GA for Planning Cable Telecommunication Networks

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Abstract

Optical fiber networks over synchronous digital hierarchy operating on ATM mode appear as one of the best alternatives for the new high-speed telecommunications networks. The network management requires planning studies so that the designed networks allow efficient operation in real time. A quantitative model evaluating the global topological design of an optical fiber network over synchronous digital hierarchy is described. The model includes the economic aspects involved in the project, the civil works, the capacity evaluation costs and the introduction of reliability conditions. A genetic approach based on the exploitation of the Kuhn-Tucker optimality conditions is proposed to solve the model.

Keywords: evolutionary computing, telecommunication network planning, Kuhn -Tucker optimality conditions.

1. Introduction

In the near future telecommunication networks will become the necessary infrastructure for the new *information highways*. Networks with higher capacity and speed will be needed to give solution to the set of transport and traffic volume requirements. The future network will be known as Broadband Integrated Services Digital Network (B-ISDN), and will let transmit more information, of different types (audio, video, data, etc.), at a greater speed and with more interconnection possibilities.

The broadband evolution is strongly linked to the optical fiber technology development (Hecht, 1993). Light permits greater capacity than electric current but it requires the use of more expensive equipment. In fact, it has been only used for the connection of large telephone exchanges.

In other line, future networks will appear linked to the new switching and transmission technologies. In the eighties, the European standard committee CCITT recommended the adoption of Asynchronous Transfer Mode (ATM) as switching mode and the Synchronous Digital Hierarchy (SDH) as transmission mode for the future broadband. The concept of switching and/or multiplexing centers (or nodes) appears joined to the ATM technology. We name these nodes as hubs (Sexton et al., 1997).

The problem addressed here deals with the design and dimensioning of an optical fiber backbone network among a set of urban nodes. As initial data, we are given the input traffic as the demand measured in Erlangs for all origin-destination urban communication pairs. This Erlang traffic can be converted into the number of synchronous transport module (STM) levels required for each point-to-point connection. So, the output of the conversion process is the number STM-*u* levels required for each of the origin-destination pairs. According to the European standard, the SDH standard divides the traffic units into STM-1 level equivalent to 155.52 Mbps, that can be assembled into STM-4 level equivalent to 622.08 Mbps and into STM-16 level equivalent to 2488.32 Mbps. STM-*u* has been widely used as the planning and design basic demand unit of such optical fiber backbone networks

over Synchronous Digital Hierarchy. Next table 1 summarizes the relation among the European and USA standards and their correspondences with the capacity measured as Mbps and optical levels.

Table 1. European and USA transmission standards

Optical level	Capacity (Mbps)	SDH (ITU) ^a	SONET (ANSI) ^b
OC ^c -1	51.84	STS ^d -1	-
OC-3	155.52	STS-3	STM ^e -1
OC-9	466.56	STS-9	-
OC-12	622.08	STS-12	STM-4
OC-18	933.12	STS-18	-
OC-24	1244.16	STS-24	-
OC-36	1866.24	STS-36	-
OC-48	2488.32	STS-48	STM-16

^a Synchronous Digital Hierarchy (International Telecommunication Union)

^b Synchronous Optical NETwork (American National Standards Institute)

^c Optical Carrier level

^d Signal Transport Signal level

^e Synchronous Transport Module level

So, we will have to deal with a two-stage problem. Initially, we state the conduit network locating ditches wherein cables can be placed. After that we consider the transmission link routing topology in the conduit infrastructure considering reliability as multirouting to avoid failures in the network, and the transmission devices location at some specific urban nodes. The transmitters connect a low hierarchical level flow into a high hierarchical level flow at higher speed.

Optical fiber networks are well suited to deal with broadband networks due to their wide transmission capability, typically named hubbing topologies are indicated for this optical fiber backbone networks (Cardwell et al., 1989; Kerner et al., 1986). We consider a hubbing topology based on two levels. In the first level (the low speed network) flows are first transported from the initial switching office to the regional hub to switch. After, in the second level (the high speed network) all the arrived demands to the hub are sorted and rerouted to their specific destinations. To deal with this arrangement in the hubs, transmission equipment such as Digital Cross-connect System (DCS) have to be installed at each hub.

The output of the problem must be the location of transmission nodes (hubs), the locations of transmission links and their capacities as STM- u total flow, the paths to transport the origin-destination pair demand as STM- u levels and the costs associated to the planning and dimensioning process in the network.

Several references based on soft computing techniques can be found dealing with the real time network operation. Diverse authors (Ghanwani, 1998; Wang et al., 1995) have shown the use of neural approaches to solve the real time traffic routing problem. Aboelela et al., 1999 present a fuzzy multiobjective optimization model to develop a routing algorithm to guarantee the various quality of service characteristics requested by the wide range of applications supported by B-ISDN. Chou et al., 1998 have developed a hybrid neural-genetic procedure to deal with the bandwidth allocation of virtual paths in ATM networks. However, not many soft computing references appear with respect to the planning and dimensioning stage. In spite of it, the global telecommunication networks topological design has been dealt with in the bibliography extensively with the use of traditional operational research approaches. Chang et al., 1993; Chang et al., 1995 have developed communication models among hubs solved by means of lagrangian relaxation techniques. Cox et al., 1993 have implemented an exhaustive cost evaluation model for the US WEST in the Colorado area solved with the commercial optimization software CPLEX. More recently, Yoon et al., 1998 have developed with success an important model for hierarchical networks at tending to hubs and terminal nodes broached with a dual ascent procedure.

The rest of the paper deals with the global topological design for the optical fiber backbone of a two - level hierarchical network. Section 2 establishes the approach to construct the model, the capacity to be assigned to the links and the traffic routing at minimum cost. The global model is presented as an *NP-Hard* problem reducible to the Steiner problem, proved *NP-Hard* by Garey et al., 1979. Section 3 presents a solution method of evolutionary type whose results are shown in section 4. Finally, we review the main aspects in section 5.

2. Formulation

Before formulating the model it is important to clarify the differences between the terms graph and telecommunications network such as we use in this paper. In our model we will deal with graphs on two levels. One level will be related to the topological graph and the other will be related to the telematic net.

Graph: map abstraction equivalent to the transport system. It includes the urban nodes, the junction nodes (intersections), and the direct connections among all these nodes (*arcs*).

Network: represents the objective telecommunication network. Hence it identifies as nodes only those representing hubs or terminal nodes, while the transmission *links* are different from arcs and represent the *hub-hub connections* or the *terminal-hub connections*.

Figure 1. Graph versus telematic network

Note in the figure the difference between the arc concept and the link concept. Links will be the direct connections between one hub and the terminals of its region (within -region links) or the connection among hubs from different regions (between the region links). On the other hand arc is the joining between each pair of adjacent nodes independently of their switching capacity. These nodes could be hubs, terminals or simple intersections in the transport system. On the contrary the link term appears related to the infrastructure for telematic services transmission.

So in figure 1, the arc $(1, H_1)$ coincides with a terminal-hub transmission link. On the contrary, the arc $(2, 3)$ does not shape a transmission link by itself, but it is a part of the hub-hub transmission link (H_1, H_2) . Even more, note how the arc $(3, H_2)$ shapes a terminal-hub by itself and is part of the hub-hub transmission link (H_1, H_2) .

Summarizing, all the directed feasible paths between two hubs in the potential graph will shape the set of links among the regions (the inter-regional set). All these feasible links will be constituted by a succession of nodes and undirected arcs from the initial hub to the final hub. These kinds of links shape the upper level network and represent a high speed network with full switching and

multiplexing capacities. All the feasible paths between each terminal within-region and its regional hub will shape the set of links within each of the regions (the intra -regional set). These kinds of links shape the lower level network and have not switching capacity. The communication will have to go to the hub for switching and being sent to the correct destination.

Note, how the arc concept appears related to conduit construction and the link concept appears related to the transmission flows. Hence, we have to consider two parallel links related to both directions, i.e. , $(H_1 \rightarrow H_2)$ associated to the directed link (H_1, H_2) and $(H_2 \rightarrow H_1)$ associated to the other directed link (H_2, H_1) .

2.1. Augmented Network

In order to make easier the task of modeling the problem a fictitious node is located within each of the regions constituting the geographical network, a similar procedure is described in Yoon et al., 1998. The fictitious node is joined to all the urban nodes (candidates to be the hub) by means of fictitious arcs. The augmented network can be observed in figure 2, with the special characteristics and nomenclature that are described below.

Figure 2. Augmented network.

- (i) A fictitious node is located within each region r of the network. It will be noted as Hf_r .
- (ii) Fictitious arcs are constructed connecting all the within -region terminals j to the new fictitious hub, Hf_r .
- (iii) The cost of each fictitious arc is the hub activation cost , f_{jHf_r} .
- (iv) Each pair of communication, noted as k , will be established between one origin of communication, $o(k)$, and one destination of communication, $d(k)$. To deal with it, the origin-destination communication pairs are defined according to the following rules:

$$K_r = \{k: o(k) \in r, d(k) = Hf_r\} ; r \in R.$$

It represents the within-region r communication: the intra -regional set. The urban nodes inside a region r establish the internal connections through the regional hub. So all the communications will have the fictitious hub as their destination.

$$K_o = \{k: o(k) = Hf_r, d(k) = Hf_{r'}\} ; r, r' \in R.$$

It represents the between the region communication: the inter-regional set. The communication is established among the fictitious nodes, so the connection between users in different regions is guaranteed.

- (v) The demand is estimated in the following manner: 1) The within -region demand is determined as the total amount of STM- u levels (see table 1). 2) The demand inter regions is estimated as the STM- u levels to be installed to allow the communication between the two regions, i.e., the communication volume between every pair of regions.

With these conditions the existence of a potential path between each of the terminals inside the region and their fictitious regional hub is guaranteed. A constraint forcing the activation of only one of the candidates to be the real hub will impose the communications over this hub. The outgoing communications from each urban node is directed to the real hub in the region and switch there to be sent to the correct destination.

On the other hand, even though we define the communication for every origin -destination pair between the fictitious hubs of different regions, the effective communication will be managed and established through the real activated hubs within each region. So this will be shaping the communication between the regions: the inter -region communication.

2.2. Notation and constraints

□ Parameters:

- N set of nodes of the graph.
- A set of arcs of the graph.
- R set of regions to be defined in the network.
- $M(r)$ urban nodes (or terminals) inside the region $r \in R$.
- K set of origin-destination communication pairs. There exist two communication levels, the within each region communication, K_r , and the between the regions communication, K_o , being $K = K_o \cup \left[\bigcup_{r \in R} K_r \right]$
- $\mathcal{P}(k)$ set of feasible paths connecting the origin node of the pair $k \in K$, $o(k)$, with the destination node, $d(k)$. Each path $h \in \mathcal{P}(k)$ will be a succession of nodes and arcs from the origin, node $o(k)$ to the destination, node $d(k)$.

□ Variables:

- y_{ij} binary variable indicating if the undirected arc $\{i,j\}$ is used in the network infrastructure.
- y_{jHfr} binary variable indicating if a hub is installed at the terminal $j \in M(r)$.
- p_{kh} continuous variable indicating the origin -destination pair demand fraction, k , through the path h .

□ Data:

- γ_k origin-destination communication pair demand, k .
- δ_{ij}^{kh} binary parameter, equal to one if the path h uses the arc $\{i,j\}$ to join the pair k , and otherwise zero.
- f_{ij} civil works linear cost in the arc $\{i,j\}$. It includes the costs of ditches and conduits.
- f_{jHfr} node j activation cost as regional hub.

The equipment cost for the link e , $c_e(x_e)$, will depend on the circulating flow and the total distance covered by the link. But the elements set allowing these conditions and implying minimum cost will be determinable. There will be an optimal combination of fibers, binary regime, transmitters/receivers, regenerators, etc. (Gómez et al., 1998), as can be viewed in figure 3.

Figure 3. Components of an optical fiber cable.

Once the link has been stated the cost function $c_e(x_e)$ takes a *quasi-concave* shape (as definition in Mangasarian, 1994) depending on the flow volume and the length of the link, see figure 4.

Figure 4. Cost function of the link *versus* flow level and link length.

However the hub location will determine the definitive candidate links in the network. So the hub structure will drastically affect the network topology. *Hub structure* means a feasible arrangement of hubs in the geographical area, i.e., the location of one hub per region. So, each feasible hub combination (hub structure) will have repercussions on the arrangement of potential links with a determined costs structure, since there will appear different path sets joining each pair of nodes. In figure 5 are shown three different hub structure for a same graph A. First option, B, locates the hubs over the nodes 1, 4 and 6, second option locates the hubs in nodes 2, 3 and 5, and third option sites the hubs in nodes 2, 3 and 7. There will be more additional options until a total of 12 different hub structures, all of them will imply alternative arrangement of potential transmission links. Figure 5 depicts the potential transmission links for hub structures B, C and D.

Figure 5. Transmission links associated to its correspondent hub structure

In general, for R regions, there will exist $H=\{H_1, H_2, \dots, H_L\}$, feasible hubs structures, with $L = \prod_{r=1}^R |M(r)|$, where $|M(r)|$ indicates the total number of urban nodes in region $r \in R$.

As it has been stated each feasible hub combination (hub structure) will have repercussions on the arrangement of potential links with a determined cost structure, since there will appear different path sets joining each pair of nodes. So, new variables and data appear and are noted as follows:

- E_{H_i} set of feasible links when the structure H_i is used.
- δ_e^{kh} binary parameter, equal to one if the transmission link e is incorporated in the path h used for the communication k , and otherwise zero.
- η_i binary variable taking value equal to one if H_i is used as the hub structure.
- $x_{e_{H_i}}$ continuous variable indicating the flow level in each link e when H_i is used as the hub structure.
- $c_{e_{H_i}}(x_{e_{H_i}})$ cost of the link e when structure H_i is used and the flow over the link is x_e .

2.3 Model

The model includes the demand balance constraints (2) plus those related to the topological - technological conditions of the problem (1), (3), (4) and (5). So constraint (2) imposes that hundred per cent of the total volume of demand across all the feasible paths in the network has to reach the destination from the origin of communication. Constraint (1) determines that only one hub can be activated per region. Constraint (3) states the arcs shaping the conduit mesh as a function of the communication paths. Constraint (4) determines the flow in each transmission link as the sum of the total transported flow over all the paths being used by every origin -destination pairs of communication that include link e . Constraint (5) imposes the contemplated hub structure as the product of the binary variables associated to the correspondent structure. Although constraint (5) has been written in its non - linear form could be rewritten by means of additional binary variable and a large number of linear constraints. All these constraints can be found discussed extensively in Cortes, 2000.

Finally, the constraints are completed by means of the introduction of survivability/reliability levels (6), (7) and the network tolerated delay level (8). The diversification procedure described in Ale vras et al., 1997 has been adopted as survivability technique. Constraint (7) forces us to use as many paths as parameter $\lceil 1/\rho_k \rceil$ determines, avoiding overcharged links which would endanger the network

survivability. Constraint (6) avoids nodes managing more flow level than the same previous bound. Constraint (8) is used to forbid paths incorporating an upper number of hubs than certain maximum bound because of the switching process is the delay dominant term in fiber optical networks. So paths implying greater delays than those determined by the maximum level will not be allowed. We note these paths as $\mathcal{P}^*(H_i(k))$.

$$\begin{aligned}
\mathbf{GP}: \quad & \text{MIN} \quad \sum_{\{i,j\} \in A} f_{ij} \cdot y_{ij} + \sum_{r \in R} \sum_{j \in M(r)} f_{jHf_r} \cdot y_{jHf_r} + \sum_{H_i \subseteq H} \sum_{e_{H_i} \in E_{H_i}} c_{e_{H_i}}(x_{e_{H_i}}) \cdot \eta_i \\
& \text{s.t.:} \\
& \sum_{j \in M(r)} y_{jHf_r} = 1, \quad \forall r \in R \quad (1) \\
& \sum_{h \in \mathcal{P}(k)} p_{kh} = 1, \quad \forall k \in K \quad (2) \\
& \sum_{k \in K} \sum_{h \in \mathcal{P}(k)} p_{kh} \cdot \delta_{ij}^{kh} \leq |K| \cdot y_{ij}, \quad \forall \{i,j\} \in A \quad (3) \\
& x_{e_{H_i}} = \sum_{k \in K_0} \sum_{h \in \mathcal{P}(k)} \gamma_k \cdot p_{kh} \cdot \delta_e^{kh}, \quad \forall e_{H_i} \in E_{H_i}, \quad \forall H_i \subseteq H \quad (4) \\
& \eta_i = \prod_{r \in R} \left[\prod_{\substack{j \in M(r) \\ j \subseteq H_i}} y_{jHf_r} \right], \quad \forall H_i \subseteq H \quad (5) \\
& \sum_{h \in \mathcal{P}(k): j \in h} p_{kh} \leq \rho_k, \quad \forall k \in K_0, \quad \forall j \subseteq H_i, \quad \forall H_i \subseteq H \quad (6) \\
& p_{kh} \leq \rho_k, \quad \forall h \in \mathcal{P}(k), \quad \forall k \in K_0 \quad (7) \\
& p_{kh} + \eta_i \leq 1, \quad \forall h \in \mathcal{P}^*(H_i(k)), \quad \forall k \in K_0 \quad (8) \\
& p_{kh} \geq 0, \quad \forall k \in K, \quad \forall h \in \mathcal{P}(k) \\
& y_{ij} \in \{0,1\}, \quad \forall \{i,j\} \in A \\
& y_{jHf_r} \in \{0,1\}, \quad \forall j \in M(r), \quad \forall r \in R
\end{aligned}$$

GP is a mixed integer linear programming problem implying an enormous size, even for moderately size networks. For instance, a geographical area divided into 5 regions, with 20 municipalities (this will be equivalent to 28 inter-regional origin-destination pairs and 30 intra-regional origin-destination pairs), and a graph with 600 arcs implying 200 transmission links as average for each hub structure and three alternative paths for each origin-destination pair will require 1,644 binary variables, 209,974 continuous variables and 206,739 constraints excluding the nonnegativity conditions.

3. Genetic approach

Our proposal is to use a genetic algorithm (GA) to solve the model. We state the population individuals as the feasible hub structures. So the genetic encoding can be stated in accordance with figure 6, where it is determined the urban node of each region being activated as regional hub.

Figure 6. Population individual coded as the hub structure

The individual fitness is calculated by evaluation of the associated model once the hub structure is fixed in **GP**. The problem can then be reduced to a multicommodity flow problem with reliability conditions (equivalent to capacity constraints) and with a concave objective function. The model still is *NP-Hard*.

We suggest a fitness approximation based on a two-stage approach. First, the initial basic topology corresponding to the infrastructure is imposed by means of 2-trees techniques with survivability conditions, combining Steiner trees (Monma et al., 1990; Monma *et al* 1988; Wald et al., 1983). The solution from this first stage is the set of arcs (conduits) and the feasible set of paths and telema tic links for the hub structure. Once the basic infrastructure has been fixed, the second stage is to route the traffic and assign enough capacity to the links. So it corresponds with the next **RCP** problem.

$$\begin{aligned}
 \mathbf{RCP}: \quad & \text{MIN} \sum_{e \in E} c_e(x_e) \\
 \text{s.t.}: \quad & \\
 & \sum_{h \in \mathcal{P}(k)} p_{kh} = 1, \quad \forall k \in K_0 \quad (1) \\
 & x_e = \sum_{k \in K_0} \sum_{h \in \mathcal{P}(k)} \gamma_k \cdot p_{kh} \cdot \delta_e^{kh}, \quad \forall e \in E \quad (2) \\
 & p_{kh} \leq \rho_k, \quad \forall h \in \mathcal{P}(k), \quad \forall k \in K_0 \quad (3) \\
 & p_{kh} \geq 0, \quad \forall h \in \mathcal{P}(k), \quad \forall k \in K_0 \\
 & x_e \geq 0, \quad \forall e \in E
 \end{aligned}$$

A concave minimization problem, as **RCP**, is *NP-Hard*, even in such special cases as that of minimizing a quadratic concave function over very simple polytopes such as hypercubes (Pardalos et al., 1987). However it can be easily proved that the global minimum of a concave function is always attained at some vertex of the polytope (Horst et al., 1990). More accurately, the Kuhn-Tucker optimality conditions state the set of conditions and equations that the optimum satisfies in every optimization problem. The Kuhn-Tucker conditions concerning problem RCP are shown next:

$$\mathbf{RCP} : \min \sum_{e \in E} c_e(x_e) = \min C(x) = \min C(p)$$

s.t.:

$$\begin{aligned} \sum_{h \in \mathcal{P}(k)} p_{kh} &= 1, \quad \forall k \in K_0 \\ p_{kh} &\leq \delta_k, \quad \forall h \in \mathcal{P}(k), \quad \forall k \in K_0 \\ p_{kh} &\geq 0, \quad \forall h \in \mathcal{P}(k), \quad \forall k \in K_0 \end{aligned}$$

Optimality (Kuhn-Tucker)
Conditions

$$\begin{aligned} \frac{\partial C(p)}{\partial p_{kh}} - \mu_k &= u_{kh} - v_{kh} & \mu_k &\text{ free} \\ p_{kh} \cdot u_{kh} &= 0 & u_{kh} &\geq 0 \\ (p_{kh} - \delta_k) \cdot v_{kh} &= 0 & v_{kh} &\geq 0 \end{aligned}$$

Case I

$$p_{kh} = 0 \Rightarrow u_{kh} \geq 0 \text{ y } v_{kh} = 0$$

$$\frac{\partial C(p)}{\partial p_{kh}} - \mu_k = u_{kh} \geq 0$$

$$\sum_{e \in E} [c'_e(x_e)] [\gamma_k \cdot \delta_e^h] \geq \mu_k \Rightarrow \sum_{e \in E} c'_e(x_e) \cdot \delta_e^h \geq \mu'_k = \frac{\mu_k}{\gamma_k}$$

Case II.a

$$p_{kh} = \delta_k \Rightarrow u_{kh} = 0 \text{ y } v_{kh} \geq 0$$

$$\frac{\partial C(p)}{\partial p_{kh}} - \mu_k = -v_{kh} \leq 0$$

$$\sum_{e \in E} [c'_e(x_e)] [\gamma_k \cdot \delta_e^h] \leq \mu_k \Rightarrow \sum_{e \in E} c'_e(x_e) \cdot \delta_e^h \leq \mu'_k = \frac{\mu_k}{\gamma_k}$$

Case II.b

$$p_{kh} < \delta_k \Rightarrow u_{kh} = 0 \text{ y } v_{kh} = 0$$

$$\frac{\partial C(p)}{\partial p_{kh}} - \mu_k = 0$$

$$\sum_{e \in E} [c'_e(x_e)] [\gamma_k \cdot \delta_e^h] = \mu_k \Rightarrow \sum_{e \in E} c'_e(x_e) \cdot \delta_e^h = \mu'_k = \frac{\mu_k}{\gamma_k}$$

So, attending to the Horst et al., 1990 optimality necessary condition, as well as, the Kuhn -Tucker conditions two different situations can be described (case I and case II). In the first one, the origin-destination pair k does not use the path h ($p_{kh}=0$), this is case I. In the second one, the origin -destination pair k uses the path h ($p_{kh}>0$). Here two options are presented: the transmission link transports the total allowed flow according with the survivability constraint ($p_{kh}=\delta_k$) showed in case II.a; or the transmission link does not saturate the survivability constraint ($p_{kh}<\delta_k$), it is showed in case II.b.

For case I the cost variation sum associated to the links shaping path h is lower bounded by an indicator parameter, μ'_k , equal to the Kuhn-Tucker multiplier associated to the demand balance equation divided into the total demand volume relative to origin -destination pair k .

For case II.a the cost variation sum associated to the links shaping path h is upper bounded by the same indicator parameter. Finally, for case II.b the cost variation sum associated to the links shaping path h is exactly equal to the indicator parameter.

So, the optimality conditions reflect the existence of three kinds of paths for the communication. Cheap paths that must be flow saturated, expensive paths that must be flow empty and one indicator path that transports the rest of the flow to satisfy the demand equation (1) in **RCP** problem. This indicator path acts as the frontier between the cheap and expensive paths.

Then the optimality conditions can be resumed in the following condition:

$$\begin{aligned} c'_{h_s}(p_{kh}) &\leq c'_{h_i}(p_{kh}) \leq c'_{h_v}(p_{kh}) \text{ for each} \\ h &= \{h_s(\text{saturated path}), h_i(\text{indicator path}), h_v(\text{void path})\} \in \mathcal{P}(k) \end{aligned}$$

The Kuhn-Tucker multipliers incorporate the following concepts:

- $\mu_k \sim c'_{h_i}$ Kuhn-Tucker multiplier associated to the demand balance equation refers to the cost variation sum along the indicator path.
- $\nu_{kh} \sim c'_{h_i} - c'_{h_s}$ Kuhn-Tucker multiplier associated to the survivability equation refers to the difference between the cost variation sum along the indicator path and the cost variation sum along the saturated path. It must be positive in the optimum otherwise the indicator path should be flow saturated and the saturated path should not be saturated.
- $u_{kh} \sim c'_{h_v} - c'_{h_i}$ Kuhn-Tucker multiplier associated to the non-negative path variable equation refers to the difference between the cost variation sum along the empty path and the cost variation sum along the indicator path. It must be positive in the optimum otherwise the empty path should transport positive flow and the indicator path should be empty.

3.1. Routing demand and capacity assignment procedure

To route the demand and assign enough capacity we make use of the Kuhn -Tucker conditions to construct the following algorithm:

Initialization

step 1. Grade(k) connectivity in transmission links is imposed by means of *grade(k)* minimum spanning trees, $MST^{\text{grade}(k)}$.

For the simpler case of only one origin -destination pair in absence of survivability constraints, the connectivity would correspond with the shortest path problem. For several origin -destination pairs in absence of survivability constraints, the solution would correspond with the minimum spanning tree problem. As the survivability constraints exist we will impose successive survivability *grade(k)* MST. First MST will be over all the nodes in the graph, second MST will be over all the nodes with survivability of grade 2, and in the same form for the rest of nodes with upper survivability grades until the pair of nodes with maximum grade of survivability.

step 2. It is set a first traffic pattern on the network looking for the path that incorporates an upper number of switching process in hubs. With this procedure it is incorporated the hierarchically most important part of the remaining investment: the transmitter, receiver and multiplexer costs. This path will be the indicator path and will carry the remaining demand until satisfying the balance equation (1) in **RCP**, the other paths will be saturated paths.

step 3. Check the optimality conditions. In case of not satisfaction go to iterations.

Iterations

step 1. $\mathcal{H}(k)$ is the set of origin-destination pairs not satisfying the optimality conditions.

step 2. Select a pair k^* , such as:

$$k^* = \max_{k \in \mathcal{H}(k)} \left\{ \max_{h_s} \left\{ \left| c'_{h_s}(p_{kh}^0) - c'_{h_i}(p_{kh}^0) \right| \right\} \right\}$$

step 3. For selected pair, k , the demand is reassigned to the paths by order of the cost variation sum $c'_{h_s}(p_{k^*h})$.

The initialization stage states the transmission links and a first flow pattern with the purpose of taking a first dimensioning of the transmission link capacity. After that we check the optimality conditions and through diverse iteration stages we improve the dimensioning of the transmission link capacity. It is guaranteed that each iteration reduces the flow pattern non -optimality.

With these criteria to select the individual fitness, the genetic algorithm is described using uniform crossover operators, random parents selection and ranking based replacement. The main characteristics of the genetic procedure are described bellow:

1. *Uniform crossover operators.* If both parent hub structures have located the hub in the same municipality this is maintained for the offspring. Otherwise it is located in the position of each of the parents with the same $\frac{1}{2}$ probability. Figure 7 depicts such this situation.

Figure 7. Uniform crossover operator

2. *Mutation operator.* A mutation process will imply the relocation of one hub from a municipality to other, in a specific region.
3. *Random parents selection.* All the individuals from the size N population are randomly selected. This will enrich the population genetic variety.
4. *Ranking based replacement.* We propose the use of a hypergeometric function allowing more probability of replacement for individuals with worse fitness and less probability of replacement for individuals with better fitness. So, the individual in ranking position $-i$, have a replacement probability equal to $q(1-q)^i$, being q the replacement probability of the worst individual.
5. *Based on population entropy stop criterion.* We follow the Grefenstette, 1987 suggestion as a good procedure to control the convergence in genetic algorithms. So, we calculate the entropy level related to each specific region, S_r , as function of the activation frequency, fr_i , of each urban node, $i \in M(r)$, as regional hub. After that the population entropy, $S_{population}$, can be calculated as the sum of the region entropies weighted with respect to the number of municipalities inside each region.

$$S_r = - \sum_{i \in M(r)} fr_i \cdot \ln(fr_i)$$

$$S_{population} = \sum_{r \in R} \frac{|M(r)|}{T} \cdot S_r = - \sum_{r \in R} \sum_{i \in M(r)} \frac{|M(r)|}{T} \cdot fr_i \cdot \ln(fr_i)$$

The genetic algorithm pseudocode and the implementation parameters can be found in the appendix.

4. Results

All the tests have been run on a 200 MHz PC Pentium MMX workstation. Test problems were randomly generated and parameters (number of nodes, arcs, regions and origin -destination pairs) were arbitrarily selected from the corresponding ranges. We solved a total of 600 test problems, which were grouped into different ranges according to the number of nodes, the terminal density and the region density.

Our GA was put in competition with the well-known two step rule based heuristic, using the shortest path algorithm as the routing and assignment capacity procedure suggested for the heuristic in Sexton et al., 1997 (see appendix I for pseudocode). We have considered grade -2 of survivability for the inter-regional network and grade -1 of survivability for the intra-regional network for all the cases. Table 2 summarizes the results.

Table 2. Summary of computational results

Problem range ^a			GA		Heuristic		Cost reduction
Nodes	Terminal density ^b	Region density ^c	Cost ^d	Time ^e	Cost ^d	Time ^e	
≤ 40	[0,1-0,55]	[0,25-0,4]	12,9	4,7	18,1	5,1	29%
		(0,4-0,6]	11,1	5,1	14,2	5,2	22%
	(0,55-1,0]	[0,25-0,4]	20,8	21,8	28,8	24,5	28%
		(0,4-0,6]	20,2	46,6	35,1	46,8	42%
[50-70]	[0,1-0,55]	[0,25-0,4]	22,1	37,8	34,5	62,1	36%
		(0,4-0,6]	29,5	92,6	45,2	95,5	35%
	(0,55-1,0]	[0,25-0,4]	43,9	138,8	60,7	150,1	28%
		(0,4-0,6]	54,6	292,8	81,6	303,2	33%
[80-100]	[0,1-0,55]	[0,25-0,4]	26,6	101,2	62,6	147	57,5%
		(0,4-0,6]	23,7	80	33,2	99	28,6%
	(0,55-1,0]	[0,25-0,4]	54,5	460,5	82,1	495,8	33,5%
		(0,4-0,6]	62,4	720	126,4	952,3	50,6%

^a Problem range is given by the number of nodes, the terminal density and the region density which are parameters based on the number of nodes (N), arcs (A), terminals (T) and regions (R).

^b Terminal density (TD) is calculated as $TD=T/N$

^c Region density (RD) is calculated as $RD=R/T$

^d Average network deployment cost in million of euros.

^e Average execution time in seconds.

The results show a good behavior for the GA. The computation time is always inside acceptable values, with respect to the problem decision horizon, note how we are dealing with a planning problem in which civil works have to be done. Moreover, the execution times grow with the number of nodes and with the terminal density.

With respect to the network deployment cost both terminal and region densities showed as important factors. Parameters as number of arcs do not significantly affect cost or time execution.

Finally, to conclude with the comparison between our GA and the two step rule based heuristic we have to emphasize how our algorithm beats the heuristic in all the ranges tested attending to the network extension cost as well as the computational time.

We have tested our GA in randomly generated problems as we ll as in real life scenarios. Next figure 8 shows the topological network (relative to the conduits construction) for the case of the Seville province sited in Andalusia (Spain), one of the most southern regions of Europe.

Figure 8. Physical network. Seville province case.

In the figure are displayed four regions (the most cost-effective territorial division) and the inter-regional network can be distinguished from the intra-regional network by means of the thickness line. The physical network showed in the figure 8 corresponds with the logical network (relative to the transmission links) showed in the next figure 9, where each of the four regions are associated to a different colour.

Figure 9. Logical network. Seville province case

Both figures reveal the existence of a ring network for the inter-regional network and the expansion in a logical star for the transmission links (corresponding to a physical tree for the conduits) in the intra-regional network.

This network deployment implies the investment ratios showed in table 3. These ratios can be used to predict required investment levels.

Table 3. Investment ratios

Total investment in the scenario	62.21 (million of euros)
Attained survivability level	100%
Investment volume with respect to the municipality population	43.54 (€/inhabitants)
Investment volume with respect to the total family housings	126.74 (€/total family housings)
Investment volume with respect to the primary family housings	159.55 (€/primary family housings)

For the test network construction, the Statistical Institute of Andalusia supported the input social - economical databases and the Cartographical Institute of Andalusia provided the topological data in ARC/INFO format. A more detailed discussion about these topics can be found in Cortes et al., 2001.

5. Conclusions

In this paper we have dealt with the design of a hierarchical fiber transport backbone network which is typically found for supporting broadband communications. The differentiation between the graph and network concepts, and in his turn, among links, arcs and paths increases the complexity of the modeling effort and it has not been dealt historically in the telecommunications network topological design. Here, a finer formulation of the global design has been presented relating these concepts. The formulation evaluates the physical and real costs associated to the links. The final model includes survivability and grade of service constraints too, and it is showed as a *NP-Hard* problem.

Moreover, a solution procedure based on a genetic algorithm has been presented, in which the feasible hub structures are used as population individuals. The procedure showed an adequate behavior related to the network deployment cost and the execution time. It is important to recall the generality of our problem that includes traditional OR problems such as hub location, conduit network design, Steiner trees or multicommodity flow problems. The construction of good lower bounds to deal with the worst case deviation of the procedure is currently being investigated.

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Appendix I: Pseudocodes

Two step rule based heuristic

Two step rule based heuristic is a engineering procedure habitual in industrial network deployment. It is defined by Sexton et al., 1997 in the following terms:

Step 1. Design of topology

- i- Provide a direct link between two nodes if the direct traffic (i.e. the single entry in the traffic matrix) between the two nodes is greater than a threshold.
- ii- If any nodes are left isolated, connect them to the nearest connected node.
- iii- Take all nodes with more than a threshold number of links after steps 1 and 2 and make them hubs, and then provide additional links to fully interconnect all hubs.

Step 2. Dimensioning of links

- i- Apply shortest path algorithm
- ii- Route traffic from traffic matrix and dimension links accordingly.
- iii- Rebalance oversized links onto undersized links.

Genetic Algorithm

The next procedure is iterative until a maximum number of iterations (*MAX_ITER*) or a minimum level of entropy (S_{min}).

Step 1. An initial size- N population is randomly selected.

Step 2. The fitness for each individual, H_i , is calculated according with a two-stage approach:

- i. Conduit infrastructure is stated by means of 2-trees techniques with survivability conditions, combining Steiner trees.
- ii. To route the demand and assign enough capacity we make use of the Kuhn-Tucker conditions through the routing demand and capacity assignment procedure (see section 3.1.)

- Step 3. Crossover and mutation operators are randomly selected according with their probabilities.
- In case of crossover operator two parents are randomly selected from the population.
 - In case of mutation operator one parent is randomly selected from the population.
- Step 4. The replaced population individual is selected by means of a hypergeometric rule, being $q \cdot (1-q)^i$ the replacement probability for the individual in ranking position- i and q the replacement probability for the worst individual.
- Step 5. Calculate the fitness for the new individual.

Appendix II: Algorithm parameters

Next table 4 summarizes the genetic algorithm and two step rule based heuristic parameters:

Table 4. Algorithm parameters

Genetic algorithm	
Population size, N	50
Crossover probability, p_c (mutation probability, $1-p_c$)	0.8 (0.2)
Ranking based replacement probability, q	0.2/0.7 ^a
Maximum number of iterations, MAX_ITER	50
Minimum entropy level, S_{min}	0.25
Two step rule based heuristic	
Threshold traffic flow	$3/2 \times (\text{average demand level})$

^a First iterations are done with replacement probability equal to 0.2, allowing the geographical area exploration, and later iterations are done with replacement probability equal to 0.7, increasing the quality solution exploitation.

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