widehat{h}_{i}^{wr} \quad \forall i \; \forall (w, r) \in WR$$
(24)

$$\hat{y}_i^r = \sum_j \lambda_j^r y_{ij}^r \ge y_{i0}^r \quad \forall i \; \forall r \tag{25}$$

$$\hat{y}_{i}^{r} = \sum_{w:(w,r)\in WR} \hat{u}_{i}^{wr} - \hat{h}_{i}^{r} \quad \forall i \; \forall r$$
(26)

$$\sum_{r} \hat{y}_{i}^{r} = \sum_{s} \hat{x}_{i}^{s} - \hat{h}_{i}^{-} \quad \forall i$$

$$\lambda_{j}^{s} \ge 0,$$
(27)

$$\lambda_{j}^{p} \ge 0,$$

$$\lambda_{j}^{w} \ge 0,$$

$$\lambda_{j}^{r} \ge 0$$
(28)

 $\forall j \; \forall s \; \forall p \; \forall w \; \forall r$ 

$$\lambda_j^{sp} \ge 0,$$
  

$$\lambda_j^{pw} \ge 0,$$
  

$$\lambda_i^{wr} \ge 0$$
(29)

$$\forall (s, p) \in SP \ \forall (p, w) \in PW \ \forall (w, r) \in WR$$
$$\lambda_j^{sp} \ge 0,$$
$$\lambda_j^{pw} \ge 0,$$
(30)

$$\lambda_j^{wr} \ge 0 \tag{30}$$

$$\forall (s, p) \in SP \ \forall (p, w) \in PW \ \forall (w, r) \in WR$$

$$\begin{split} \widehat{\widehat{h}_{i}}^{-} &\geq 0, \\ \widehat{h}_{i}^{-} &\geq 0, \\ \widehat{h}_{i}^{s} &\geq 0, \\ \widehat{h}_{i}^{p} &\geq 0, \\ \widehat{h}_{i}^{w} &\geq 0, \\ \widehat{h}_{i}^{r} &\geq 0, \\ \widehat{h}_{i}^{r} &\geq 0 \end{split}$$
(31)

 $\forall i \; \forall s \; \forall p \; \forall w \; \forall r$ 

This model computes target values for the amount of each product entering and leaving each process, so the quantity the retailer is able to sell, is at least equal to those observed in DMU 0. The amount of each product injected at the suppliers is, however, smaller than in the observed DMU. The larger the loss reductions at the suppliers, achieved by the target solution, the lower the efficiency of DMU 0.

In particular, constraints (11) and (12) compute the reduction (with respect to the value observed for DMU 0) in the total amount of each product to be injected at the suppliers. Constraints (13) compute the target value of the amount of each product leaving each supplier for each transportation link that starts at that supplier. Constraints (14) compute the difference between the total amount of each product injected at each supplier and the total amount that leaves it. That difference represents the loss at each supplier. What leaves each supplier towards each transportation link that starts at that supplier is an intermediate product (in the NDEA terminology) and is equal to the inputs to that transportation link. Constraints (15) compute the amount of each product that reaches the destination of that transportation link. Constraints (16) compute the difference between the inputs and outputs of each transportation link, differences that represent the losses at that transportation link.

Again, the output of each transportation link between a supplier and a plant is an intermediate product (in the NDEA terminology) and is equal to the inputs to that plant. Constraints (17) compute the target outputs of each plant for each transportation link that start at that plant. Constraints (18) compute the difference between the inputs to each plant and the outputs from it. Those differences represent the product losses at each plant.

The interpretation of the rest of constraints until the products reach the retailers is analogous. Thus, constraints (24) compute the losses at each transportation link between wholesalers and retailers. Constraints (25) compute the target outputs at the retailers, which, corresponding to an inputoriented model, must be larger than or at least equal to the observed outputs. Constraints (26) compute the difference between the inputs and the outputs at each retailer. Those differences represent the product losses at each retailer. Constraints (27) compute the difference between the total amount of each product injected at the suppliers and the total amount that is sold at the retailers. Those differences represent the total product losses in the whole logistics network. Finally, constraints (28)-(31) impose the nonnegativity of the intensity and product losses variables for all the processes. Recall that the NDEA processes correspond to all the processing nodes and transportation links along the actual logistics network.

The above optimisation model assumes that material flows are continuous variables, which leads to a Linear Programme (LP). If the flows must be considered discrete, integrity constraints can be imposed, thus leading to an integer DEA model (see [53–55]).

The optimisation model has many constraints, but a modular structure. The material flow is unidirectional from the supplier to the retailers for each process (which may be a node in the supply chain or a link between two nodes), and the target inputs and outputs are computed, using linear combinations of the observed values. In the NDEA terminology (e.g., [45]) the intermediate flows, i.e., the flows between each pair of processes, are considered as free links. The model allows and measures the product losses of the target solution in each process and for the entire logistics

network. In addition, the model computes the variables  $\hat{h}_i$  which correspond to the reductions in losses, achieved by the target solution, with respect to the losses of the observed DMU 0. Actually, it is the average of these loss reductions that determines the overall efficiency score of DMU 0.

The target solution is efficient, i.e., there is no feasible operating point that delivers the required amount of products to the retailers, and has smaller losses than the target solution. Otherwise, that feasible operating point would be better than the optimum, which cannot be. Note that although the target solution is efficient, it still has losses in each process. Thus, the corresponding loss factors for each product in each logistics network node and link can be computed as follows:

Loss factor of product i in process r:

$$\widehat{\xi}_{i}^{r} = \frac{\widehat{h}_{i}^{\prime}}{\sum_{w:(w,r)\in WR}\widehat{u}_{i}^{wr}} = 1 - \frac{\widehat{\mathcal{Y}}_{i}^{r}}{\sum_{w:(w,r)\in WR}\widehat{u}_{i}^{wr}} \qquad (32)$$

Loss factor of product i in process (w,r):

$$\widehat{\xi}_{i}^{wr} = \frac{\widehat{h}_{i}^{wr}}{\widehat{z}_{i}^{wr}} = 1 - \frac{\widehat{u}_{i}^{wr}}{\widehat{z}_{i}^{wr}}$$
(33)

Loss factor of product i in process w:

$$\widehat{\xi}_{i}^{w} = \frac{\widehat{h}_{i}^{w}}{\sum_{p:(p,w)\in PW} \widehat{u}_{i}^{pw}} = 1 - \frac{\sum_{r:(w,r)\in WR} \widehat{z}_{i}^{wr}}{\sum_{p:(p,w)\in PW} \widehat{u}_{i}^{pw}}$$
(34)

Loss factor of product i in process (p,w):

$$\widehat{\xi}_{i}^{pw} = \frac{\widehat{h}_{i}^{pw}}{\widehat{z}_{i}^{pw}} = 1 - \frac{\widehat{\mu}_{i}^{pw}}{\widehat{z}_{i}^{pw}}$$
(35)

Loss factor of product i in process p:

$$\widehat{\xi}_{i}^{p} = \frac{\widehat{h}_{i}^{p}}{\sum_{s:(s,p)\in SP}\widehat{u}_{i}^{sp}} = 1 - \frac{\sum_{w:(p,w)\in PW}\widehat{z}_{i}^{pw}}{\sum_{s:(s,p)\in SP}\widehat{u}_{i}^{sp}}$$
(36)

Loss factor of product i in process (s,p):

$$\widehat{\xi}_{i}^{sp} = \frac{\widehat{h}_{i}^{sp}}{\widehat{z}_{i}^{sp}} = 1 - \frac{\widehat{\mu}_{i}^{sp}}{\widehat{z}_{i}^{sp}}$$
(37)

Loss factor of product i in process s:

$$\widehat{\xi}_{i}^{s} = \frac{\widehat{h}_{i}^{s}}{\widehat{x}_{i}^{s}} = 1 - \frac{\sum_{p:(s,p)\in SP}\widehat{z}_{i}^{sp}}{\widehat{x}_{i}^{s}}$$
(38)

The loss factors along the whole logistics network can be computed as follows:

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|              |              |            |                      | Pro          | duct 1     |              |            |              |              |
|--------------|--------------|------------|----------------------|--------------|------------|--------------|------------|--------------|--------------|
| Peri         | iod 1        | Peri       | od 2                 | Peri         | od 3       | Peri         | od 4       | Peri         | od 5         |
| $x_{11}^{s}$ | $y_{11}^r$   | $x_{12}^s$ | $\mathcal{Y}_{12}^r$ | $x_{13}^{s}$ | $y_{13}^r$ | $x_{14}^{s}$ | $y_{14}^r$ | $x_{15}^{s}$ | $y_{15}^{r}$ |
| 4,450        | 2,709        | 4,055      | 2,402                | 4,397        | 2,994      | 5,328        | 2,959      | 3,058        | 4,130        |
| 6,691        | 3,538        | 3,802      | 2,419                | 4,383        | 3,542      | 4,072        | 3,582      | 11,856       | 3,571        |
| 1,931        | 2,923        | 3,102      | 2,504                | 6,020        | 4,292      | 4,492        | 3,281      | 0            | 2,680        |
|              | 2,828        |            | 2,953                |              | 3,188      |              | 2,728      |              | 3,454        |
|              |              |            |                      | Pro          | duct 2     |              |            |              |              |
| Peri         | iod 1        | Peri       | od 2                 | Peri         | od 3       | Peri         | od 4       | Peri         | od 5         |
| $x_{11}^{s}$ | $y_{11}^{r}$ | $x_{12}^s$ | $y_{12}^r$           | $x_{13}^{s}$ | $y_{13}^r$ | $x_{14}^s$   | $y_{14}^r$ | $x_{15}^{s}$ | $y_{15}^{r}$ |
| 2,992        | 4,097        | 10,381     | 2,971                | 4,225        | 2,732      | 3,987        | 3,919      | 6,917        | 1,922        |
| 7,266        | 4,027        | 0          | 2,373                | 6,175        | 3,128      | 0            | 3,761      | 1,796        | 2,074        |
| 4,995        | 2,751        | 0          | 2,405                | 4,649        | 4,584      | 11,539       | 2,536      | 0            | 1,858        |
|              | 3,148        |            | 1,993                |              | 3,803      |              | 3,780      |              | 2,231        |

TABLE 2: Amount of each product upstream and corresponding amounts that reach customers in each period.

Overall loss factor of product i:

$$\widehat{\xi}_i = \frac{\widehat{h}_i^-}{\sum_s \widehat{x}_i^s} = 1 - \frac{\sum_r \widehat{y}_i^r}{\sum_s \widehat{x}_i^s} = 1 - \frac{\sum_r \widehat{y}_i^r}{x_{i0} - \widehat{h}_i^-}$$
(39)

Average overall loss factor:

$$\widehat{\xi} = \frac{1}{m} \sum_{i} \widehat{\xi}_{i} \tag{40}$$

Note that since for each product the proposed NDEA approach computes input and output targets at each level of the supply chain, from these targets product-specific loss factors can be computed. The overall loss factor is an unweighted average of these product-specific loss factors. Note also that since, for each product,  $\sum_r \hat{y}_i^r \ge \sum_r y_{i0}^r$  and  $\hat{h}_i^- \ge 0$ , the corresponding loss factor of the target solution is always smaller than that of DMU 0, i.e.,

$$\zeta_{i0} = 1 - \frac{\sum_{r} y_{i0}^{r}}{x_{i0}} \ge 1 - \frac{\sum_{r} \hat{y}_{i}^{r}}{x_{i0} - \hat{h}_{\cdot}^{-}} = \hat{\xi}_{i}$$
(41)

## 4. Illustration

In this section the proposed approach will be illustrated, with a small problem randomly generated. The problem involves data from five periods, for a logistics chain with three suppliers, two plants, two wholesalers, and four retailers. Two products flow through the network. The way the product flows have been generated, is the following. For each period, a general reference loss value *grefloss* in the interval [0.95%;1.05%] is randomly generated. Once the general reference loss for the current period is determined, a specific reference loss value in each facility/transportation link is computed as *srefloss=grefloss*×r where  $r \in [0.95;1.05]$ . The final loss for each node/link in the supply chain is generated around its specific reference loss value as *loss=srefloss*×r where  $r \in [0.95;1.05]$ . Starting from the suppliers, where the

amount of each product injected in the network is generated randomly in the interval [10,000; 15,000], the loss factor of each node/link is successively applied, thus reducing the downstream flows. After each node, the outgoing flow is split randomly among the different outgoing links after choosing uniformly how many of these outgoing arcs to use.

Note that as the products go through the nodes (supplier, plant, wholesaler, and retailer) plus three transportation links (SP, PW, and WR) and the average loss factors are around 1%, the overall loss factors for each product should be around 7%. However, given the random variability present in the data, there will be some instances in which the loss factors in some node(s) or link(s) will be lower, which is what the proposed approach will detect, i.e., the periods in which the losses were minimal for each process. Those efficient operation instances are then used by the NDEA model to benchmark the observations and compute both the efficiency scores and target efficient product flows for each period.

For information, as it is not feasible to show all the data of all the periods, Table 2 shows, for each period, the amounts of each product injected upstream by each of the three suppliers and the corresponding amounts that reach downstream to the clients at each of the four retailers. Figure 2 shows the material flows of the two products for one of the periods, namely, period 1. Note that, in period 1, there was no material flow of some products in some links.

Note that the observed flows in period 1 involve losses in each node and each arc. Figure 3 shows the corresponding loss factors for each product at each node/arc. The total inputs, outputs, and losses of each product in the entire network for each observed period are shown in Table 3. Also note that, because of the way the data have been generated, the loss factors are similar for both products, but vary for the different periods.

The above flow balance (and loss) figures correspond to the observed data. Now let us look at the target values computed by the proposed NDEA efficiency assessment model. The model is solved for each period 0, considering the amount of products sold by each retailer in each period  $y_{i0}^r$ and finding corresponding material flows that minimize the



FIGURE 2: Observed material flows along the logistic network in period 1.

amount of products injected at the suppliers. For determining the target material flows in each period, the model uses the observed data for all periods, to define the PPS, which is the feasible region within which the optimal operating points are computed. Table 4 shows the total inputs and outputs of the efficient projection for each period. It indicates that the losses are lower than those that occurred in the observations. The table also shows the efficiency score  $\theta_0$  for each period, i.e., the optimal value of the objective function (10).

Recall that the efficiency score measures the reduction of the losses of the target solution with respect to those observed for that period. Note that the efficiency scores are rather high in all periods. This is because, when generating the data, the variability in the losses that occurred in the different periods was low, which means that relative efficiency improvement, although possible, cannot be large. However, as shown in Table 4, the loss factors in some periods (e.g., periods 1 and 4) are significantly lower than those observed in those periods (shown in Table 3). It is to be expected that a higher variability in performance, with a more efficient operation in some periods and more inefficiency in others, would allow the model to identify and correct those inefficiencies and, correspondingly, compute lower efficiency scores for the inefficient periods.

Figures 4 and 5 show the target flows and loss factors that result when assessing the efficiency of period 1. The corresponding efficient material flows and product loss factors can be compared with those of Figures 2 and 3, which correspond to the observed data. As shown in the first row of Table 4, the overall loss factors of this solution are 0.0650 for product 1 and 0.0686 for product 2, which are both lower than those observed in period 1 (shown in the first row of Table 3). The results of projecting the other periods are similar. In all periods, the target solution reduces the spoilage losses of all products, in some cases significantly.



FIGURE 3: Observed loss factors of each product in each node and each link in period 1.

## 5. Experimental Design

In order to further study the influence of the variability of the observed losses on the performance of the proposed approach, an experimental design has been carried out. Two factors have been considered, namely, the loss variability between periods (intertemporal variability, FACT1) and the loss variability between the different nodes and arcs (intraperiod variability, FACT2). Two levels (low/high variability) have been considered for each factor. For each period the general reference loss value grefloss is randomly generated in the interval 1%±0.05% for FACT1=1 (its low level) and in the interval 0%~2% for FACT1=2 (the high level of variability between periods). Similarly, the specific reference loss value in each facility/transportation link is computed as srefloss=grefloss×r where  $r \in [0.95;1.05]$  for FACT2=1 (low level) and  $r \in [0.80; 1.20]$  for FACT2=2 (high level of variability). Lastly, similarly to the illustration of Section 4, the final loss for each node/link in the supply chain is generated around its specific reference loss value as loss=srefloss×r' where r'∈[0.95;1.05].

For each factor-level combination, 10 random instances have been generated, providing a total of  $4\times10=40$  instances. A single product has been considered. Three types of response variables have been recorded. One is the efficiency scores of the different periods (theta). The second is the loss factors of the observations (OLFACTOR) and the third is the loss factors of the corresponding target solutions, found by the proposed approach (TOLFACTOR). The number of periods has been set at 52, making a total of  $40\times52=2080$  observations for each response variable.

As can be seen in Figure 6, theta values clearly depend on the loss variability between periods. For low intertemporal variability (FACT1=1), theta values vary within a narrow range (95.81%-98.53%); i.e., there are no great variations in the efficiency scores of the different observations. However, when the variability between the different periods is high (FACT1=2), the variability of theta values has a larger dispersion, with some observations even reaching 100 efficiency. The effect of FACT2 is negligible when FACT1=2, although for FACT1=1, it seems that a high intraperiod variability (FACT2=2) slightly reduces the dispersion of the efficiency scores.

A boxplot drawing confirms this situation (Figure 7), with small variations for theta in the case of FACT1=1 and larger variations in the case of FACT1=2 and with the intraperiod variability (FACT2) not showing any significant effect on the theta values. As regards the overall losses of the target solution obtained by the NDEA model (TOLFACTOR), when FACT1=1 (low intertemporal variability) TOLFACTOR reaches an average of 4.39%, and for the high level of FACT1 the average goes down to 0.17% (see also Table 4). No significant differences can be seen on TOLFACTOR depending on FACT2 according to Figure 6. Table 5 also shows an interaction between FACT1 and FACT2 in the value of theta. Thus, while for the low intertemporal variability (FACT1=1) the efficiency is higher for the high level of FACT2, the opposite occurs for high intertemporal variability (FACT1=2).

The loss reduction effectiveness of the proposed approach can be seen in Figure 8. For each of the four combinations of factors and levels, the losses in the observations are higher than the losses in the target solution (i.e., all points lie below the bisector line). For FACT1=1 (low level) the OLFACTOR values are around 7%, while the corresponding TOLFACTOR values are in the range 2%-5%. Larger loss reductions occur when the intertemporal variability is higher, with observed losses between 1% and 13%, and TOLFACTOR is always below 2%. This is because when the observations have large variability, some instances involve large losses, while others

|          |         |                       |            |            | an partoa.    |                    |         |
|----------|---------|-----------------------|------------|------------|---------------|--------------------|---------|
|          |         | $\sum_{s} x_{ij}^{s}$ | $\sum_{r}$ | $y_{ij}^r$ |               | Losses             |         |
| Period j | Prod. 1 | Prod. 2               | Prod. 1    | Prod. 2    | Prod. 1       | Prod. 2            | Average |
| 1        | 13,072  | 15,253                | 11,998     | 14,023     | 1,074 (8.22%) | $1,230 \ (8.06\%)$ | 8.14%   |
| 2        | 10,959  | 10,381                | 10,278     | 9,742      | 681 (6.21%)   | 639 (6.16%)        | 6.18%   |
| 3        | 14,800  | 15,049                | 14,016     | 14,247     | 784 (5.30%)   | 802 (5.33%)        | 5.31%   |
| 4        | 13,892  | 15,526                | 12,550     | 13,996     | 1,342 (9.66%) | 1,530 (9.85%)      | 9.76%   |
| 5        | 14,914  | 8,713                 | 13,835     | 8,085      | 1.079 (7.23%) | 628 (7.21%)        | 7.22%   |
| Average  | 13,527  | 12,984                | 12,535     | 12,019     | 992 (7.32%)   | 966 (7.32%)        | 7.32%   |
|          |         |                       |            |            |               |                    |         |

TABLE 3: Observed overall loss factors in each period.

| Effic. Score $(\theta_{c})$ |         | 0.9887            | 0.9962        | 0.9993        | 0.9776          | 0.9984            | 0.9921        |  |
|-----------------------------|---------|-------------------|---------------|---------------|-----------------|-------------------|---------------|--|
|                             | Average | 6.68%             | 5.71%         | 5.18%         | 7.21%           | 6.97%             | 6.35%         |  |
| Losses                      | Prod. 2 | 1,035.3~(6.86%)   | 598.7 (5.78%) | 784.9 (5.22%) | 1,169.6 (7.70%) | 599.5(6.89%)      | 837.6 (6.49%) |  |
|                             | Prod. 1 | $840.1\ (6.50\%)$ | 614.1 (5.63%  | 759.9 (5.14%) | 913.1 (6.73%)   | 1,049.6 $(7.05%)$ | 835.4 (6.21%) |  |
| $\widehat{y}_{i}^{r}$       | Prod. 2 | 14,054.9          | 9,756.0       | 14,254.7      | 14,028.8        | 8,101.2           | 12,039.1      |  |
|                             | Prod. 1 | 12,076.3          | 10,290.0      | 14,029.5      | 12,649.8        | 13,837.2          | 12,576.6      |  |
| $\widetilde{x}_i^s$         | Prod. 2 | 15,090.2          | 10,354.7      | 15,039.5      | 15,198.4        | 8,700.7           | 12,876.7      |  |
| , M                         | Prod. 1 | 12,916.5          | 10,904.2      | 14,789.4      | 13,562.9        | 14,886.9          | 13,411.9      |  |
|                             | Period  | 1                 | 2             | 3             | 4               | 5<br>2            | Average       |  |

TABLE 4: Total inputs, outputs, and losses of target solution in each period.





FIGURE 4: Efficient material flows along the logistic network in period 1.

|                 | Theta (%)           | OLFACTOR (%)        | TOLFACTOR (%)       |
|-----------------|---------------------|---------------------|---------------------|
|                 | (global aver. 95.6) | (global aver. 6.63) | (global aver. 2.28) |
| FACT1.1         | 97.50               | 6.79                | 4.39                |
| FACT1.2         | 93.69               | 6.47                | 0.17                |
| FACT2.1         | 95.74               | 6.69                | 2.49                |
| FACT2.2         | 95.45               | 6.60                | 2.07                |
| FACT1.1×FACT2.1 | 97.84               | 6.79                | 4.73                |
| FACT1.1×FACT2.2 | 97.15               | 6.79                | 4.05                |
| FACT1.2×FACT2.1 | 93.65               | 6.58                | 0.25                |
| FACT1.2×FACT2.2 | 93.74               | 6.35                | 0.10                |

TABLE 5: Average value of the three response variables for the different factor levels.



FIGURE 5: Target loss factors of each product in each node and each link in period 1.



FIGURE 6: Theta values for the 2080 observations.

involve small losses. The high-loss instances will be assessed as inefficiency, while the low-loss instances will determine the EF, setting the benchmark at a low level of loss.

The practical implications of the results of the experiments carried out are that the advantages of benchmarking the product flows of different periods are higher if the performance of the supply chain in terms of product losses is uneven, with loss factors that vary from one period to another. In that case the identification and deployment of the best practices can bring about significant improvements (average reduction of loss factor from 6.79% to 0.17%). However, since the DEA methodology assesses relative efficiencies, if the product losses are steady its usefulness for reducing them is more limited (average reduction of loss factor from 6.79% to 4.39%). The variability of the product losses in the different levels of the supply chain also has a much smaller influence. A high variability corresponds to the case when the product losses affect some links more than others, while the low variability corresponds to product losses uniformly spread along the supply chain. The results indicate that the proposed approach is able to detect and remove the inefficiencies in both cases, reducing the product losses in all levels of the supply chain.

## 6. Conclusions

In this paper an input-oriented NSBM DEA approach has been proposed to assess the efficiency of a supply chain in terms of product losses along the network. The proposed approach has a number of innovative features. Thus, each processing node and transportation link has been mapped to an NDEA process. Also, the inputs and outputs of each process represent the product flows that enter and leave the node or link. Since output flows are lower than the corresponding input flows, the processes involve product losses, which can be higher or lower depending on how the process was carried out (e.g., product handling, storage, and refrigeration). Using the observed data regarding material flows (and losses) in previous periods, the model can identify the best practices (i.e., the periods in which each network node and link performed best) and thus estimate an EF, against which each observation can be benchmarked. This allows an efficiency assessment of the spoilage efficiency in each period and provides target material flows, so that the losses along the network are minimal. This can be of great help for those in charge of managing supply chains in which product losses are significant (e.g., food supply chains).



FIGURE 7: Boxplots for theta and TOLFACTOR for the two factors considered.



FIGURE 8: Relationship between the TOLFACTOR and OLFACTOR for each combination of each factor level.

The NDEA methodology is nonparametric and datadriven. It only needs data on the flow of products along the chain to be recorded. For traceability reasons, these data are available in food and other perishable products supply chains. In the end this is what Big Data is all about: taking advantage of the large quantity of data available and processing it (often using sophisticated models) to help improve operations. In this way, the proposed approach can assess the efficiency of the observed product flows and compute efficient (i.e., minimum loss) targets based on the best practices along the supply chain.

According to the experimental results, when the intertemporal variability in the loss is low, the differences between the observations are not large and their efficiency scores are therefore rather similar. With this understanding, although overall losses can be reduced, the reduction is limited, among other things, because there is little inefficiency in the observed data. However, if the intertemporal variability of the observed losses is large, then the proposed NDEA model can detect and remove the larger inefficiencies present in the data, distinguishing between those processes that are efficient and those that are inefficient. Thus, the variability in the efficiency scores is much higher, and the reduction in the overall losses, with respect to the observed values, is significantly higher. Although the proposed approach can handle multiple products, the experimental design reported in Section 5 considers just a single product in order to study the effects on each product. Studying the effects of these factors on multiple products, effects that may depend on whether the products are similar or not in terms of their corresponding loss factors, is a topic for further research.

From a managerial point of view, most of the spoilage during transportation is related to highly perishable or fragile products. If we take the case of fruits and vegetables, for instance, long trips and numerous handlings and transhipments imply a higher rate of products arriving at destination in nonoptimal conditions. Obviously, a natural (and effective) way of reducing this spoilage rate is to consider shorter and simpler logistics networks. However, climate restrictions and high international demand make it sometimes necessary to deliver this sort of products over long routes. Our analysis is able to identify the minimum losses that can be expected from the best practices over these long networks, providing targets to reduce the current product loss ratios.

As a limitation of the proposed approach, we must note that the DEA methodology cannot be expected to eliminate all losses due to spoilage. What the proposed approach aims to achieve is to compute relatively efficient material flows, i.e., flows where the losses are minimal and feasible, i.e., consistent with the best practice observed in the different nodes and arcs of the network. In other words, the proposed approach benchmarks the logistics network against itself in each period, and the best performance ever, in terms of losses, is used for computing the target material flows.

Another limitation of the present study is that transportation costs at each arc and processing costs at each node have not been taken into account. Thus, a topic for further research is to extend the analysis to include not only spoilage but also operating costs. The proposed NDEA methodology is rather flexible and should be able to accommodate this, as well as other features (such as additional operational constraints), into the model. The ultimate goal is to increase the sustainability of the logistics network.

## **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

## **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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