Vertical externalities with lump-sum taxes: how much difference does unemployment make?*

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Abstract

This paper analyses how the existence of unemployment affects the conventional approach to vertical externalities. We discuss the optimality rule for the provision of public inputs both in an unitary and a federal country. Our findings show that decentralizing the spending responsability on public inputs can bring its optimality rule closer to the production efficiency condition. Moreover, we describe the inability of the federal government, behaving as Stackelberg leader, to replicate the unitary outcome, unless to have new policy instruments at government's disposal.

Keywords: Public inputs, unemployment, vertical externalities.

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1 Introduction

The usual approach to vertical externalities establishes that sharing taxes between different levels of government has an impact on efficiency. From the seminar contribution by Keen (1998), a number a papers has dealt with this issue, offering various solutions to internalize this problem as well (see, for instance, Boadway and Tremblay, 2006). A common issue in all these contributions is assuming distortionary taxation. In fact, it is clear that vertical tax externalities only appear as households' decisions are influenced by distorting taxes; otherwise, the marginal cost of public funds is not affected by lump-sum taxes decided by one level of government and, consequently, the impact of fiscal policies across different tiers of government does not take place.

Another common feature in this literature is that the labor market is competitive, with the labor force matching exactly the demand for labor. Papers such as Dahlby and Wilson (2003) and Kotsogiannis and Martinez (2008) give a central role to the labor supply and demand for labor in determining equilibria but always with labor market clearing. In such a world, there is no scope for one of the conventional fiscal policies aimed at fighting against unemployment, namely the provision of public inputs. In fact, to the best of our knowledge, no paper so far has dealt with vertical expenditure externalities (caused by the provision of productivity-enhancing public expenditures in a federal context) in the presence of unemployment. This has not been the case when horizontal externalities are involved; Ogawa et al (2006) study the implications of labor market imperfections on capital tax competition at the same level of governments.

This paper precisely combines vertical externalities and labor market imperfections in a single model. Indeed, we build a theoretical framework in which the federal government is in charge of unemployment benefits and the states provide a public input with positive effects on demand for labor. Taxes are assumed to be lump-sum because we are interesting in focussing on the efficiency implications derived from the expenditure side of government decisions rather than on vertical tax externalities. Anyway, we will show that ignoring distortionary taxation as a policy variable may play a crucial role for correcting the vertical externality.

The following contributions can be summarized from our results. Firstly, we prove that, in spite of using exclusively lump-sum taxes to finance governments (and thus no space for tax externalities), a vertical expenditure externality arises when unemployment exists. This confirms a previous result found in the literature (Dahlby and Wilson, 2003; Martinez, 2008), namely, that both vertical (tax and expenditure) externalities are independent of each other. The provision of public inputs creates a positive vertical impact on federal revenues as long as this type of public spending increases the demand for labor and, therefore, it reduces the resources needed at federal level for paying unemployment benefits. And this occurs without the co-occupancy of elastic tax bases.

Moreover, we also see how the optimality rule for the provision of public inputs at state level is closer to the production efficiency condition than the optimal condition in a unitary country with a non-clearing labor market. In a sense, one could say that more federalism does not necessarily leads to more inefficiency. Particularly, in the presence of a distortion (in the labor market, resulting in unemployment), it could be positive for efficiency to bring in a new distortion (that coming from the vertical expenditure externality).

Secondly, we have studied whether the federal government is able to replicate the equilibrium of an unitary country. As usual, we have assumed that the upper level of government knows the states' reaction functions and, behaving as Stackelberg leader, tries to achieve the centralized outcome. Our result deviates from previous papers as long as we conclude that the policy variables available for the federal government are not effective instruments to get the unitary equilibrium. We guess here that the fact of using exclusively lump-sum taxes prevents from affecting decisions taken by governments and households, in an attempt to internalize the effects from states' policy.

In a sense, this result can be placed on the discussion initiated by Sato (2000) about the capability of federal government to replicate second-best results depending on the federal instruments available. Precisely, as result of taking into consideration a new policy instrument**,** i. e., a public input provided by the federal government that is complement to that offered by the states, the upper level of government is able to replicate the second-best outcome of an unitary country.

The structure of the paper is as follows. Section 2 describes the main features of the model and the different versions of the optimality rule for the provision of public inputs, taking account whether the country is federal or not. Sections 3 and 4 evaluate the ability of the federal government to replicate the unitary outcome with the policy instruments available and with a complementary public input, respectively. Finally, section 5 concludes.

2 The Basic Model

This section aims to show two points. First, to characterize the equilibrium in a centralized country with unemployment; this will allow us not only to see how the optimal rule for the provision of public inputs must be modified with respect to a situation with full employment, but also having a benchmark scenario to compare with federal equilibria. Second, to highlight that the fiscal decisions taken by one level of government (particularly that with spending responsabilities on public inputs) will affect other levels of government; consequently, vertical expenditure externalities will arise despite of using exclusively lump-sum taxes.

The theoretical framework consists of firms, households and two different tiers of government: the federal level and k subnational states. Firms are identical across the country and, for the sake of simplicity, we assume that their number is normalized to one in each state. All of them produce a single good on the basis of the following production function:

$$F(N, K, G) = N^{\alpha} K^{1-\alpha} G^{\beta}, \tag{1}$$

where N is labor, K a fixed factor and G a public input. Such a production technology allows us to qualify the public input as factor-augmenting[1]. In this context, the public spending will increase the return to the fixed production factor K, which we normalized to one, in which case the profit can be expressed as:[2]

$$\pi = F(N, G) - wN, \tag{2}$$

where w is the wage rate. Profit maximization implies to define the first-order condition $w = F_N(N, G)$, that implictly defines the following function for labor demand:

$$N(w, G) = \alpha^{\frac{1}{1-\alpha}} G^{\frac{\beta}{1-\alpha}} w^{-\frac{1}{1-\alpha}} \tag{3}$$

Combining equations (2) and (3), the profit function can be obtained:

$$\pi(w, G) \tag{4}$$

We consider that all households have the same preferences for consumption c across the federation and described by a utility function $u(c)$, which is increasing in c. Each state is populated by three types of consumers: a firm-owner, employed and unemployed workers,

[1] An alternative approach would imply a production function with constant returns to scale in all the inputs (private and public). This would be the case of firm-augmenting public input. It would create economic rents that, in terms of the model we develop here, would not exhibit substantial differences with respect to what we obtain below.

[2] The return to labor is not affected by the public input, although this would be the normal situation with factor-augmenting public inputs. This is not the case here because we are interested in considering the impact of the public input on employment, and the demand for labor we obtain below implies that the wage rate is independent of G. In a model with full-employment, however, we should set up $w(M, g)$.

which are denoted by superindices "f", "e" and "u", respectively. The firm-owner, endowed with the production factor K, is who receives the profit in return for hiring the fixed factor to the firm. His budget-constraint is defined by $c^f = \pi - \tau^f$, where τ^f is a lump-sum tax. Regarding the other two types of consumers, we insert here a distinction between the total labor force available for working M and the number of households that effectively are employed N. Obsviously, full employment is characterized by $M = N$. The budget constraint for an employed worker is $c^e = w - \tau^e$, where τ^e is a lump-sum tax, while workers without jobs faces $c^u = b$, where b denotes a net of tax unemployment benefit.

In a centralized country, for the policy variables $\left\{\tau^f, \tau^e, b, G\right\}$, the government maximizes a utilitarian welfare function

$$W = kNu^e + k\left[M - N\right]u^u + ku^f \tag{5}$$

subject to the following budget constraint:

$$kN\tau^e + k\tau^f - kG - k\left[M - N\right]b = 0 \tag{6}$$

In a situation where there is no unemployment, the first-order conditions are as follows:

$$FOC\left(\tau^f\right) : \lambda = \left(u^f\right)' \tag{7}$$

$$FOC\left(\tau^e\right) : \lambda = \left(u^e\right)' \tag{8}$$

$$FOC\left(G\right) : F_G = 1 \tag{9}$$

$$FOC\left(\lambda\right) : N\tau^e + \tau^f - G = 0, \tag{10}$$

where λ is the Lagrange multiplier. The two first equations show the usual result from optimization with lump-sum taxes and transfers: private marginal utility (of each type of consumer) must be equal to social welfare cost of taxation, which is represented here by the Lagrange multiplier of government budget constraint. The equation (9) is the standard production efficiency condition in the provision of public inputs. Finally, (10) is the budget constraint of central government, where the last term of LHS in (6) has been dropped as $M = N$.

Let us turn to the equilibrium with unemployment. For institutional reasons (i. e., the existence of a minimum wage), the rate wage is assumed to exceed the market-clearing wage and, consequently, $M > N$. Things dramatically change for the optimal provision of G when unemployment appears; additionally, the first-order condition for the unemployment benefit b also must be taken into consideration:

$$FOC\left(b\right) : \lambda = \left(u^u\right)' \tag{11}$$

$$FOC\,(G):\frac{N_G\,(u^e-u^u)}{\lambda}+N_G\tau^e+N_Gb+F_G=1. \tag{12}$$

Let us consider now the case of different tiers of governments. We assume that the federal level is in charge of providing the unemployment benefit while the states provide the public inputs[3]. Both levels of government share the tax on employed workers (with the tax rates T^e and t^e chosen by the federal and states governments, respectively; $\tau^e = T^e + t^e$). The revenues collected from the tax on profits are assigned in a proportion θ (which is exogenously determined) to the states ($0 \leq \theta \leq 1$), while the tax rate τ^f is exclusively decided by the federal government.

Under such a framework, let us assume that the states behave as Nash players, that is, each subnational government ignores the impact of its fiscal decisions on federal revenues. Therefore, the optimization problem to be solved by the states is:

$$\begin{aligned} Max \qquad & W = Nu^e\,(w-\tau^e)+(M-N)\,u^b(b)+u^f\left(\pi-\tau^f\right) \qquad (13)\\ & s.t. \qquad Nt^e+\theta\tau^f-G+S=0 \\ & N=N\,(w,G) \\ & w^o>w^e, \end{aligned}$$

where S is a vertical lump-sum from the federal government to states. Last inequality refers to the distorsion existing in the labor market, which is the reason for unemployment. First-order conditions for t^e, G and λ give:

$$FOC\,(t^e):\lambda=(u^e)' \tag{14}$$

$$FOC\,(G)\ :\ \frac{N_G\,(u^e-u^u)}{\lambda}+N_Gt^e+F_G-1=0\equiv\Omega \tag{15}$$

$$FOC\,(\lambda)\ :\ Nt^e+\theta\tau^f-G+S=0\equiv\Psi \tag{16}$$

Expression (14) sets up an identical rule for chosing the optimal tax rate on employed workers in a centralized country than in a world with two tiers of government. This is a direct consequence of using lump-sum taxes. Even in the presence of tax sharing between different levels of government, if the households' behavior is not affected by taxes, there is no scope for vertical tax externalities.

By contrast, and leaving aside the discussion on the optimal levels of G (see Martinez and Sjongren (2009) for a further analysis), expression (15) shows the main difference by comparing

[3] This distribution of spending responsabilities is not crucial for the results, which would be symmetric with an inverse vertical assignment of public expenditures. Anyway, the scheme we follow here is in line with the mainstream of theory of fiscal federalism.

it to the expression (12). The term $N_G t^e$ differs from its equivalent in (12), namely, $N_G(\tau^e + b)$. As long as the federal government sets up a non-negative tax rate T^e on employed workers, the fact of having states deciding on G leads to reduce the overprovision bias that the presence of unemployment creates in the provision of public inputs. In other words, expression (15) is closer to (9) than equation (12).[4]

In this regard, and contrary to the conventional view in previous literature on vertical externalities, we guess here that more federalism may lead to more efficiency in the design of fiscal policies. To see this in an extreme case, assume that all rent taxes accrue to the states ($\theta = 1$); the federal government needs to be financed by a negative fiscal grant (from states) and/or by charging a positive tax rate T^eon workers. This latter solution involves an optimal rule for the provision of public inputs closer to the production efficiency condition, minimizing the differential effect that the presence of unemployment creates in the discussion on optimality.

Consequently, the behavior of federal government becomes a crucial issue to determine the effect of unemployment on the achivement of production efficiency condition in the provision of public inputs. This is what we study in the next section.

3 The ability of federal government to replicate the centralized outcome

A usual way of correcting vertical (tax and expenditure) externalities is assuming a federal government behaving as Stackelberg leader. In such a context, the sequence of the game is as follows. Firstly, the federal government decides on T^e, τ^f, S and, residually, on b, taking into consideration the states' reaction to changes in federal policy variables. Secondly, the states choose G and t^e, taken as exogenous all the decision variables of the upper-level of government.

[4]It is straightforward to show that with full employment no vertical (tax and expenditure) externalities appear.

Consequently, the optimization problem of the federal government is:

$$Max \qquad W = kNu^e\left(w - \tau^e\right) + k\left(M - N\right)u^b(b) + ku^f\left(\pi - \tau^f\right) \tag{17}$$

$$s.t. \qquad kNT^e + k(1-\theta)\tau^f - k(M-N)b - kS = 0$$

$$G = G(T^e, \theta, \tau^f, S, M, N) \tag{18}$$

$$t^e = t^e(T^e, \theta, \tau^f, S, M, N) \tag{19}$$

$$N = N\left(w, G\right)$$

$$w^o > w^e.$$

Expressions (18) and (19) are the states' reaction functions. Therefore, for solving the federal problem, some information on comparative statics of these reaction functions is required. To do that, we start from the first-order conditions of states (15) and (16)[5]. Differentiating totally Ω and Ψ (and ignoring superindex "e" for sake of simplicity in the notation), we have:

$$\Omega_G dG + \Omega_t dt + \Omega_T dT + \Omega_S dS + \Omega_{\tau^f} d\tau^f = 0$$

$$\Psi_G dG + \Psi_t dt + \Psi_T dT + \Psi_S dS + \Psi_{\tau^f} d\tau^f = 0$$

This two-equation system can be expressed using a matricial form as follows (and after solving for dG and dt):

$$\begin{pmatrix} dG \\ dt \end{pmatrix} = -\begin{pmatrix} \Omega_G & \Omega_t \\ \Psi_G & \Psi_t \end{pmatrix}^{-1} \begin{pmatrix} \Omega_T & \Omega_S & \Omega_{\tau^f} \\ \Psi_T & \Psi_S & \Psi_{\tau^f} \end{pmatrix} \begin{pmatrix} dT \\ dS \\ d\tau^f \end{pmatrix} \tag{20}$$

Matricial manipulation on (20) shows that:

$$\frac{dG}{dT} = G_T = A(\Psi_t\Omega_T - \Omega_t\Psi_T) \tag{21}$$

$$\frac{dG}{dS} = G_S = A(\Psi_t\Omega_S - \Omega_t\Psi_S) \tag{22}$$

$$\frac{dG}{d\tau^f} = G_{\tau^f} = A(\Psi_t\Omega_{\tau^f} - \Omega_t\Psi_{\tau^f}) \tag{23}$$

$$\frac{dt}{dT} = t_T = A(-\Psi_g\Omega_T - \Omega_g\Psi_T) \tag{24}$$

$$\frac{dt}{dS} = t_S = A(-\Psi_g\Omega_S - \Omega_g\Psi_S) \tag{25}$$

$$\frac{dt}{d\tau^f} = t_{\tau^f} = A(-\Psi_g\Omega_{\tau^f} - \Omega_g\Psi_{\tau^f}) \tag{26}$$

[5]The first-order condition (14) can be ignored in this analysis. In a sense, this expression does not admit any influence from federal variables and, consequently, it does not matter at this point. Anyway, expression (14) can be easily inserted in (15) without modifying substantially the analysis below.

where A is $-\frac{1}{\Omega_G\Psi_t-\Psi_G\Omega_t}$.

Turning back to the federal problem, it is clear that its budget constraint can be written as $b=\frac{NT^e+(1-\theta)\tau^f-S}{(M-N)}$. Plugging this into the objective function (17), we obtain the first-order conditions for the policy variables of the federal government:

$$FOC(T^e): -N(u^e)'(1+t_T)+(M-N)(u^b)'(b_T+b_Tt_T+b_GG_T) \quad (27a)$$
$$+(u^f)'(F_NN_GG_T-wN_GG_T)-u^bN_GG_T=0$$
$$FOC(\tau^f): -N(u^e)'(t_{\tau^f})-u^bN_GG_{\tau^f}+(M-N)(u^b)'(b_{\tau^f}+b_tt_{\tau^f}+b_GG_{\tau^f}) \quad (27b)$$
$$+(u^f)'(F_NN_GG_{\tau^f}-wN_GG_{\tau^f})-(u^f)'=0$$
$$FOC(S): -N(u^e)'(t_S)+(M-N)(u^b)'(b_S+b_tt_S+b_GG_S)-u^bN_GG_S \quad (27c)$$
$$+(u^f)'(F_NN_GG_S-wN_GG_S)=0$$
$$FOC(\lambda): kNT^e+k(1-\theta)\tau^f-k(M-N)b-kS=0 \quad (27d)$$

Taking into account that b can be residually obtained from the above four equation-system, we symplify (27a)-(27d) and the following result is achieved:

$$t_T = 0 \quad (28)$$
$$t_{\tau^f} = -\frac{\theta}{N} \quad (29)$$
$$t_S = -\frac{1}{N}, \quad (30)$$

where $w=F_N(N,G)$, (21)-(23) and the corresponding partial derivatives of Ω and Ψ (according to (15) and (16)) have been used. What is implicitly established in (28)-(30) is the inability of federal government to affect states' behavior. In fact, not only the federal tax rate on employed workers T^e has no effect on the equivalent state tax rate t^e (equation (28)), but also none of the policy variables of upper level of government has any impact on the state provision of public inputs. Indeed, from expressions (21)-(23), it is clear that $G_T=G_{\tau^f}=G_S=0$, that is, there is no way through which the federal government can modify the provision of public inputs. The unique impact of the federal policy variables (τ^f and S on t^e) is trivial: an increase (decrease) in some of them reduces (rises) the state tax rate in a magnitude given by the number of employed workers N. Therefore, the highest level of government is not able to replicate not only the first-best outcome of (9) but also the optimality rule for the provision of public inputs in an unitary country with unemployment[6].

[6] Anyway, we must be aware that the first-best values for T^e and t^e are guaranteed in each scenario as long as they are lump-sum taxes.

4 New instruments for federal government: complementary public inputs

Things may be different if federal government is also in charge of providing public inputs, namely G^F, which is assumed to be complementary with the state public input (now G^S). Production, profit and labor demand functions have to be conveniently modified:

$$F\left(N, K, G^F, G^S\right) = N^\alpha K^{1-\alpha} \left(G^F\right)^\gamma \left(G^\beta\right)^\beta \quad (31)$$

$$\pi = F\left(N, G^F, G^S\right) - wN, \quad (32)$$

$$N\left(w, G^F, G^S\right) = \alpha^{\frac{1}{1-\alpha}} \left(G^F\right)^{\frac{\gamma}{1-\alpha}} \left(G^S\right)^{\frac{\beta}{1-\alpha}} w^{-\frac{1}{1-\alpha}}. \quad (33)$$

As before, combining (32) and (33), the profit function can be written as follows:

$$\pi\left(w, G^F, G^S\right).$$

In an unitary country, the government maximizes $W = kNu^e + k[M-N]u^u + ku^f$, subject to $kN\tau^e + k\tau^f - kG^F - kG^S - k[M-N]b = 0$. In a situation characterized by full employment ($M = N$), the optimal provision of public inputs is given by the standard production efficiency condition: $F_{G^F} = F_{G^S} = 1$. By contrast, when unemployment appears as a result of non market-clearing wage rate, the first-order conditions for G^F and G^S are, respectively:

$$FOC\left(G^F\right) : \frac{N_{G^F}\left(u^e - u^u\right)}{\lambda} + N_{G^F}\tau^e + N_{G^F}b + F_{G^F} = 1 \quad (34)$$

$$FOC\left(G^S\right) : \frac{N_{G^S}\left(u^e - u^u\right)}{\lambda} + N_{G^S}\tau^e + N_{G^S}b + F_{G^S} = 1 \quad (35)$$

It is trivial to show that when the government is concerned with the level of employment, if the effect of, say, the state public input on labor demand is higher than the equivalent effect by the federal public input ($N_{G^S} > N_{G^F}$), then the optimal amount of G^S will exceed G^F.

In a decentralized environment, in which both federal and state government behave as Nash competitors, the first-order conditions for G^F and G^S are, respectively:

$$FOC\left(G^F\right) : \frac{N_{G^F}\left(u^e - u^u\right)}{\lambda} + N_{G^F}T^e + N_{G^F}b + F_{G^F} = 1 \quad (36)$$

$$FOC\left(G^S\right) : \frac{N_{G^S}\left(u^e - u^u\right)}{\lambda} + N_{G^S}t^e + N_{G^S}b + F_{G^S} = 1 \quad (37)$$

Comparing these expressions with (34) and (35), it is clear that both types of public inputs will be underprovided if governments set positive tax rates on employed workers. Under these circumstances, the levels of G^F and G^S will be below the optimal ones derived from a centralized setting. In other words, each level of government decides a level of public input without considering its impact on other jurisdictions. Consequently, we are in the presence of a double vertical expenditure externality from each level of government to the other.

The question now is whether the federal government, behaving as Stackelberg leader, is able to replicate the second-best outcome. Recall that the answer to this question was "no" in a setting in which the federal instruments were T^e, τ^f, S and b. With the federal government also providing a public input, its optimization problem is now:

$$\begin{aligned} Max \qquad & W = kNu^e\left(w-\tau^e\right) + k\left(M-N\right)u^b(b) + ku^f\left(\pi-\tau^f\right) && (38) \\ s.t. \qquad & kNT^e + k(1-\theta)\tau^f - k(M-N)b - kG^F - kS = 0 && \\ & G^S = G(T^e, \theta, \tau^f, S, M, N, G^F) && (39) \\ & t^e = t^e(T^e, \theta, \tau^f, S, M, N, G^F) && (40) \\ & N = N\left(w, G^S, G^F\right) && \\ & w^o > w^e. && \end{aligned}$$

Note that the states' reaction functions (39) and (40) now include a new argument: the federal public input G^F. As before, we first need to know some comparative statics of these functions. Equations (21)-(26) are still valid in the new context -with a slight change: the term A must be substituted by A' -(see below)- and we only have to add the corresponding response of G^S and t^e to the new federal policy instrument G^F. Particularly, we can write:

$$\frac{dG^S}{dG^F} = A'(\Psi_t\Omega_{G^F} - \Omega_t\Psi_{G^F}) \qquad (41)$$

$$\frac{dt}{dG^F} = A'(-\Psi_{G^S}\Omega_{G^F} - \Omega_{G^S}\Psi_{G^F}), \qquad (42)$$

where A' is $-\frac{1}{\Omega_{G^S}\Psi_t - \Psi_{G^S}\Omega_t}$.

In this regard, a principal difference with respect to the previous framework appears. While in Section 3 the federal government only had a very limited (and trivial) impact on state tax rate t (recall expressions (28)-(30) and the fact that $G_T = G_{\tau^f} = G_S = 0$), things are quite different now. Consider first the case of $\frac{dG^S}{dG^F}$; after some algebra manipulations it can be seen that the effect of changes in the federal public input on the state provision of public inputs is given by:

$$\frac{dG^S}{dG^F} = -\frac{\left(N_{G^SG^F}\left(u^e-u^u\right)/\lambda\right) + N_{G^SG^F}t + F_{G^SG^F}}{\left(N_{G^SG^S}\left(u^e-u^u\right)/\lambda\right) + N_{G^SG^S}t + F_{G^SG^S}} > 0.$$

This means that an increase in the federal public input encourages the provision of the state public input. That is, there is an additional channel through which the federal government can affect states' behavior. In the case of the state tax rate it happens something similar:

$$\frac{dt}{dG^F} = -\frac{t}{N}\left(N_{G^S}\frac{dG^S}{dG^F} - N_{G^F}\right) \lesseqgtr 0.$$

But here the effect of federal public input on state policy variable is not so clear. Indeed, an increase in the federal public input may lead to either an increase or a decrease in the state tax rate on employed workers. Anyway, it is worth to noting that again federal government may affect states' behavior, which was not possible under the previous assumptions.

Given this, the first-order condition for the optimal provision of G^F is as follows:

$$FOC\left(G^F\right): \frac{N_{G^F}\left(u^e - u^u\right)}{\lambda} + \frac{N_{G^S}\frac{dG^S}{dG^F}\left(u^e - u^u\right)}{\lambda} - N\frac{dt}{dG^F} + N_{G^F}T + N_{G^S}\frac{dG^S}{dG^F}T + b(N_{G^F} + N_{G^S}\frac{dG^S}{dG^F}) = 1. \tag{43}$$

After some algebra manipulations, it can be shown that to replicate the second-best condition (34) requires to hold:

$$N_{G^S}\frac{dG^S}{dG^F}\left(\frac{u^e - u^u}{\lambda} - \tau + b\right) = F_{G^F}. \tag{44}$$

If the individual utility is assumed to be linear ($u(c) = c$), it is straightforward to prove that both sides of expression (44) have the same sign. Hence, to replicate the second best outcome for the provision of public inputs is a real possibility when federal government can spend money in public inputs which are complementary to state public inputs.

5 Concluding Remarks

Vertical externalities use to involve challenges for efficiency in federal countries. Sharing taxes between different levels of government or the provision of certain public expenditures with effects on other tiers of government's revenues, imply deviations from the optimality rules which would be obtained in a centralized world. However, the presence of vertical (tax and expenditures) externalities can be disregarded if lump-sum taxes are used. Indeed, the idea of governments affecting fiscal decisions taken by others requires distorting taxes able to modify households' behavior.

All these general statements have to be qualified in the presence of unemployment, and this has been what we have done in this paper. Particularly, we have built a simple model with lump-sum taxes and unemployment in which the optimal rule for the provision of public inputs depends on whether the structure of the country is federal or not. Indeed, while there is no scope for vertical tax externalities (the fact of using lump-sum taxes here is crucial), a deviation from the second-best outcome takes place when states are in charge of the provision of productivity-enhancing public factors and the federal government finances unemployment benefits.

We have confirmed that the optimality condition for the provision of public inputs must consider the impact of this type of public expenditures on employment and, consequently, on public spending in unemployment benefits. As the production efficiency condition for public inputs is not satisfied even in the case of a centralized country, we have analyzed what would occur when states behaving as Nash players take part in the game. Since subnational governments do not take into account the effect of their public expenditures on unemployment benefits, the overprovision of public inputs (compared to the first-best case with full employment) is lower with a federal structure than in an unitary country. In a sense, one can say that considering a new distortion (a vertical expenditure externality) in a world with previous distortions (unemployment) may improve the efficiency in the sense of coming close the state behavior to the production efficiency in public inputs.

When we have wondered about the capability of federal government to replicate the outcome of an unitary country, we have assumed that the upper level of government behaves as Stackelberg leader, considering the states reaction's functions. Under such a scenario, we have concluded that, unlike previous papers, federal government is not able to internalize the vertical expediture externality. Federal policy variables have no impact on states' decision variables.

In part, this is caused by using lump-sum taxes; indeed, distortionary taxation can affect agents' behavior and this is the way through which all the effects of public inputs can be internalized. By contrast, when the federal government is also in charge of providing a public input which is complementary to the state public input, it is possible to replicate the second-best outcome for the optimal provision of such as public inputs.

A number of issues arises for further research. Asymmetries at regional level in the federation can be taken into consideration. Given our federal budget constraint, the characterization of equilibria may then involve that not all the resources collected by the upper level of government in a region must be spent in such territory; consequently, some possibilities for horizontal redistribution arise and even for explicit equalization schemes. Also under this framework, in the presence of mobile production factors, phenomena of tax competition may take place, with the consequent effects on efficiency and regional labor markets.

As policy implication we would underline how important the coordination of different levels of government is to get social welfare gains. Indeed, the design of federal and state fiscal policies must take into account the magnitude of their cross effects on tax revenues of others liers of government. Particularly, this is specially true in the case of public infrastructure because this type of government expenditure is very vulnerable to public spending cuts and the visibility of its benefits. Improving efficiency here, for instance, increasing the coordination in the provision of public transport infrastructure (some of them provided at regional level, say, roads; others by the federal government, say, railways) means more social welfare, part of it in terms of employment.

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