NUCLEAR STRUCTURE INFORMATION OBTAINED FROM CHARGE RADII

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We analyze the nuclear structure information obtained from measurements of charge radii in even-even and even-odd nuclei in terms of the interacting boson model. The observed deviations from the smooth behavior, $R = R_0 A^{1/3}$, appear to be well described by this model.

On-line spectroscopy provides accurate measurements of isotope and isomer shifts in long isotopic chains. These measurements can be used to test nuclear structure models. The tests can be performed in two steps: in the first step, one can analyze the data in terms of a collective model and see whether or not a particular collective model is able to describe the data; if the first step is successful, one can then attempt a derivation of the collective parameters obtained in the previous analysis from a microscopic theory. In this letter, we address the first question and point out that the interacting boson model, in the form in which proton and neutron degrees of freedom are explicitly introduced (IBM-2 for even-even nuclei [1] and IBFM-2 for odd-even nuclei [2]), provides a natural collective framework for the analysis of the data, especially if one wants to discuss simultaneously even-even and odd-even nuclei. In fact, the model has built in, from the very beginning, by its separation into bosons (pairs) and fermions (single particles), the main effect observed experimentally, i.e. *odd-even staggering*.

We begin by briefly reviewing the formalism needed in this analysis. Nuclear charge radii can be written as

$$\langle r^2 \rangle_\pi = \langle r^2 \rangle_\pi^{(c)} + \alpha_{\pi\pi} N_\pi + \alpha_{\pi\nu} N_\nu + \beta_{\pi\pi} \langle n_{d\pi} \rangle + \beta_{\pi\nu} \langle n_{d\nu} \rangle, \quad (1)$$

for even-even nuclei [3,4], and

$$\langle r^2 \rangle_\pi = \langle r^2 \rangle_\pi^{(c)} + \alpha_{\pi\pi} N_\pi + \alpha_{\pi\nu} N_\nu + \beta_{\pi\pi} \langle n_{d\pi} \rangle + \beta_{\pi\nu} \langle n_{d\nu} \rangle + \sum_{j\rho} \gamma_{\pi\rho}^{j\rho} \langle n_{j\rho} \rangle \quad (2)$$

for even-odd proton ($\rho = \pi$) and even-odd neutron ($\rho = \nu$) nuclei. Here N_π (N_ν) denote the total boson numbers, $n_{d\pi}$ ($n_{d\nu}$) the number operators for d bosons, and $n_{j\pi}$ ($n_{j\nu}$) the number operators for fermions in the orbit j . The brackets indicate expectation values of the appropriate operators in the state under consideration (usually the ground state) and $\langle r^2 \rangle_\pi^{(c)}$ denotes the square charge radius of the closed shell. These formulas apply for nuclei from the beginning to the middle of a major shell. For nuclei from the middle to the end of the shell, $\langle r^2 \rangle_\pi^{(c)}$ is replaced by the full closed shell, $\alpha_{\pi\pi}$, $\alpha_{\pi\nu}$ and the γ are replaced by $-\alpha_{\pi\pi}$, $-\alpha_{\pi\nu}$ and minus the γ and bosons and fermions are counted as holes.

From eqs. (1) and (2), one can obtain all isotope and isomer shifts. For example, the isotope shift (even-even minus even-even) is given by

$$\Delta \langle r^2 \rangle_\pi^{(E-E)} = \alpha_{\pi\nu} + \beta_{\pi\pi} \Delta \langle n_{d\pi} \rangle + \beta_{\pi\nu} \Delta \langle n_{d\nu} \rangle. \quad (3)$$

The last term in eq. (3) represents the contributions to proton radii coming from changes in the neutron deformation. If one assumes that proton and neutron deformations are identical, the last two terms can be combined together, by changing $\beta_{\pi\pi}$ to an effective

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value $\beta_{\pi\pi}^{\text{eff}}$ and setting $\beta_{\pi\nu} = 0$. For this reason, the last term in eq. (3) has been usually neglected, except in the calculation of Bijker et al. [5] where it has been explicitly considered. Furthermore, in most regions $\alpha_{\pi\nu}$ and $\beta_{\pi\pi}$ can be considered independent of proton and neutron number, and thus all (even—even minus even—even) isotope shifts can be obtained in terms of only two parameters. Similarly, isomer shifts in even—even nuclei are given by

$$\delta\langle r^2 \rangle_{\pi}^{(E-E)} = \beta_{\pi\pi} \delta\langle n_{d\pi} \rangle + \beta_{\pi\nu} \delta\langle n_{d\nu} \rangle. \quad (4)$$

When $\beta_{\pi\nu} = 0$, they can be calculated in terms only of $\beta_{\pi\pi}$. Isotope shifts in the even—even Xe—Ba—Ce nuclei have been analyzed in this way in refs. [3,4].

Here we want to study the implications that eqs. (1) and (2) have for odd—even nuclei. We begin by considering the isotope shifts (odd—even minus odd—even). Using eq. (2) we obtain

$$\begin{aligned} \Delta\langle r^2 \rangle_{\pi}^{(O-O)} &= \alpha_{\pi\nu} + \beta_{\pi\pi} \Delta\langle n_{d\pi} \rangle + \beta_{\pi\nu} \Delta\langle n_{d\nu} \rangle \\ &+ \sum_{j\pi} \gamma_{\pi\pi}^{j\pi} \Delta\langle n_{j\pi} \rangle, \end{aligned} \quad (5)$$

for odd-proton nuclei, and

$$\begin{aligned} \Delta\langle r^2 \rangle_{\pi}^{(O-O)} &= \alpha_{\pi\nu} + \beta_{\pi\pi} \Delta\langle n_{d\pi} \rangle + \beta_{\pi\nu} \Delta\langle n_{d\nu} \rangle \\ &+ \sum_{j\nu} \gamma_{\pi\nu}^{j\nu} \Delta\langle n_{j\nu} \rangle, \end{aligned} \quad (6)$$

for odd-neutron nuclei. If $\gamma_{\pi\pi}^{j\pi}$ and $\gamma_{\pi\nu}^{j\nu}$ do not depend on $j\pi$ and $j\nu$, the last terms in eqs. (5) and (6) drop out since $\sum_{j\rho} \langle n_{j\rho} \rangle = 1$ for all states and one sees that odd—even minus odd—even isotope shifts have identical structure as even—even minus even—even isotope shifts. The assumption that the γ do not depend on $j\pi$ or $j\nu$ is only met if the single-particle orbitals occupied by the odd-particle originate from the same major shell and may, in general, not be correct. Isomer shifts can be obtained in a similar way as

$$\begin{aligned} \delta\langle r^2 \rangle_{\pi}^{(O-O)} &= \beta_{\pi\pi} \delta\langle n_{d\pi} \rangle + \beta_{\pi\nu} \delta\langle n_{d\nu} \rangle \\ &+ \sum_{j\rho} \gamma_{\pi\rho}^{j\rho} \delta\langle n_{j\rho} \rangle, \end{aligned} \quad (7)$$

$\rho = \pi, \nu$. They appear to be much more sensitive to small changes in the γ than the isotope shifts.

A very interesting phenomenon is provided by the occurrence of intruder configurations [6]. Here again

an advantage of the interacting boson model is that it can take into account explicitly these configurations [7]. A result of the presence of intruder configurations is that one has extra contributions due to the difference in radii of the two basic configurations. If the intruder configuration is a two-particle—two-hole *proton* configuration, as it appears to be the case in the region around the Sn isotopes, the radius of this configuration can be written in even—even nuclei as

$$\begin{aligned} \langle r^2 \rangle_{\pi}^{(\text{Intruder})} &= \langle r^2 \rangle_{\pi}^{(c)} + \alpha_{\pi\pi}^{(I)} N_{\pi} + \alpha_{\pi\nu} N_{\nu} \\ &+ \beta_{\pi\pi} \langle n_{d\pi} \rangle + \beta_{\pi\nu} \langle n_{d\nu} \rangle, \end{aligned} \quad (8)$$

indicating that the proton parameter $\alpha_{\pi\pi}$ has changed due to the presence of the intruder state. If the intruder configuration would have been a *neutron* configuration, one would have expected the change to occur in the neutron parameter $\alpha_{\pi\nu}$. A consequence of eq. (8) is that the (even—even minus even—even) isotope shift, becomes, when going from a normal to an intruder configuration,

$$\begin{aligned} \Delta\langle r^2 \rangle_{\pi}^{(\text{Normal-Intruder})} &= \eta_{\pi\pi} + \alpha_{\pi\nu} + \beta_{\pi\pi} \Delta\langle n_{d\pi} \rangle \\ &+ \beta_{\pi\nu} \Delta\langle n_{d\nu} \rangle, \end{aligned} \quad (9)$$

where $\eta_{\pi\pi} = [\alpha_{\pi\pi} N_{\pi} - \alpha_{\pi\pi}^{(I)} (N_{\pi} + 2)]$. Both here and in eq. (8) we have used the notation appropriate to the treatment of the intruder configuration by increasing the boson number by two units (one-proton—boson particle and one-proton—boson hole) [7]. The parameter $\alpha_{\pi\pi}^{(I)}$ reflects then the change in the average radius. It is worthwhile noting that the presence of the extra term $\eta_{\pi\pi}$ produces a sudden jump in the isotope shift when the intruder configuration becomes the ground state configuration. A similar effect occurs in the (odd—even minus odd—even) isotope shift, which can be written as

$$\begin{aligned} \Delta\langle r^2 \rangle_{\pi}^{(\text{Normal-Intruder})} &= \theta_{\pi\pi} + \alpha_{\pi\nu} + \beta_{\pi\pi} \Delta\langle n_{d\pi} \rangle \\ &+ \beta_{\pi\nu} \Delta\langle n_{d\nu} \rangle, \end{aligned} \quad (10)$$

and in the (odd—even minus odd—even) isomer shift, which can be written as

$$\delta\langle r^2 \rangle_{\pi}^{(\text{Normal-Intruder})} = \theta_{\pi\pi} + \beta_{\pi\pi} \delta\langle n_{d\pi} \rangle + \beta_{\pi\nu} \delta\langle n_{d\nu} \rangle, \quad (11)$$

where

$$\theta_{\pi\pi} = \sum_{j\pi} \gamma_{\pi\pi}^{j\pi} \langle n_{j\pi} \rangle + \left(\sum_{j\pi} \gamma_{\pi\pi}^{j\pi} \langle n_{j\pi} \rangle \right)^{(I)} - \alpha_{\pi\pi}.$$

Again the presence of the extra term $\theta_{\pi\pi}$ produces a sudden jump in the isotope shift when the intruder configuration becomes the ground state configuration.

We have applied our analysis to the (even—even and even—odd) ^{54}Xe and (odd—even) ^{55}Cs nuclei, for which extensive data exist and for which we had previously performed extensive calculations including intruder configurations [8]. Our results are shown in figs. 1 and 2. They have been obtained by assuming all γ independent of $j\pi$ or $j\nu$, $\gamma_{\pi\nu}^{j\nu} \equiv \gamma_{\pi\nu}$, $\gamma_{\pi\pi}^{j\pi} \equiv \gamma_{\pi\pi}$ and $\beta_{\pi\nu} = 0$. They are thus given in terms only of four parameters, $\alpha_{\pi\nu} = 0.057 \text{ fm}^2$, $\beta_{\pi\pi} = 0.230 \text{ fm}^2$, $\theta_{\pi\pi} = -0.104 \text{ fm}^2$ and $\gamma_{\pi\nu} = 0.093 \text{ fm}^2$. The parameters $\alpha_{\pi\nu}$ and $\beta_{\pi\pi}$ are consistent with those previously reported in ref. [4] ($\alpha_{\pi\nu} = 0.062 \text{ fm}^2$, $\beta_{\pi\pi} = 0.224 \text{ fm}^2$). The value $\beta_{\pi\pi}$ reported in ref. [3] differs from the value 0.224 by a factor of $\sqrt{5}$ because of a different definition of this coefficient. The interesting features of the data are the startling effect produced by the intruder proton configuration $g_{9/2}$ in ^{55}Cs at neutron number 66 and the large odd—even staggering in ^{54}Xe . Both

Fig. 1. Isotope shifts (even—even minus even—even, EE, and even—odd minus even—even, OE) in ^{54}Xe . The experimental data are from refs. [9,10]. The values expected from a smooth increase of charge radii $R = R_0 A^{1/3}$, $R_0 = 1.2 \text{ fm}$, are shown for comparison.

Fig. 2. Isotope shifts (odd—even minus odd—even, 00) in ^{55}Cs . The experimental data are from ref. [11]. The values expected from $R = R_0 A^{1/3}$ are shown also here for comparison.

effects are reasonably well reproduced by the calculations. The values of $\langle n_{d\pi} \rangle$ which enter here were obtained from refs. [8,12] and have *not* been further adjusted. It is interesting to note that, because of systematic errors, the data points shown in figs. 1 and 2 may be shifted somewhat upwards or downwards. The most important information is thus contained in their behavior as a function of neutron number. The same calculation also produces the isomer shifts shown in table 1. The important features here are the small isomer shifts for normal configurations (^{133}Cs and ^{129}Xe) and the large isomer shifts between normal and intruder configurations (^{121}Cs and ^{119}Cs). The calculations appear to reproduce these features (one parameter $\theta_{\pi\pi}$ for three data points), although they are unable to reproduce precisely the experimental values. This may be due to our assumption $\gamma_{\pi\pi}^{j\pi} \equiv \gamma_{\pi\pi}$ and $\gamma_{\pi\nu}^{j\nu} \equiv \gamma_{\pi\nu}$, which appears to introduce an error of the order of $\approx 0.010 \text{ fm}^2$.

In conclusion, we have presented results of detailed calculations of isomer and isotope shifts in the Xe—Cs isotopes. Similar calculations are also available for other isotopes in this region and for several isotopic chains in the Os—Pt region. A glance at figs. 1 and 2 shows how dramatically different are the observed

Table 1
Isomer shifts in $_{55}\text{Cs}$ and $_{54}\text{Xe}$.

Nucleus	Neutron number	States	$\delta\langle r^2 \rangle_{\text{exp}}^{\text{a)}}$ (fm^2)	$\delta\langle r^2 \rangle_{\text{th}}^{\text{c)}}$ (fm^2)
Cs	78	$5/2^+ - 7/2^+$	0.004 ^{b)}	0.006
Cs	66	$9/2^+ - 3/2^+$	0.185 ± 0.002	$0.185^{\text{c)}}$
Cs	64	$3/2^+ - 9/2^+$	-0.198 ± 0.007	-0.179
Xe	75	$3/2^+ - 1/2^+$	0.001 ^{b)}	-0.006

a) Refs. [11,13]. b) From Mössbauer effect. c) Used to extract $\theta_{\pi\pi}$.

isotope shifts from the behavior expected from a smooth increase of the charge radii, $R = R_0 A^{1/3}$. In particular, fig. 1 shows clearly the odd-even staggering and how it is reproduced by the calculation. This figure and fig. 2 also point to the predictive power of the model calculations, since there are, in both figures, calculated values for nuclei not yet measured. It would be interesting to test these predictions. It also would be interesting to compare the experimental results with calculations based on other collective models, for example those of the droplet model [14]. Up to now, what is done in terms of this model is to reduce the contribution arising from $R = R_0 A^{1/3}$ by a quantity $\eta \approx 0.5$ and then to extract the deformation β by fitting each point in figs. 1 and 2 individually. This approach does not provide tests of the model. Finally, it would be interesting to perform microscopic calculations in order to derive the values of the parameters $\alpha_{\pi\nu}$, $\beta_{\pi\pi}$, $\theta_{\pi\pi}$ and $\gamma_{\pi\nu}$. We have made some attempts to do so on the basis of the microscopic theory of the interacting boson model [15], but further work is still needed to clarify fully the situation.

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