Probing nucleon-nucleon interactions in breakup of the one-neutron halo nucleus ^{11}Be on a proton target

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A comparison between full few-body Faddeev/Alt-Grassberger-Sandhas (Faddeev/AGS) and continuum-discretized coupled-channels calculations is made for the resonant and nonresonant breakup of ^{11}Be on proton target at 63.7 MeV/u incident energy. A simplified two-body model is used for ^{11}Be which involves an inert ^{10}Be(0^+) core and a valence neutron. The sensitivity of the calculated observables to the nucleon-nucleon potential dynamical input is analyzed. We show that with the present NN and N-core dynamics the results remain a puzzle for the few-body problem of scattering from light exotic halo nuclei.

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Halo nuclei are few-body light nuclear systems that appear at the neutron drip line. The study of these nuclei is providing new theoretical challenges to the nuclear reaction theory, a key tool to interpret and extract reliable nuclear structure information from the new generation of very precise experimental measurements.

When describing the scattering of stable nuclei from halo nuclei it is crucial to handle its few-body character. In addition it is necessary to treat all opening channels (elastic, inelastic, transfer, and breakup) on equal footing. Yet, it is fair to say that a tight control on the underlying physics of the reaction mechanisms has not been achieved. We aim to shed light on the relevant dynamical aspects of the reaction framework.

Recently the resonant and nonresonant breakup resulting from the scattering of ^{11}Be from a proton target at 63.7 MeV/u was measured at MSU in order to obtain information on the ^{11}Be continuum by nuclear excitation [1]. The experimental results were compared with the calculated observables using the Faddeev/Alt-Grassberger-Sandhas (Faddeev/AGS) scattering framework [2–4] and making use of a simplified two-body model for ^{11}Be in terms of an inert ^{10}Be(0^+) core and a valence neutron [5]. Due to the experimental energy resolution, the breakup angular distribution results contained integrated contributions from ranges of relative neutron-core energy. It was found in Ref. [5] that in the case where the relative core-neutron energy is integrated around the resonance $E_r = 1.275$ MeV in the energy range $E_{\rm rel} = $ 0–2.5 MeV the Faddeev/AGS approach reproduced fairly well the shape distribution of the data although underestimating the maximum value of the breakup observable. A large discrepancy was found between the predictions of the Faddeev/AGS approach and those made by the continuum-discretized coupled-channels (CDCC) framework done in the works of Refs. [1,6]. Given the previous $p+^{11}$Be benchmark calculations [7] one should expect some differences between Faddeev/AGS and CDCC results, however, the different dynamic input may be responsible for the disagreement between the predictions

of [5] and [1,6] as well. In this work we aim to make a fair comparison between the two scattering frameworks using the same dynamical and structure inputs in order to establish the source of the discrepancy and shed light on the sensitivity of the observables to some key aspects of the dynamic input. In any three-body reaction approach to our working example of the breakup of ^{11}Be by a proton target, one needs as input the three-pair potentials: nucleon-nucleon (NN), N-core, and p-core. In the late 1990s high-precision NN potentials which reproduce the NN data below 350 MeV laboratory energy with χ^2/datum $\sim$1 were developed, such as the CD Bonn [8] and AV18 [9] potentials. However, puzzling three-body problems remain (such as the A_y discrepancy) where the current NN potentials are unable to reproduce the data. It is relevant to know to what extent our three-body observables are sensitive to this underlying dynamical input.

The Faddeev/AGS is a nonrelativistic three-body multiple scattering framework that can be used to calculate all relevant three-body observables. We use here the odd-man-out notation for the three interacting particles (1,2,3) which means, for example, that the potential between the pair (2,3) is denoted as v_1. According to this reaction framework, one needs to evaluate the operators $U^{\beta\alpha}$, whose on-shell matrix elements are the transition amplitudes. These operators are obtained by solving the three-body AGS integral equation [3,4]

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_{\gamma} G_0 U^{\gamma\alpha}, \qquad (1)$$

with $\alpha, \beta, \gamma = (1, 2, 3)$ ($\beta = 0$ in the final breakup state). Here, $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$ and the pair transition operator is

$$t_{\gamma} = v_{\gamma} + v_{\gamma} G_0 t_{\gamma}, \qquad (2)$$

where G_0 is the free resolvent $G_0 = (E + i0 - H_0)^{-1}$, and E is the total energy of the three-particle system in the center-of-mass (c.m.) frame.

In our calculations Eq. (1) is solved exactly in momentum space after partial-wave decomposition and discretization of all momentum variables. The Padé method [10] is used to sum the multiple-scattering series. We include the nuclear interaction between all three pairs, and the Coulomb interaction between the proton and ^{10}Be, following the technical developments implemented in Refs. [11,12] and the breakup observables are calculated as summarized in detail in Ref. [13].

The three-body CDCC [14,15] reaction framework consists in solving the Schrödinger equation in a model space in which the three-body wave function is expanded in the internal states (bound and continuum resonant and nonresonant states) of the two-body projectile H_p. In practical calculations, the continuum spectrum has to be truncated in excitation energy and discretized into a set of square-integrable functions. The most widely used discretization method is the called *binning* method, in which the continuum is divided into a set of energy intervals; for each interval, or *bin*, a representative function, $\phi_\alpha(\mathbf{r})$, is constructed by superposition of the *true* scattering states within the bin interval. The total three-body wave function is expanded in terms of these representative functions as

$$\Psi_{\mathbf{K}_0}^{\text{CDCC}}(\mathbf{R}, \mathbf{r}) = \sum_{\alpha=0}^{N} \phi_\alpha(\mathbf{r})\omega_\alpha(\mathbf{R}), \qquad (3)$$

where $\alpha = 0$ refers to the projectile ground state, $\mathbf{K}_0$ is the incident wave number of the projectile in the center-of-mass frame, $\mathbf{R}$ the relative distance between the center-of-mass of the projectile and the target, and $\mathbf{r}$ the relative distance between the valence particle and the core. The wave functions $\omega_\alpha(\mathbf{R})$ of the projectile-target relative motion are solutions of the coupled-channels equations

$$[E_\alpha - T_R - V_{\alpha\alpha}(\mathbf{R})]\omega_\alpha(\mathbf{R}) = \sum_{\beta \neq \alpha} V_{\alpha\beta}(\mathbf{R})\omega_\beta(\mathbf{R}), \qquad (4)$$

where $E_\alpha = E - \varepsilon_\alpha$ and the coupling potentials are

$$V_{\alpha\beta}(\mathbf{R}) = \langle \phi_\alpha | \sum_{j=C,v} V_{jt}(\mathbf{R}, \mathbf{r}) | \phi_\beta \rangle. \qquad (5)$$

The CDCC method was originally developed in order to incorporate the effect of the breakup channels in deuteron induced reactions. The proton-target and neutron-target interactions used in these calculations are usually taken as optical potentials adjusted to reproduce the elastic scattering at the same energy per nucleon, i.e., $E_p \approx E_n \approx E_d/2$. For nucleon-nucleus scattering these optical potentials are in many cases well represented by a simple, local, L-independent interaction comprising a central and, maybe, a spin-orbit term [16]. This has been also the standard choice in the application of the CDCC method to the scattering of other weakly bound nuclei (^{6,7}Li, ^{11}Be, ^{8}B, etc.) by medium-mass or heavy targets. However, this simple prescription might not be appropriate to describe the scattering of halo nuclei on protons, because in this case one of the fragment-target interactions is the NN potential, which is known to be strongly L dependent. Furthermore, the absence of an imaginary (absorptive) part in the NN interaction makes less clear the formal justification

of the CDCC method as an accurate approximation of a three-body scattering problem [17].

In this context, the calculations presented in this work arise from a twofold motivation. First, we aim to study the importance of using a realistic NN interaction in the description of the scattering of one-neutron halo nuclei by a proton target. In addition, by comparing the CDCC with the Faddeev/AGS calculations using the same three-body Hamiltonian we check to what extent the CDCC method provides an accurate reaction framework to the solution of a three-body scattering problem, following previous work done in Ref. [7].

In order to study the sensitivity of the calculated observables to the underlying NN potential, we have compared the Faddeev/AGS calculations using the realistic CD Bonn [8] and AV18 [9] NN potentials with those obtained making use of a simple L-independent potential. For the latter, we first consider a simple Gaussian potential $v_{pn}(r) = -v_0 \exp[-(r/r_0)^2]$, with $v_0 = 72.15$ MeV and $r_0 = 1.484$ fm. These parameters are adjusted to fit the deuteron binding energy and low-energy $^3S_1$ proton-neutron phase shifts. This parametrization, here denoted as G_{TE}, has been used in several works to generate the deuteron states in d+A CDCC calculations (see, e.g., Ref. [18]). The scattering phase shifts associated to this potential for S and P waves are compared in Fig. 1 with those obtained with the realistic CD Bonn potential. It is seen that the $^3S_1$ phase shifts are well described by the Gaussian parametrization up to a center-of-mass energy of $\approx$20 MeV, but they differ strikingly from the realistic phase shifts for the singlet S wave and the P waves.

In addition, we have performed some additional Faddeev/AGS calculations considering several L-dependent modified Gaussian potentials: the G_{SE} (G_{P1}) where the S (P) partial waves were modified taking the parameters from a Gaussian potential adjusted to reproduce the singlet scattering length [18] and all the other partial waves kept the same as the G_{TE} potential. Finally, we also consider the G_{P2} example
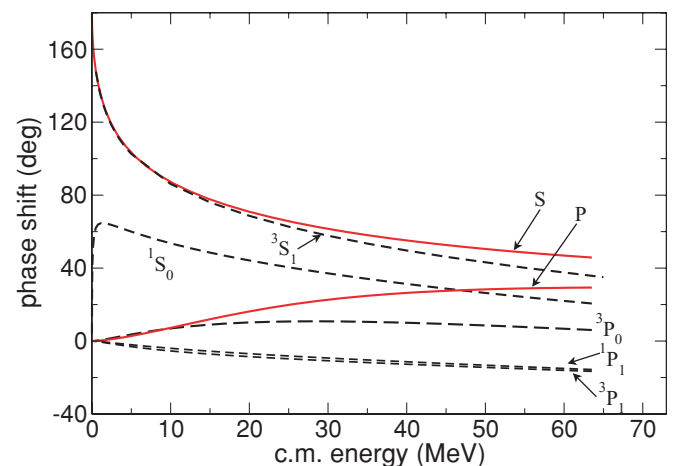

FIG. 1. (Color online) NN phase shifts for S and P partial waves obtained from the CDBonn potential (dashed lines) and Gaussian potential (solid lines) as indicated by the labels, where the depth and range parameters were adjusted to fit the deuteron binding energy and denoted in the text as G_{TE} [18].
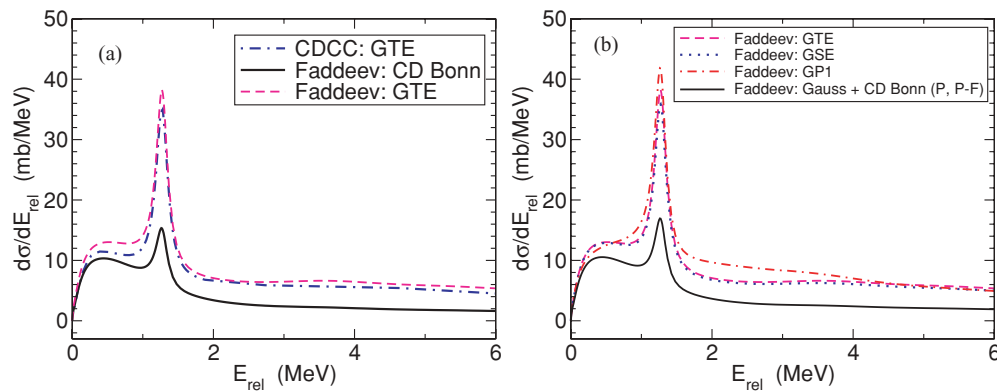
FIG. 2. (Color online) Energy spectrum for $p(^{11}\text{Be},p)^{10}\text{Be }n$ at 63.7 MeV/u integrated over the whole angular range. The NN interactions are described in the text.

case where the P and P-F partial waves were taken from the realistic CD Bonn potential and all the other partial waves kept the same as the G_{TE} potential. The proton-core and neutron-core pair interactions are taken as described in Ref. [5].

In the solution of the Faddeev/AGS equations we include n-p partial waves with relative orbital angular momentum $L \leqslant 6$, p-^{10}Be with $L \leqslant 21$, and n-^{10}Be with $L \leqslant 6$. Three-body total angular momentum is included up to 40.

As for the CDCC equations we discretized the n-^{10}Be continuum using energy bins for partial waves $L \leqslant 4$, and up to a maximum excitation energy of 40 MeV, above breakup threshold. The coupled equations were solved for total angular momentum up to 60, and the solutions matched to their asymptotic form at a distance of 60 fm. Both Coulomb and nuclear couplings were included and treated on equal footing. For the energy spectrum the number of bins was increased for excitation energies below 2 MeV in order to provide a finer description of the resonance region.

In Fig. 2 we represent the calculated energy spectrum $d\sigma/dE_{\text{rel}}$ that emerges by integrating the semi-inclusive cross section over the solid angle $d\Omega_{\text{c.m.}}$. As follows from Fig. 2(a) the Gaussian L-independent potential G_{TE} (dashed line) considerably overpredicts the nonresonant background and also the resonant peak with respect to the Faddeev/AGS result calculated with the CD Bonn potential (solid line). The predicted results using CD Bonn are indistinguishable from those which make use of the AV18 [9] potential and are not represented in the graph. The CDCC result (dashed-dotted line) that uses the same G_{TE} potential is very close to the corresponding Faddeev/AGS calculation except at the very low relative energies in a region around 1 MeV, where the CDCC cross section is somewhat smaller. The sensitivity to the underlying NN low partial waves interactions is shown in Fig. 2(b) which shows that this observable is sensitive to a realistic treatment of the NN potential in particular of the P waves.

In Fig. 3 we show the breakup angular distribution $d\sigma/d\Omega_{\text{c.m.}}$. We have not included very small angles $\theta_{\text{c.m.}} < 5°$ where there are no data and the convergence of the Faddeev/AGS results with respect to the Coulomb screening radius is slow. Due to the energy resolution of the experimental

setup, the relative core-neutron energy is integrated around the resonance $E_r = 1.275$ MeV in the energy range $E_{\text{rel}} = 0$–2.5 MeV in the upper part of the figure. The angular distribution calculated with the Faddeev/AGS approach and using the CD Bonn NN potential (solid curve) shown in Fig. 3(a) describes the overall shape distribution of the data reasonably well, although clearly underestimating the data around $\theta_{\text{c.m.}} = 20°$ by about 40%. The dependence on the calculated observable on the proton-core optical potential was studied. Using a potential that fits the elastic proton core data [6] increases the calculated observable at $\theta_{\text{c.m.}} = 20°$ by less than 10%. The origin of this disagreement needs to be further investigated. The Faddeev/AGS results are again indistinguishable when using the AV18 [9] potential. It follows that these three-body observables are insensitive to the choice of the realistic NN potential and probe essentially the NN scattering on-shell behavior.

On substituting the realistic potential by a L-independent Gaussian G_{TE} potential the Faddeev/AGS results for the calculated angular distribution (dashed solid line) are significantly enhanced at small center-of-mass angles and overall become similar to the predictions of the CDCC approach (dashed-dotted line). As seen in Fig. 3(b), in this case, the angular distribution is very sensitive to a realistic treatment of the NN potential, in particular to the NN P waves.

In the lower part of Fig. 3 we show the breakup angular distribution $d\sigma/d\Omega_{\text{c.m.}}$, where the relative core-neutron energy is integrated over the energy range $E_{\text{rel}} = 2.5 - 5.0$ MeV. In this case the Faddeev approach using the CD Bonn potential does not reproduce the data. As in the upper case the Faddeev calculation using the GTE Gaussian interaction overestimates the realistic calculation and overall becomes similar do the CDCC approach. Again the nonresonant breakup at higher relative energies is also very sensitive to a realistic treatment of the NN low partial waves.

Overall we can say that in our case study, there is a fairly good agreement between the two microscopic reaction formalisms, the Faddeev/AGS and the CDCC, better than in the benchmark calculation of Ref. [7]. We attribute this difference to the fact that, in the referred work, the breakup observables are studied with respect to the ^{10}Be core (integrated with
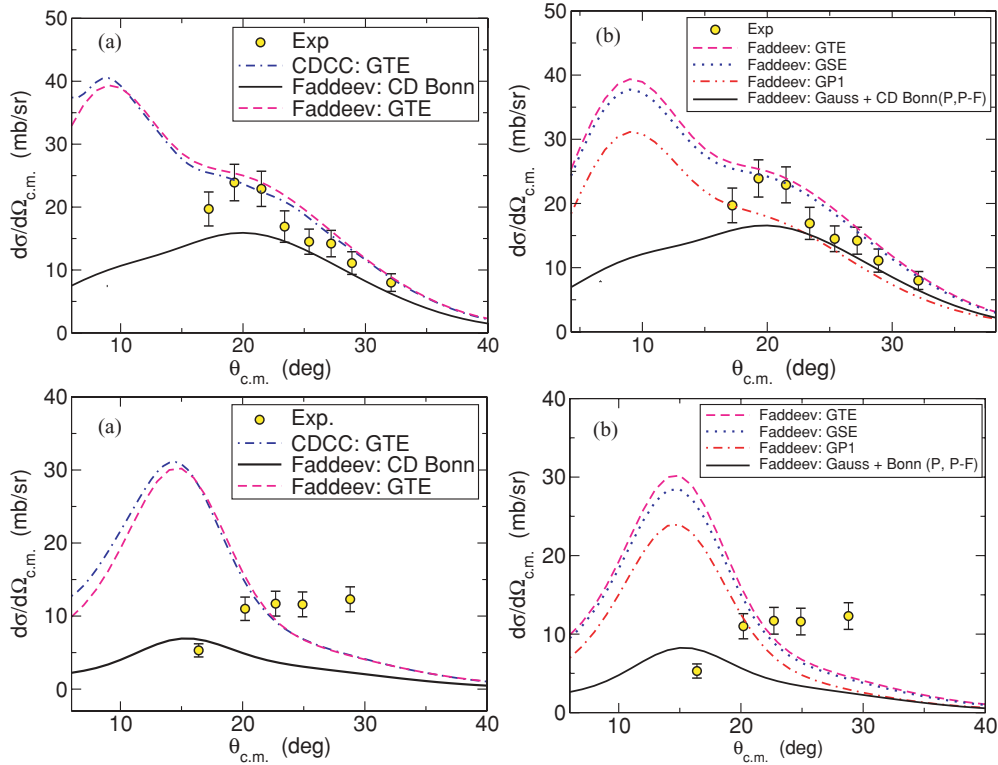
FIG. 3. (Color online) Angular distribution for the breakup $p(^{11}\text{Be},p)^{10}\text{Be}\ n$ at 63.7 MeV/u integrated over the energy range $E_{\text{rel}} = 0\text{–}2.5$ MeV (upper part) and integrated over the energy range $E_{\text{rel}} = 2.5\text{–}5.0$ MeV (lower part). The NN interactions are described in the text.

respect to the neutron angle). That calculated inclusive breakup observable was found to be dominated by configurations with small p-n angular momentum. These configurations are difficult to treat in a CDCC framework based in the expansion of the internal states of the n-^{10}Be [Eq. (3)] and hence the slow convergence found in Ref. [7] for those observables. In the present work, by contrast, we study selected regions dominated by small n-core relative energies and angular momenta, for which the convergence of the CDCC approach is expected to be much faster. Similar conclusions were achieved in the work of Ref. [19] for the breakup of ^{8}B on the ^{58}Ni target.

In conclusion, we have performed full Faddeev-type and CDCC calculations for the breakup of ^{11}Be on a proton target at 63.7 MeV/u incident energy. A simplified two-body model for ^{11}Be consisting of an inert ^{10}Be(0^+) core and a valence neutron has been used. We have shown that the Faddeev results using a L-independent p-n potential of Gaussian form (with the depth and range parameters adjusted to fit the deuteron binding energy) considerably differ from those obtained with a realistic CD Bonn potential. The former are found to overestimate the resonant and nonresonant breakup angular distributions calculated with the realistic NN interaction. These results strongly suggest that, at least in this energy regime, these breakup observables are very sensitive to the underlying NN interaction, particularly to the P wave.

We have also compared the Faddeev calculations with CDCC calculations, using in both cases the simple NN L-independent Gaussian parametrization. The angular and

energy breakup distributions are fairly similar, displaying only some small differences in the magnitude of the breakup cross section at small excitation energies. The good agreement between the two formalisms in this case study is better than in the previous benchmark calculation Ref. [7].

Since existing CDCC codes usually rely on simple central, L-independent fragment-target interactions, extensions of these codes, in order to incorporate realistic NN potentials in the calculation of the coupling potentials would be mandatory for future applications of the CDCC method in order to extract physically meaningful information from nucleon-nucleus scattering data.

We conclude that one can only extract reliable information from the breakup of halo interaction if a realistic potential is used. Different realistic NN potentials predict the same breakup observables which then essentially probe the NN scattering on-shell behavior. In addition, the breakup angular distribution integrated around the resonance underestimates significantly the data by about 40% and clearly does not reproduce this observable when integration is made at a higher relative core-neutron energy range. Further work should be performed to understand the physics of this discrepancy. With the present NN and N-core dynamics these results remain a puzzle for the few-body problem of scattering from light exotic halo nuclei.

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