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IMPROVING THE PERFORMANCE OF ORBITALLY-STABILIZING CONTROLS FOR THE FURUTA PENDULUM

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Abstract: This paper presents an approach to compute suboptimal nonlinear H_∞ controls for orbital stabilization of a certain class of underactuated systems. The control objective is to locally asymptotically stabilize the system around a predefined target orbit with a certain degree of L_2-Gain disturbance attenuation. Additionally, to solve the Hamilton-Jacobi-Bellman equation (HJBE) arising in the nonlinear H_∞ framework, an approximate solution is provided, based on a Galerkin-like approximation scheme. In particular, the methodology is applied to the well-known Furuta Pendulum system and simulation results are shown.

Keywords: Orbital stabilization, Non-linear H_∞ Control, Underactuated Systems, Galerkin approximation, Hamilton-Jacobi-Bellman equation.

1. INTRODUCTION

This paper considers the problem of characterization of periodic orbits using nonlinear H_∞ state feedback controllers, for a class of underactuated systems.

Orbital stabilization problems stem from the need of inducing stable oscillatory behavior in certain applications. Some applications where the natural operation mode is oscillatory are for instance, walking mechanisms, where the legs of the robot are intended to move under periodic motions, or power inverters, where the objective is obtaining a stable oscillating voltage at the output.

Several approaches to obtain stable oscillations in unforced nonlinear systems are available in the literature. Recent efforts focuss on orbital stabilization for underactuated systems. Thus, a Hamiltonian approach is employed in (Aracil *et al.* 2002), where energy shaping methods (Ortega *et al.* 2001) are used to obtain stable periodic oscillation for the Inverted Pendulum. A different approach can be found in (Canudas de Wit *et al.* 2002), where the control synthesis is accomplished in two parts. First an appropriate feedback is designed in order to transform the full dimensional system into one with lower dimension (zero-dynamics). Then, a controller is designed such that the low-dimension subsystem reaches a specified target orbit (Gorgnard and Canudas de Wit 2002).

One of the main drawbacks of these techniques is that none offer a clear indication of the performance of the system, and when the performance is unacceptable, they do not suggest a method to systematically enhance the performance. Nonlinear Optimal control methods would clearly fill this gap, but to best of our knowledge, little work has been done regarding the orbital stabilization problem.

One of these scarce works is presented in (Chung and Hauser 1994), where a nonlinear H_∞ approach is employed. In this work, under the assumption of the existence of a limit cycle for the system, and using a particular change of coordinates (generalized angle and generalized radius deviation) (Hauser and Chung 1994), a nonlinear L_2-Gain attenuation problem is solved to induce locally asymptotically stable periodic orbits. One major drawback of this approach is that, depending on the system and the target orbit, requires to find case-by-case the above mentioned coordinate transformation.

In this paper a setup for the computation of nonlinear state-feedback H_∞ controls to obtain locally asymptotically stable limit cycles is also presented. The controller synthesis is here formulated to induce a certain L_2-Gain attenuation level for the mapping $\omega \mapsto z$, where ω represents disturbances acting on the system and z is an error signal that weights the control action and deviations from the target orbit.

As a difference with other orbital stabilization methods available in the literature, this procedure gives an indication of the performance of the system. Particularly, as will be shown for the Furuta pendulum application, the proposed methodology yields a similar control law previously obtained in (Aracil *et al.* 2002) based on energy-shaping arguments. In this work, the authors introduced some *ad-hoc* controller constants, for which a computation criteria is given in this paper.

On the other hand, while linear H_∞ control is well developed and efficient algorithms for controller synthesis are available, the much more complicated nonlinear H_∞ control problem involves solving a Hamilton-Jacobi-Bellman partial differential equation (HJBE) which lacks, up to date, of a computational algorithm to obtain exact explicit solutions for this problem.

To overcome this difficulty an adaptation of a Galerkin-like approximation scheme for the solution of HJBE proposed in (Beard and McLain 1998) is introduced in this paper. Our contribution consists of introducing a novel type of approximating functions for the storage function, $V(x)$, that renders oscillatory behavior for the overall controlled system, while keeping (if such control exists) the required L_2-Gain attenuation level.

The paper is organized as follows. In section 2, the proposed problem is presented. In section 2.1 the problem is stated for second-order systems and the proposed controller synthesis methodology is explained, while section 2.2 is devoted to an adaptation of a Galerkin-like approximation scheme which is proposed for the solution of the HJBE for the orbital stabilization problem. To illustrate the performance of this methodology, an application to the Furuta Pendulum system is shown in section 3 and finally, some conclusions and additional commentaries are summarized in section 4.

2. PROBLEM FORMULATION

2.1 Problem Statement

In this section, the dissipative theory of nonlinear systems is applied to derive standard results of the nonlinear H_∞ control theory for later use. The dissipative theory (Van der Schaft 1989) generalizes the idea of passivity, and provides a means of robust stabilization for nonlinear systems. In particular, in this paper, this framework will be applied for robust stabilization around periodic orbits.

Let's consider first an affine-input second-order system of the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2) + g_1(x1, x2)u + g_2(x_1, x_2)\omega \end{aligned} \tag{1}$$

where $f(x)$, $g_1(x)$ and $g_2(x)$ are smooth functions, $x = [x_1, x_2]^T \in \mathcal{R}^2$, is the state of the system, $u \in \mathcal{R}$ is the control action and $\omega \in \mathcal{R}$ represents exogenous perturbations acting on the system.

With this set up, the objective is finding a state feedback stabilizing control $u(x)$ that drives the system to a target periodic orbit $\eta_t(x_1, x_2)$ defined in $\mathcal{R}^2$.

The target orbit can, in general, be described in terms of an orbit generator $\beta_d(x_1, x_2)$ such that η_t is an invariant set with respect to

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \beta_d(x_1, x_2) \end{aligned} \tag{2}$$

Designing the controller referred to a general target orbit as in (2) is shown to require a rather cumbersome coordinate transformation (Hauser and Chung 1994) that will be avoided by the moment in our formulation.

The approach proposed in this paper focusses initially on *harmonic target orbits*, by taking $\beta_d(x_1, x_2) = -\omega_c^2 x_1$. As will be shown, this simplifications allows as to greatly simplify the controller synthesis procedure and some considerations for the general case will be addressed later.

Taking β_d in this form, transforms system (2) into, what is called a *harmonic orbit generator*. It is fairly easy to show that the *harmonic orbit generator* exhibits a family of structurally-unstable elliptic-shaped orbits, η, which fulfill the whole state-space.

Thus, taking $\beta_d(x_1, x_2) = -\omega_c^2 x_1$ is not sufficient to fix an specific *harmonic target orbit*. Additionally it is necessary to specify the amplitude of oscillations μ. This way, the following definition is introduced

Definition 1 Let η be the set of harmonic limit cycles of the *harmonic orbit generator* previously introduced. The *harmonic target orbit*, $\eta_0(\omega_c, \mu)$, is defined as the elliptic-shaped orbit in η whose phase portrait matches $\eta_0(\omega_c, \mu)(x) = \{x \in \mathcal{R}^2 : \omega_c^2 x_1^2 + x_2^2 - \mu^2 = 0\}$.

Thus, if a state-feedback control transformation of the form

$$u = g_1(x_1, x_2)^{-1}(-x_1 + f(x_1, x_2)) + v \tag{3}$$

is applied to system (1) which, assuming that $g_1(x_1, x_2)^{-1}$ is well defined in the domain of interest, transforms system (1) into

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\omega_c^2 x_1 + g_2(x1, x2)\omega + v \end{aligned} \tag{4}$$

It is worth to mention that system (4), simplifies to an *harmonic orbit generator* for ($v = 0$, $\omega = 0$).

Our control objective will be now to find an state feedback control law $v(x)$ that locally asymptotically stabilizes the transformed system (4) around the target orbit $\eta_0(\omega_c, \mu)$.

At this point, the following definition is also required

Definition 2 A function V defined on a neighborhood $\mathcal{B}$ of $\eta_0(\omega_c, \mu)$ such that $V(x) : \mathcal{B} \subset \mathcal{R}^n \to \mathcal{R}$ is positive definite on $\eta_0(\omega_c, \mu)$ if $V(x) = 0 \ \forall \ x \in \eta_0$ and $V(x) > 0 \ \forall \ x \in \mathcal{B} \backslash \eta_0(\omega_c, \mu)$ on an open neighborhood of $\eta_0(\omega_c, \mu)$.

According to the dissipative theory formalism, system (4) is said to be dissipative around the target orbit $\eta_0(\omega_c, \mu)$ with respect to the supply rate $r(\cdot, \cdot)$, if there exists a positive definite energy storage function $V(x)$ on $\eta_0(\omega_c, \mu)$, satisfying the following dissipation inequality.

$$0 \le V(x(t)) \le \int_0^t r(\omega(\xi), z(\xi))d\xi \tag{5}$$

for all t and for all $x(\cdot)$, $z(\cdot)$ and $\omega(\cdot)$ satisfying equation (4), where $z(\cdot)$ is the penalized output, that in our case will be taken

$$z = \begin{bmatrix} h(x) \\ \sigma v \end{bmatrix} \tag{6}$$

with the output function $h(x) = \omega_c^2 x_1^2 + x_2^2 - \mu^2$ that, as can be easily shown, vanishes on the target orbit $\eta_0(\omega_c, \mu)$.

Thus, z can be interpreted as a signal that penalizes the control action, and the deviation from the target orbit, $\eta_0(\omega_c, \mu)$, by means of the output function, $h(x)$. The weighting coefficient σ is introduced for the trade-off between performance and control effort.

If the supply rate $r(\cdot, \cdot)$ is taken to be

$$r(\omega, z) = \frac{1}{2}\gamma^2 \omega^T \omega - \frac{1}{2} z^T z \tag{7}$$

substituting in (5) and taking $t \to \infty$ yields

$$\frac{1}{2}\int_0^\infty \|z(\tau)\|^2 d\tau < \frac{\gamma^2}{2}\int_0^\infty \|\omega(\tau)\|^2 d\tau \tag{8}$$

The last equation represents, by definition, an L_2-Gain attenuation problem for the mapping $\omega \mapsto z$, where the lowest γ satisfying equation (8) is called the H_∞-norm of the system.

This problem can be solved in the Nonlinear H_∞ framework according to the following theorem

Theorem 1 Consider the nonlinear system (4) and suppose there exist a positive definite function $V(x)$ on $\eta_0(\omega_c, \mu)$ satisfying

$$\begin{gathered} \frac{\partial V}{\partial x}F(x) + \frac{1}{2}\frac{\partial V}{\partial x}\left(\frac{1}{\gamma^2}G_2(x)G_2^T(x) - \frac{1}{\sigma^2}G_1(x)G_1^T(x)\right)\frac{\partial^T V}{\partial x} + \\ \frac{1}{2}H^T(x)H(x) \le 0 \end{gathered} \tag{9}$$

where

$$F(x) = \begin{bmatrix} x_2 \\ f(x) \end{bmatrix} \quad G_1(x) = \begin{bmatrix} 0 \\ g_1(x) \end{bmatrix}$$

$$G_2(x) = \begin{bmatrix} 0 \\ g_2(x) \end{bmatrix} \quad H(x) = \begin{bmatrix} 0 \\ h(x) \end{bmatrix}$$

then the control $v = -\frac{1}{\sigma^2}G_1^T(x)\frac{\partial V}{\partial x}$ renders the L_2-Gain locally around $\eta_0(\omega_c, \mu)$ for the mapping $\omega \mapsto z$ in system (4) less than γ.

Conversely, if there exists a smooth feedback $v(x) = \Gamma(x)$, such that there exists a smooth storage function, $V(x)$, positive definite on $\eta_0(\omega_c, \mu)$ with supply rate as in (7), then $V(x)$ is also a solution to the HJBE (9).

For a proof of the theorem, the reader is referred to (Van der Schaft 1989) and (Chung and Hauser 1994).

2.2 Solutions of HJBE for orbital stabilization

Using the definitions in section 2.1, the control objective can be more precisely defined in the nonlinear H_∞

framework. Thus, our control objective will be to find a state feedback stabilizing control law $v(x)$, such that the L_2-gain of the mapping $\omega \mapsto z$ in system (4) around the target orbit $\eta_0(\omega_c, \mu)$ is less than a certain γ.

Theorem 1. gives us the tool to find, if it exits, such a control law. All its needed is finding a storage function $V(x)$ satisfying the HJBE (9).

In this section an algorithm to obtain an approximate solution of the HJBE (9) for the orbital stabilization problem is proposed. The method, based on the algorithm described in (Beard and McLain 1998), constructs an approximate solution to the HJBE by using a two-step successive Galerkin approximation scheme.

The algorithm can be stated as follows:

Initialization: Let $v^{(0)} = 0$ be the initial control law that stabilizes the undriven system (4) on the target orbit $\eta_0(\omega_c, \mu)$.
For $i = 0 : \infty$, *Set* $\omega^{(i,0)} = 0$

For $j = 0 : \infty$, *Solve* (*see equation* (10))

$$\frac{\partial V^{(i,j)}}{\partial x}(F(x) + G_1 v^{(i)} + G_2 \omega^{(i,j)}) + |h|^2 + \\ +\sigma^2 |u^{(i)}|^2 - \gamma^2 |\omega^{(i,j)}|^2 = 0$$

$$Set\ \omega^{(i,j+1)} = \frac{1}{\gamma^2} G_2^T \frac{\partial V^{(i,j)}}{\partial x}$$

$$Set\ v^{(i+1)} = -\frac{1}{\sigma^2} G_1^T \frac{\partial V^{(i,\infty)}}{\partial x}$$

The algorithm consists of two nested iterations corresponding to the min - max problem associated with the nonlinear H_∞ control paradigm. The inner loop iterates to compute an approximate solution of the nonlinear HJBE as the solution to a subsidiary linear partial differential equation, which can be easily solved in terms of a Galerkin-like expansion for $V(x)$. The inner loop also computes the worst possible disturbance for a given control. The outer loop updates the control loop to improve the performance for a given worst case disturbance.

In order to obtain an implementable algorithm, the solution to the linear equation (2.2) is approximated by a Galerkin-like expansion scheme, namely

$V^{(i,j)} = \sum_{k=1}^{\infty} c_k^{(i,j)} \phi_k(x)$.

The coefficients $c_k^{(i,j)}$ are thus computed to induce minimum projection error onto the linear space spanned by $\{\phi_k(x)\}_1^\infty$.

It is worth to mention that this algorithm is meaningful in this context since, as opposed to other numerical methods available in the literature, it is a projection algorithm and allows us to take control on the final shape of the solution.

Our contribution to this numerical procedure lays on the selection of the approximating functions, $\phi_k(x)$. In order to obtain a solution that induces orbital stabilization of the target harmonic orbit, $\eta_0(\omega_c, \mu)$, the approximating functions are selected such that $\{\phi_k(x)\}_{k=1}^\infty$ is a complete set of positive definite on $\eta_0(\omega_c, \mu)$, basis functions on $\mathcal{R}^2$.

In particular, functions of the form $\phi_k(x) = (\omega_c^2 x_1^2 + x_2^2 - \mu^2)^{2k}$ with $\mu > 0$, have been tested. Figure (1) shows the shape of these approximating functions.

Fig. 1. Shape of approximating functions, $\phi_k(x) = (\omega_c^2 x_1^2 + x_2^2 - \mu^2)^{2k}$ with $\rho^2 = \omega_c^2 x_1^2 + x_2^2$ for different values of k

Thus, the Galerkin approximation to equation (2.2) is obtained as the truncated expansion $V_N^{(i,j)} = \sum_{k=1}^{N} c_k^{(i,j)} \phi_k(x)$ where N is the desired *degree of approximation*, and the coefficients $c_k^{(i,j)} \in \mathcal{R}$ are obtained as the solution of the linear algebraic equations (10).

Where Ω is a compact set of the stability region of the initial stabilizing control, $v^{(0)}$, around the harmonic target orbit $\eta_0(\omega_c, \mu)$.

The resulting feedback control can be written as

$$v_N^{(i,j)} = -\frac{1}{\sigma^2} \sum_{k=1}^{N} c_k^{(i,j)} g_1^T \frac{\partial \phi_k(x)}{\partial x} \qquad (11)$$

3. ILLUSTRATIVE EXAMPLE: THE FURUTA PENDULUM

The method proposed in this paper is presented by its application to the well known Furuta Pendulum system

$$\int_{\Omega}\left(\frac{\partial V^{(i,j)}}{\partial x}(F(x)+G_1u^{(i)}+G_2\omega^{(i,j)})+|H|^2+|u^{(i)}|^2-\gamma^2|\omega^{(i,j)}|^2\right)\phi_k(x)dx=0 \quad (10)$$

(See (Åström and Furuta 2000) for a description). Let the lenght of the pendulum be l, the mass of the weight M, the mass of the pendulum m, its moment of inertia J_p. The length of the arm is R. The angle of the pendulum θ is defined to be zero in the upright position, and the angle of the arm, ϕ, is positive when the arm is moving in positive clockwise direction. Additionally, u, is the control torque applied to the system.
By introducing the coefficients

$$a = J_p + Ml^2 \qquad c = Mrl$$
$$b = J + MR^2 + mR^2 \qquad d = lg(M + m/2)$$

The equations of this system take the form (Åström and Furuta 2000)

$$a\ddot{\theta} - a\dot{\phi}^2\sin(\theta)\cos(\theta) + c\ddot{\phi}\cos(\theta) - d\sin(\theta) = 0 \quad (12)$$
$$c\ddot{\theta}\cos(\theta) - c\dot{\theta}^2\sin(\theta) + 2a\dot{\theta}\dot{\phi}\sin(\theta)\cos(\theta) +$$
$$+(b + a\sin^2(\theta))\ddot{\phi} = u$$

The objective of the control strategy proposed here is to obtain stable harmonic oscillations of the pendulum around the upright position, while keeping bounded arm displacements.

Thus, if we intend to induce stable harmonic oscillations for this, systems, equations (12) must be transformed. The first step consists of partially linearizing the system by taking from the first equation in (12)

$$\ddot{\theta} = \dot{\phi}^2\sin(\theta)\cos(\theta) - \frac{c}{a}\ddot{\phi}\cos(\theta) + \frac{d}{a}\sin(\theta)$$

which together with the second equation in (12) yields

$$c\left(\dot{\phi}^2\sin(\theta)\cos(\theta) - \frac{c}{a}\ddot{\phi}\cos(\theta) + \frac{d}{a}\sin(\theta)\right)\cos(\theta) -$$
$$-c\dot{\theta}^2\sin(\theta) + 2a\dot{\theta}\dot{\phi}\sin(\theta)\cos(\theta) +$$
$$+(b + a\sin^2(\theta))\ddot{\phi} = u \quad (13)$$

making $\dot{\phi_2} = v$ in this last equation, the system can be posed in this much more convenient form

$$\dot{\theta}_1 = \theta_2 \quad (14)$$
$$\dot{\theta}_2 = \frac{d}{a}\sin(\theta_1) - \frac{c}{a}\cos(\theta_1)v + \sin(\theta_1)\cos(\theta_1)\dot{\phi}^2$$
$$\dot{\phi}_1 = \phi_2$$
$$\dot{\phi}_2 = v$$

where the term $\sin(\theta)\cos(\theta)\dot{\phi}^2$ represents the influence of the rotating effect in the pendulum dynamics. Taking this influence term as an external perturbation acting on the system, the first two equations in (14) can be cast into the form in (1) by taking

$$f(\theta_1) = \frac{d}{a}sin(\theta_1) \quad (15)$$
$$g_1(\theta_1) = -\frac{c}{a}\cos(\theta_1)$$
$$g_2(\theta_1) = \sin(\theta_1)\cos(\theta_1)$$

with the perturbation term $\omega = \dot{\phi}^2$.

Leaving for the moment the arm dynamics, (ϕ_1, ϕ_2), for later discussion, the controller synthesis procedure in this paper can be applied to pendulum dynamics, (θ_1, θ_2) in (14).

First, a transformation as in (3) is employed yielding

$$\dot{\theta}_1 = \theta_2 \quad (16)$$
$$\dot{\theta}_2 = -\omega_c^2\theta_1 + v + g_2(\theta_1)\omega$$

As it has already been discussed, this system exhibits a set of structurally unstable harmonic orbits for the undriven disturbance-free system ($v = 0$ and $\omega = 0$).

Among these orbits, the one with period 2 seconds ($\omega_c = \pi$) and amplitude $\mu = 1$ was selected as the harmonic target orbit.

With this setup, the nonlinear H_∞ controller was synthesized selecting a disturbance level $\gamma = 1.5$ and a *degree of approximation* $N = 3$, yielding the following control law

$$v(\theta_1, \theta_2) = -\theta_2(1.96((\theta_1^2 + \theta_2^2)^2 - 1) -$$
$$+0.63((\theta_1^2 + \theta_2^2)^2 - 1)^3 +$$
$$+0.023((\theta_1^2 + \theta_2^2)^2 - 1)^5) \quad (17)$$

it can be shown that the arm displacement , ϕ, exhibits an oscillating behavior with a drift. In order to eliminate this drift, an additional damping term, $-k_a\phi_2$ with $K_a > 0$, is included. With this term, a stable oscillatory behavior is obtained for both the pendulum and the arm dynamics as it is shown in figures 2 and 3.

It is worth to mention that when the *degree of approximation* is taken $N = 1$, the controller structure takes the form

$$u(\theta_1, \theta_2) = \frac{a}{c\cos(\theta_1)}(\omega_c^2\theta_1 + \frac{d}{a}\sin(\theta_1) -$$
$$-4c_1\frac{c}{a}\cos(\theta_1)(\omega_c^2\theta_1^2 + \theta_2^2 - \mu^2)\theta_2) \quad (18)$$

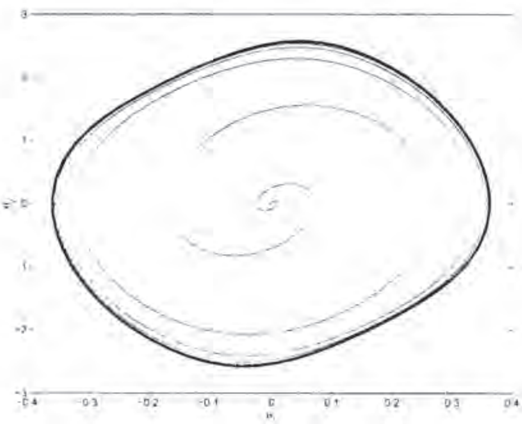

Fig. 2. Phase portrait for the Furuta Pendulum system. Angular displacement θ

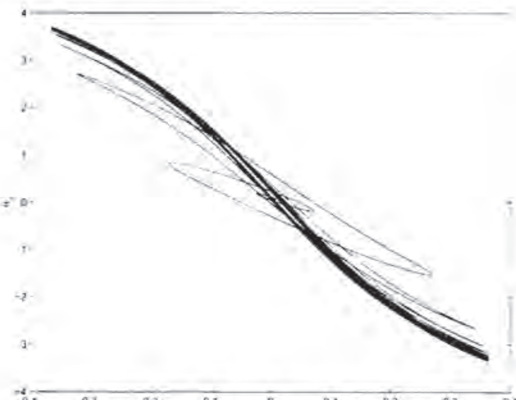

Fig. 3. Phase portrait for the Furuta Pendulum system. Arm displacement ϕ

where c_1 represents the coefficient of the approximating function in (11).

This control law is essentially the same presented in (Aracil *et al.* 2002), except for the damping term $4c_1\frac{c}{a}\cos(\theta_1)$, which was a constant term in the referenced paper. The advantage of the procedure introduced in this paper, is that such coefficient is no longer introduced *ad-hoc*, but a procedure for its computation is given.

4. CONCLUSIONS

In this paper, a technique to obtain stable robust oscillations in nonlinear systems has been presented. As an improvement with respect to similar techniques, the problem is formulated in the nonlinear H_∞ framework in terms of a certain L_2-Gain level of disturbance rejection, which gives an indication of the performance of the system.

Additionally, to solve the Hamilton-Jacobi-Bellman equation arising in the associated nonlinear H_∞ problem, an approximate solution is also provided based on a Galerkin-like approximation scheme.

The method can be applied to second order systems, as well as for certain kind of higher order underactuated systems. In particular, in this paper simulation results are shown for the Furuta Pendulum system. For this system and under certain assumptions, the method is shown to yield a very similar control law obtained by energy-shaping arguments in (Aracil *et al.* 2002), where the control law was expressed in terms of some constants, for which no selection criteria was given. As a main contribution of this work, a procedure is given to obtain such constant with a L_2-Gain disturbance attenuation criteria.

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