Contour Approximation with P Systems

Rodica Ceterchi[1], David Orellana-Martín[2], and Gexiang Zhang[3]

[1] University of Bucharest
Faculty of Mathematics and Computer Science
14 Academiei St, 010014 Bucharest, Romania
[2] Research Group on Natural Computing
Dept. of Computer Science and Artificial Intelligence
Universidad de Sevilla
E.T.S.I Informática, Avda. Reina Mercedes s/n 41012, Sevilla, Spain
[3] Research Center for Artificial Intelligence, Chengdu University of Technology,
Chengdu 610059 China

Summary. We model the problem of contour approximation using Hilbert's space filling curve, with a novel type of parallel array rewriting rules. We further use their pattern to introduce a special type of tissue P system, with novel features, among which is controlling their behavior with input. We propose some further developments.

1 Introduction

Space-filling curves (SFCs) were studied by mathematicians as a curiosity, since Peano discovered the first one in 1980 [7]. A year after this, Hilbert presented a much simpler curve [3]. Many properties and aspects of them were studied, and many other versions appeared in the literature, see for instance the monograph [8]. Lately, interesting applications to problems in Computer Science have been developed [1].

The finite approximations of the Hilbert curve can be described by words over a four letter alphabet $\{u, r, d, l\}$, letters which stand for the four directions in which a writing head can move in the lattice plane and draw a unit line. Formal language instruments have been used to describe families of SFC words.

In a series of papers we have studied the generation of such words with parallel rewriting controled by P systems, and we have proposed to model more complex applications of them. This short paper (rather a sketch) tries to accomplish this last purpose.

In the paper [2] we have proposed parallel array rewriting for the generation of Hilbert words. In Section 3 we modify the rules, introducing an external control, in order to generate contour approximations, after the ideas of [1] presented briefly in Section 2. Section 4 illustrates the generation of tissue P systems to model the rules.

In Section 5 a new variant of tissue P systems where in each transition step it can obtain inputs from external systems. Section 6 is devoted to a P system of the above variant capable of generating a contour approximation based on the Hilbert Curve.

Finally, in Section 7 we propose further developments of the ideas of this paper, and in **??** we indicate a possible development for crisis management.

2 Problem presentation

In his monograph [1], Bader proposes to use SFCs to maintain data about 2D objects. Figure 1 from his monograph illustrates the fact that, among several possible traversals of the quadtree associated to a 2D closed contour, a traversal based on the Hilbert SFC is more suited for applications which process the data in the quadtree nodes, since it has the locality property.

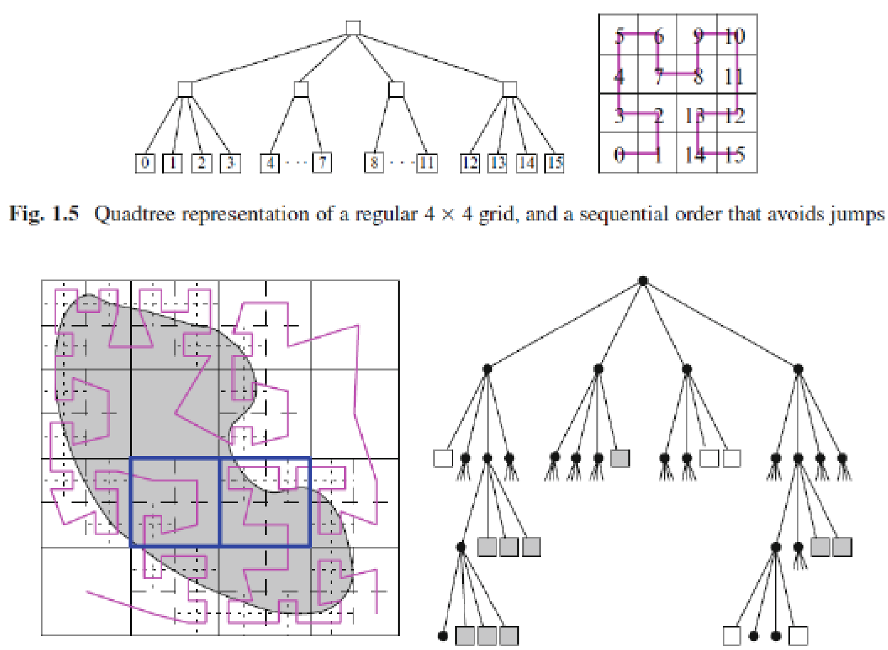

Fig. 1. Figures extracted from monograph [1] (Ch. 1, Pg. 6)

As Figure 2 illustrates, the approximation of the contour is obtained by pasting together pieces of the Hilbert curve, of different orders. Each piece of the Hilbert SFC is obtained in the usual manner, by repeated subdivisions of each sub-square where necessary, that is only for those sub-squares that cross the contour (border) of the 2D picture.

In the following, we propose to model the contour approximation generated by this method, first with arrays, next with P systems.

3 Contour generation with array rewriting

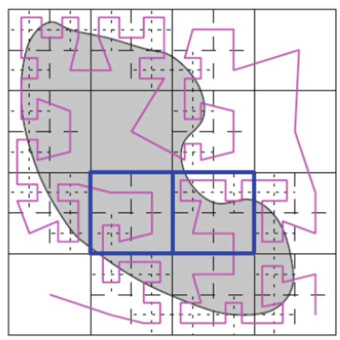

Fig. 2. The approximation of the contour is based on the pasted pieces of the Hilbert curve generated in the process [1]

Note: A combination of term rewriting rules with some constraints (called control) specifying the possible rewrite positions.

Consider the alphabet of non-terminals

$$\bar{N} = \{Ud, Ur, Ru, Rl, Ld, Lr, Du, Dl\},$$

where each element is a 1×1 array.

Denote by Γ the array morphism of the eight rewriting rules bellow:

$$U* \rightarrow \begin{matrix} Ur & Ud \\ Ru & L* \end{matrix} \qquad with\ * = d, r \tag{1}$$

$$R* \rightarrow \begin{matrix} D* & Rl \\ Ur & Ru \end{matrix} \qquad with\ * = u, l \tag{2}$$

$$L* \rightarrow \begin{matrix} Ld & Dl \\ Lr & U* \end{matrix} \qquad with\ * = d, r \tag{3}$$

$$D* \to \begin{matrix} R* & Ld \\ Du & Dl \end{matrix} \qquad with * = u, l \tag{4}$$

Denote by F the array morphism of the eight rewriting rules below:

$$Ud \to d, Ur \to r, Ld \to d, Lr \to r, Ru \to u, Rl \to l, Du \to u, Dl \to l \tag{5}$$

We have shown in [2] that $F(\Gamma^n(Ur))$ = the nth Hilbert word $H_n r$ in array representation.

We will now enlarge the set of non-terminals with one more symbol, #, standing for the blank array, and the set of Γ rules, as follows:

$$U^1* \to \begin{matrix} Ur & Ud \\ Ru & L* \end{matrix} \qquad with * = d, r \tag{6}$$

$$R^1* \to \begin{matrix} D* & Rl \\ Ur & Ru \end{matrix} \qquad with * = u, l \tag{7}$$

$$L^1* \to \begin{matrix} Ld & Dl \\ Lr & U* \end{matrix} \qquad with * = d, r \tag{8}$$

$$D^1* \to \begin{matrix} R* & Ld \\ Du & Dl \end{matrix} \qquad with * = u, l \tag{9}$$

$$U^0* = \#^0 \to \begin{matrix} \#^0 & \#^0 \\ \#^0 & \#^0 \end{matrix} = \#^1 \qquad with * = d, r \tag{10}$$

$$R^0* \to \#^1 \qquad with * = u, l \tag{11}$$

$$L^0* \to \#^1 \qquad with * = d, r \tag{12}$$

$$D^0* \to \#^1 \qquad with * = u, l \tag{13}$$

where $\#^0$ stands for the blank 1×1 array. Since we will have parallel rewriting rules, and we want to keep the growing dimension of the array, we will also have rules

$$\#^0 \to \begin{matrix} \#^0 & \#^0 \\ \#^0 & \#^0 \end{matrix} = \#^1 \tag{14}$$

which rewrite blanks to blanks. Two succesive applications of the rule above will produce $\#^2$, the 4×4 array filled with blanks, and so on.

(The notation $\#^n$ will be useful when we pass to the string representation.)

Each application of array rewriting rules (6)-(9) will correspond to a **division** of a square into four subsquares.

A division step will be followed by a **recognizer** step: each subsquare 'checks' whether it intersects the contour, in which case it gets a 1 superscript, or not, in which case it gets a 0 superscript, and will be rewritten accordingly at the next derivation.

Let us illustrate this with the case of the Figure 2 above:

$$U^1* \to \begin{array}{cc} Ur & Ud \\ Ru & L* \end{array} \leadsto \begin{array}{cc} U^1r & U^1d \\ R^1u & L^1* \end{array} \to \begin{array}{cccc} Ur & Ud & Ur & Ud \\ Ru & Lr & Ru & Ld \\ Du & Rl & Ld & Dl \\ Ur & Ru & Lr & U* \end{array} \leadsto \begin{array}{cccc} U^1r & U^1d & U^1r & U^0d \\ R^1u & L^0r & R^1u & L^0d \\ D^1u & R^1l & L^1d & D^1l \\ U^0r & R^1u & L^1r & U^1* \end{array} \to$$

$$\to \begin{array}{cccccccc} Ur & Ud & Ur & Ud & Ur & Ud & \#^0 & \#^0 \\ Ru & Lr & Ru & Ld & Ru & Lr & \#^0 & \#^0 \\ Du & Rl & \#^0 & \#^0 & Du & Rl & \#^0 & \#^0 \\ Ur & Ru & \#^0 & \#^0 & Ur & Ru & \#^0 & \#^0 \\ Ru & Ld & Dl & Rl & Ld & Dl & Rl & Ld \\ Du & Dl & Ur & Ru & Lr & Ud & Du & Dl \\ \#^0 & \#^0 & Du & Rl & Ld & Dl & Ur & Ud \\ \#^0 & \#^0 & Ur & Ru & Lr & Ur & Ru & L* \end{array} \leadsto \begin{array}{cccccccc} Ur & Ud & Ur & Ud & U^0r & U^0d & \#^0 & \#^0 \\ Ru & L^0r & R^0u & Ld & Ru & L^0r & \#^0 & \#^0 \\ Du & R^0l & \#^0 & \#^0 & Du & R^0l & \#^0 & \#^0 \\ Ur & R^0u & \#^0 & \#^0 & Ur & R^0u & \#^0 & \#^0 \\ Ru & Ld & D^0l & R^0l & Ld & Dl & Rl & L^0d \\ D^0u & Dl & Ur & R^0u & L^0r & U^0d & Du & D^0l \\ \#^0 & \#^0 & Du & Rl & L^0d & D^0l & Ur & U^0d \\ \#^0 & \#^0 & U^0r & Ru & Lr & Ur & Ru & L^0* \end{array} \to \cdots$$

After 3rd division-recognizer steps.

We generate the *nw* subsquare of the picture, illustrated below.

We have marked only the 0 superscript of non-terminals, for more clarity.

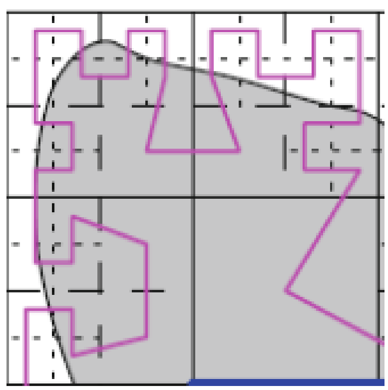

Fig. 3. Detail of the *nw* subsquare of the initial picture.

4 Membrane division rules

We will introduce dynamic P systems, which 'grow' by repeated membrane division rules, which will correspond to the subdivisions of the squares.

4.1 In string format

We have from [2] the linearization procedure which allows to pass from the 2D array representation to the linear one.

$$[U] \to [Ru]_{sw}[Ur]_{nw}[Ud]_{ne}[L*]_{se} \qquad [R] \to [Ur]_{sw}[Ru]_{se}[Rl]_{ne}[D]_{nw}$$

$$[L] \to [Dl]_{ne}[Ld]_{nw}[Lr]_{sw}[U]_{se} \qquad [D] \to [Ld]_{ne}[Dl]_{se}[Du]_{sw}[R]_{nw}$$

4.2 In 2D array format

Each nonterminal array is in a membrane which has the possibility of dividing itself into 4 membranes organized in a tissue manner.

The array rewriting rules become division rules for membranes, resulting in tissue P systems:

$$[U^1*] \to \begin{matrix}[Ur] & [Ud]\\ [Ru] & [L*]\end{matrix} \qquad with\ * = d, r \tag{15}$$

$$[R^1*] \to \begin{matrix}[D*] & [Rl]\\ [Ur] & [Ru]\end{matrix} \qquad with\ * = u, l \tag{16}$$

$$[L^1*] \to \begin{matrix}[Ld] & [Dl]\\ [Lr] & [U*]\end{matrix} \qquad with\ * = d, r \tag{17}$$

$$[D^1*] \to \begin{matrix}[R*] & [Ld]\\ [Du] & [Dl]\end{matrix} \qquad with\ * = u, l \tag{18}$$

An example of 3 succesive subdivisions, which will finally lead to the picture:

$$[U^1*] \to \begin{matrix}[Ur] & [Ud]\\ [Ru] & [L*]\end{matrix} \rightsquigarrow \begin{matrix}[U^1r] & [U^1d]\\ [R^1u] & [L^1*]\end{matrix} \to \begin{matrix}[Ur] & [Ud] & [Ur] & [Ud]\\ [Ru] & [Lr] & [Ru] & [Ld]\\ [Du] & [Rl] & [Ld] & [Dl]\\ [Ur] & [Ru] & [Lr] & [U*]\end{matrix} \rightsquigarrow \begin{matrix}[U^1r] & [U^1d] & [U^1r] & [U^0d]\\ [R^1u] & [L^0r] & [R^1u] & [L^0d]\\ [D^1u] & [R^1l] & [L^1d] & [D^1l]\\ [U^0r] & [R^1u] & [L^1r] & [U^1*]\end{matrix} \to$$

$$\to \begin{matrix}[Ur] & [Ud] & [Ur] & [Ud] & [Ur] & [Ud] & [\#^0] & [\#^0]\\ [Ru] & [Lr] & [Ru] & [Ld] & [Ru] & [Lr] & [\#^0] & [\#^0]\\ [Du] & [Rl] & [\#^0] & [\#^0] & [Du] & [Rl] & [\#^0] & [\#^0]\\ [Ur] & [Ru] & [\#^0] & [\#^0] & [Ur] & [Ru] & [\#^0] & [\#^0]\\ [Ru] & [Ld] & [Dl] & [Rl] & [Ld] & [Dl] & [Rl] & [Ld]\\ [Du] & [Dl] & [Ur] & [Ru] & [Lr] & [Ud] & [Du] & [Dl]\\ [\#^0] & [\#^0] & [Du] & [Rl] & [Ld] & [Dl] & [Ur] & [Ud]\\ [\#^0] & [\#^0] & [Ur] & [Ru] & [Lr] & [Ur] & [Ru] & [L*]\end{matrix} \rightsquigarrow \begin{matrix}[Ur] & [Ud] & [Ur] & [Ud] & [U^0r] & [U^0d] & [\#^0] & [\#^0]\\ [Ru] & [L^0r] & R^0u & Ld & Ru & L^0r & \#^0 & \#^0\\ [Du] & [R^0l] & [\#^0] & [\#^0] & [Du] & [R^0l] & [\#^0] & [\#^0]\\ [Ur] & [R^0u] & [\#^0] & [\#^0] & [Ur] & [R^0u] & [\#^0] & [\#^0]\\ [Ru] & [Ld] & [D^0l] & [R^0l] & [Ld] & [Dl] & [Rl] & [L^0d]\\ [D^0u] & [Dl] & [Ur] & [R^0u] & [L^0r] & [U^0d] & [Du] & [D^0l]\\ [\#^0] & [\#^0] & [Du] & [Rl] & [L^0d] & [D^0l] & [Ur] & [U^0d]\\ [\#^0] & [\#^0] & [U^0r] & [Ru] & [Lr] & [Ur] & [Ru] & [L^0*]\end{matrix} \to \cdots$$

With labels on membranes:

$$[U] \to [Ru]_{sw}[Ur]_{nw}[Ud]_{ne}[L*]_{se} \qquad [R] \to [Ur]_{sw}[Ru]_{se}[Rl]_{ne}[D]_{nw}$$

$$[L] \to [Dl]_{ne}[Ld]_{nw}[Lr]_{sw}[U]_{se} \qquad [D] \to [Ld]_{ne}[Dl]_{se}[Du]_{sw}[R]_{nw}$$

$$[U^1*] \to \begin{matrix} [Ur]_{nw} & [Ud]_{ne} \\ [Ru]_{sw} & [L*]_{se} \end{matrix} \qquad \textit{with } * = d, r \tag{19}$$

$$[R^1*] \to \begin{matrix} [D*]_{nw} & [Rl]_{ne} \\ [Ur]_{sw} & [Ru]_{se} \end{matrix} \qquad \textit{with } * = u, l \tag{20}$$

$$[L^1*] \to \begin{matrix} [Ld]_{nw} & [Dl]_{ne} \\ [Lr]_{sw} & [U*]_{se} \end{matrix} \qquad \textit{with } * = d, r \tag{21}$$

$$[D^1*] \to \begin{matrix} [R*]_{nw} & [Ld]_{ne} \\ [Du]_{sw} & [Dl]_{se} \end{matrix} \qquad \textit{with } * = u, l \tag{22}$$

4.3 Labels for membranes, and memory

In the above we have used labels $\{sw, nw, ne, se\}$ standing for the obvious notation for corners of a square: southwest, northwest, etc.

Of course, binary labels could be used instead, with interesting properties. For instance:

$$sw = 00,\ nw = 01,\ ne = 11,\ se = 10.$$

This has the property that any 2 adjacent squares have labels differing in only 1 bit (Gray code on 2 bits). Many binary codes can be associated to SFCs.

We will **concatenate (properly!) labels at every derivation step**, such that each membrane: on one hand inherits the label of its 'parent', and gets a label stating what 'son' it is. In this way, membranes have **memory**. Division rules (with labels) will be of the form:

$$[\ ]_\alpha \to [\ ]_{\alpha 00}\ [\ ]_{\alpha 01}\ [\ ]_{\alpha 11}\ [\ ]_{\alpha 10}$$

5 Tissue P systems with evolutional communication rules, extended division rules and external inputs

Definition 1. *Let $\Pi = (\Gamma, H, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, \mathcal{I})$ be a tissue P system with evolutional communication rules, extended division and r external inputs rules of degree q, where:*

1. *Γ is a finite alphabet;*
2. *H is the set of labels $\{1, \ldots, q\}$;*
3. *$\mathcal{M}_1, \ldots, \mathcal{M}_q$ are multisets over Γ;*
4. *$\mathcal{R}$ is the set of rules of the following forms:*
   a) $[\,u\,]_{h_1}\,[\,v\,]_{h_2} \to [\,v'\,]_{h_1}\,[\,u'\,]_{h_2}, h_1, h_2 \in H, u, v, u', v' \in M_f(\Gamma), |u| + |v| > 0, |u| = 0 \to |u'| = 0, |v| = 0 \to |v'| = 0$ (evolutional communication rules);

b) $[\,a\,]_h \rightarrow [\,a_1\,]_{h_1} \dots [\,a_s\,]_{h_s}, h, h_1, \dots, h_s \in H, a, a_1, \dots, a_s \in \Gamma$ *(extended division rules);*

5. $\mathcal{I}$ *be a set of elements* $(\Gamma_i, H_i, \mathcal{R}_i), 1 \leq i \leq r$ *such that:*
   a) $\Gamma_i \subseteq \Gamma$*;*
   b) $H_i \cap H = \emptyset \wedge H_i \cap H_j = \emptyset, i \neq j$*.*
   c) $\mathcal{R}_i$ *is the set of rules of the following form:*
      i. $[\,u\,]_{h_1} [\,v\,]_{h_2} \rightarrow [\quad]_{h_1} [\,u'\,]_{h_2}, h_1 \in H_i, h_2 \in H, u \in M_f(\Gamma_i), v, u' \in M_f(\Gamma), |u| > 0$ *(evolutional communication rules);*

A tissue P system with communication rules, extended division rules and r inputs

$$\Pi = (\Gamma, H, \mathcal{M}_1, \dots, \mathcal{M}_q, \mathcal{R}, \mathcal{I})$$

of degree q can be viewed as a set of q cells such that $\mathcal{M}_1, \dots, \mathcal{M}_q$ represent the multisets of objects initially placed in the q cells of the system.

A rule of the type $[\,u\,]_{h_1} [\,v\,]_{h_2} \rightarrow [\,v'\,]_{h_1} [\,u'\,]_{h_2}$ is called an *evolutional communication rule*. A rule of the type $[\,a\,]_h \rightarrow [\,a_1\,]_{h_1} \dots [\,a_s\,]_{h_s}$ is called an *extended division rule*. The length of evolutional communication rules is defined by $|u| + |v| + |u'| + |v'|$. The length of extended division rules is defined by $s + 1$. These rules were introduced in [9], and more deeply investigated in [4, 5, 6]

An *instantaneous description* or a *configuration* at an instant t of a tissue P system with evolutional communication rules and extended division rules is described by the cells present and the corresponding multisets of objects over Γ associated with all the cells present in the system (not in the inputs). The *initial configuration* is $((1, \mathcal{M}_1), \dots, (q, \mathcal{M}_q))$.

A rule $[\,u\,]_{h_1} [\,v\,]_{h_2} \rightarrow [\,v'\,]_{h_1} [\,u'\,]_{h_2}, h_2 \in H$ is *applicable* to a configuration $\mathcal{C}_t$ at an instant t if there exist a cell labelled by h_1 containing the multiset u and a cell labelled by h_2 containing the multiset v. When applying such a rule, the objects specified by u and v disappear from their respective cells and multisets v' and u' appear in h_1 and h_2, respectively. If $|u| = 0$ (respectively, $|v| = 0$), then $|u'| = 0$ (resp., $|v'| = 0$) must be satisfied (this would correspond to symport rules). If $h_2 \in H_1 \cup \dots \cup H_r$, the rule is *applicable* to a configuration $\mathcal{C}_t$ at an instant t if there exist a cell labelled by h_1 containing the multiset u and the cell of the external input i such that $h_2 \in H_i$ contains the multiset v. The behaviour of the application of the rule is similar to when $h_2 \in H$.

A rule $[\,a\,]_h \rightarrow [\,a_1\,]_{h_1} \dots [\,a_s\,]_{h_s}$ is *applicable* to a configuration $\mathcal{C}_t$ at an instant t if there exists a cell labelled by h containing an object a. When applying such a rule, the cell h is divided in s new cells labelled by $h_i (1 \leq i \leq s)$, where a is changed to a_i in the corresponding cell and the rest of the contents is replicated in each cell.

We can think that the external inputs are independent systems that are computing a function. In each computational step, they will have different contents, that will be stated when the system is defined. In this sense, the contents of each cell of the system has to be defined for every configuration.

The rules from $\mathcal{R}$ of a tissue P system with evolutional communication rules, extended division rules and external inputs are applied in a non-deterministic maximally parallel manner (at each step we apply a multiset of rules which is maximal; that is, no further applicable rule can be added), with the following important remark: if a cell is divided, then the division rule is the only one which is applied to that cell at that step; that is, extended division rules interrupts the communication of that cell with others in that step. The new cells resulting from division will be able to interact with other cells from the next step.

Let us fix a tissue P system with evolutional communication rules, extended division rules and r inputs Π. We say that configuration $\mathcal{C}_t$ yields configuration $\mathcal{C}_{t+1}$ in one *transition step*, denoted by $\mathcal{C}_t \Rightarrow_\Pi \mathcal{C}_{t+1}$ if we can pass from $\mathcal{C}_t$ to $\mathcal{C}_{t+1}$ by applying the rules from $\mathcal{R}$ as follows: A transition step is divided in two micro-steps.

1. First, rules from $\mathcal{R}_i, 1 \leq i \leq r$ are applied in a maximally parallel and non-deterministic way. The "input systems" cannot receive any new contents from the main system. This first step is denoted as $\mathcal{C}_t \rightsquigarrow \mathcal{C}'_t$;
2. Second, rules from $\mathcal{R}$ are applied as stated above. This is denoted as $\mathcal{C}'_t \to \mathcal{C}_{t+1}$;

We say that a *transition step* $\mathcal{C}_t \Rightarrow_\Pi \mathcal{C}_{t+1}$ is a transition $\mathcal{C}_t \rightsquigarrow \mathcal{C}'_t \to \mathcal{C}_{t+1}$.

A *computation* of Π is a (finite or infinite) sequence of configurations such that:

1. the first term of the sequence is the initial configuration of the system;
2. each non-initial configuration of the sequence is obtained from the previous configuration by applying rules of the system in a maximally parallel manner with the restrictions previously mentioned; and
3. if the sequence is finite (called *halting computation*) then the last term of the sequence is a *halting configuration* (a configuration where no rule of the system is applicable to it).

All computations start from an initial configuration and proceed as stated above.

If $\mathcal{C} = (\mathcal{C}_0, \ldots, \mathcal{C}_p)$ of Π ($p \in \mathbb{N}$) is a halting computation, then the *length* of $\mathcal{C}$, denoted by $|\mathcal{C}|$ is p; that is, $|\mathcal{C}|$ is the number of non-initial configurations which appear in the finite sequence $\mathcal{C}$. We denote by $\mathcal{C}_t(i), i \in H$, the multiset of objects over Γ contained in all membranes labelled by i (by applying extended division rules different membranes with the same label can be created) at configuration $\mathcal{C}_t$. We denote $\mathcal{C}^*_t$ the multiset $\Sigma_{h\in H}\mathcal{C}_t(h)$

6 Generating contour approximations with P systems

We will use a P system of the type introduced in Section 5. It interacts with a 2D picture with contour as described by Figure 4

Let n be the number of iterations of the Hilbert curve we want to describe, let $L = \{00, 01, 10, 11\}^n$ the set of all words of length at most $2n$ over $\{00, 01, 10, 11\}$ and $\overline{N} = \{Ud, Ur, Ru, Rl, Ld, Lr, Du, Dl\}$. We consider the tissue P system

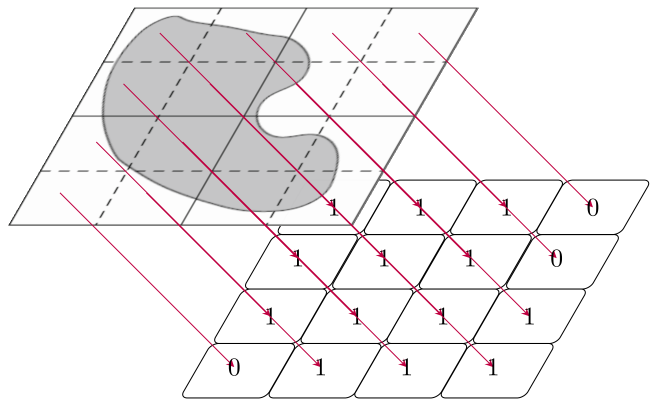

Fig. 4. In each step, the external input system *pic* interacts with Π in such a way that a symbol 0 or 1 is sent to the corresponding cell in the system.

$$\Pi = (\Gamma, H, \mathcal{M}_\lambda, \mathcal{R}, \mathcal{I})$$

with evolutional communication rules, extended division rules and 1 external input defined as follows:

1. Working alphabet: $\Gamma = \overline{N} \cup \{U^\alpha d, U^\alpha r, R^\alpha u, R^\alpha l, L^\alpha d, L^\alpha r, D^\alpha u, D^\alpha l \mid \alpha \in \{0,1\}\} \cup \{0,1\}$, being λ the empty string;
2. $H = \{00, 01, 10, 11\}^*$ (that is, the set of all words over $\{00, 01, 10, 11\}$) is the set of labels;
3. $\mathcal{M}_\lambda = \{U^1 r\}$
4. The set $\mathcal{R}$ consists of the following rules:
   a) Rules to divide the cells with intersections:
      $[U^1 d]_h \rightarrow [Ru]_{h00}\ [Ur]_{h01}\ [Ud]_{h11}\ [Ld]_{h10}$
      $[U^1 r]_h \rightarrow [Ru]_{h00}\ [Ur]_{h01}\ [Ud]_{h11}\ [Lr]_{h10}$
      $[R^1 u]_h \rightarrow [Ur]_{h00}\ [Du]_{h01}\ [Rl]_{h11}\ [Ru]_{h10}$
      $[R^1 l]_h \rightarrow [Ur]_{h00}\ [Dl]_{h01}\ [Rl]_{h11}\ [Ru]_{h10}$
      $[L^1 d]_h \rightarrow [Lr]_{h00}\ [Ld]_{h01}\ [Dl]_{h11}\ [Ud]_{h10}$
      $[L^1 r]_h \rightarrow [Lr]_{h00}\ [Ld]_{h01}\ [Dl]_{h11}\ [Ur]_{h10}$
      $[D^1 u]_h \rightarrow [Du]_{h00}\ [Ru]_{h01}\ [Ld]_{h11}\ [Dl]_{h10}$
      $[D^1 l]_h \rightarrow [Du]_{h00}\ [Rl]_{h01}\ [Ld]_{h11}\ [Dl]_{h10}$
   b) Rules to divide the cells without intersections:

$[U^0 d]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[U^0 r]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[R^0 u]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[R^0 l]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[L^0 d]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[L^0 r]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[D^0 u]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[D^0 l]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$
$[\#^0]_h \rightarrow [\#]_{h00}\ [\#]_{h01}\ [\#]_{h11}\ [\#]_{h10}$

5. $I = (\Gamma_{pic}, H_{pic}, \mathcal{R}_{pic})$, where:
   a) $\Gamma_{pic} = \{0, 1\}$
   b) H_{pic} is the set of the elements from H but with superscript *pic*.
   c) The set $\mathcal{R}_{pic}$ consists of the following rules:
      i. Rules to communicate if there is an interesection in a specific area:

$$\left.\begin{array}{l}
[a]_h^{pic}\ [Ud]_h \rightarrow [\ ]_h^{pic}\ [U^a d]_h \\
[a]_h^{pic}\ [Ur]_h \rightarrow [\ ]_h^{pic}\ [U^a r]_h \\
[a]_h^{pic}\ [Ru]_h \rightarrow [\ ]_h^{pic}\ [R^a u]_h \\
[a]_h^{pic}\ [Rl]_h \rightarrow [\ ]_h^{pic}\ [R^a l]_h \\
[a]_h^{pic}\ [Ld]_h \rightarrow [\ ]_h^{pic}\ [L^a d]_h \\
[a]_h^{pic}\ [Lr]_h \rightarrow [\ ]_h^{pic}\ [L^a r]_h \\
[a]_h^{pic}\ [Du]_h \rightarrow [\ ]_h^{pic}\ [D^a u]_h \\
[a]_h^{pic}\ [Dl]_h \rightarrow [\ ]_h^{pic}\ [D^a l]_h
\end{array}\right\} for\ a \in \{0, 1\}, h \in H$$

$$[0]_h^{pic}\ [\#]_h \rightarrow [\ ]_h^{pic}\ [\#^0]_h$$

   d) The contents of a cell in this system will be 0 if there is no intersection in the corresponding area of the picture, and 1 otherwise. In each configuration there will exist the cells corresponding to the resolution of the system.

7 Conclusions, Open problems, Suggestions for further developments

The present paper proposes a new variant of parallel array rewriting rules, capable to generate approximations of irregular contours, based on conecting pieces of Hilbert words of different 'resolutions'.

It proposes also a new variant of tissue P systems with evolutional communication rules, extended division rules and external inputs. In this variant, division rules are allowed to change the labels of the new created cells. The capability of receive input from an external source allows these systems to get more precision of a picture in each transition step.

Further developments are possible along several lines.

- to find means of effectively representing in a graphical manner the entire approximation, the problem being the segments which connect pieces of the SFC;

- other variants of P systems and refinements of the proposed one
- use the external input as a catalyst to allow or forbid the system to evolve.
- making use of the array representation in string format;
- taking into account the versatility of P systems, to use this small model as a template for complex applications, which involve the manipulation of spatial data; an example would be looking for applications in robotics (global path planning).

Acknowledgements

The present paper is work in progress.

It was supported in part by the research project TIN2017-89842-P, cofinanced by Ministerio de Economía, Industria y Competitividad (MINECO) of Spain, through the Agencia Estatal de Investigación (AEI), and by Fondo Europeo de Desarrollo Regional (FEDER) of the European Union.

The work of GZ was supported by the National Natural Science Foundation of China (61972324, 61672437, 61702428), by Beijing Advanced Innovation Center for Intelligent Robots and Systems (2019IRS14) and Artificial Intelligence Key Laboratory of Sichuan Province (2019RYJ06).

On a more personal note, RC would like to thank Luis Valencia-Cabrera for asking a very interesting question.

References

1. M. Bader. Space-filling Curves - An Introduction with applications in Scientific Computing. Texts in Computational Science and Engineering. Springer-Verlag (2013).
2. R. Ceterchi, L. Zhang, L. Pan, K. G. Subramanian, G. Zhang. Generating Hilbert Words in Array Representation with P Systems, presented at ACMC2019, 14-16 November 2019, Xiamen, China (submitted)
3. D. Hilbert. Über die stetige Abbildung einer Linie auf ein Flächenstück. Math. Annln. 38 459–460 (1891).
4. D. Orellana-Martín, L. Valencia-Cabrera, B. Song, L. Pan, M.J. Pérez-Jiménez. Tuning frontiers of efficiency in tissue P systems with evolutional communication rules. *Complexity*, to be published.
5. D. Orellana-Martín, L. Valencia-Cabrera, B. Song, L. Pan, M.J. Pérez-Jiménez. Narrowing Frontiers with Evolutional Communication rules and Cell Separation. In D. Orellana, Gh. Păun, A. Riscos, L. Valencia (eds.), *Proceedings of the Sixteenth Brainstorming Week on Membrane Computing*, Sevilla, Spain, January 30 - February 2 2018, pp. 123-162.
6. L. Pan, B. Song, L. Valencia-Cabrera, M.J. Pérez-Jiménez. The Computational Complexity of Tissue P Systems with Evolutional Symport/Antiport Rules. *Complexity*, vol. 2018, Article ID 3745210, 21 pages, 2018 (`10.1155/2018/3745210`).
7. G. Peano. Sur une courbe qui remplit toute une aire plane. Math. Annln. 36 157–160 (1890).

8. H. Sagan. Space-filling curves. Springer-Verlag. New York. 1994.
9. B. Song, C. Zhang, L. Pan. Tissue-like P systems with evolutional symport/antiport rules. *Information Sciences*, **378** (2017), 177-193 (`doi: 10.1016/j.ins.2016.10.046`).