On Bi-Polarization and the Middle Class in Latin America:

A Look at the First Decade of the Twenty First Century.

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Abstract

This paper proposes a new index and graphical representation of the change in bi-polarization and in the relative importance of the middle class that took place in a given country during a given period. These tools extend in fact the concepts of inter-distribution income inequality and Lorenz curves by making a distinction between overall, "pure growth based" and "shape related" distributional changes.

The empirical illustration is based on data covering 17 Latin American countries in 2000 and 2009, obtained from the Latinobarómetro surveys for these years. The standard of living of individuals was derived on the basis of correspondence analysis. It appears that these new tools help understanding the changes that took place in the distribution of standards of living during the period analysed.

Key Words: bipolarization, generalized Lorenz dominance, interdistributional income inequality, Latin America, Latinobarómetro.

J. E. L. Classification: D31 – O15 - O54

1. Introduction

In a very recent paper C. Kenny (2011) writes that "it is hard to find a set of characteristics or values that are consistently and uniquely middle class across countries and time". There is indeed in the literature a long list of traits that are supposed to help identifying individuals who belong to the middle class. The literature seems to have stressed the following characteristics of individuals belonging to the middle class. They are supposed to be middle aged people, who invest in their education, have a relatively small number of children (two or three?), spend more on health care, pay taxes, tend to own their own house or apartment as well as one or two cars and, as a consequence, have a significant amount of debt. They are also supposed to have some entrepreneurial spirit, to work in specific occupations, to have stable jobs and the means to avoid poverty even when facing an unexpected shock such as becoming unemployed. They are also expected to hold common values such as a belief in the virtue of democracy (free elections and free speech) and of tolerance (e.g. towards minorities) Finally it is also said that they are optimistic about the future, believe that they are doing better than their parents, belong to the middle spectrum of the distribution of incomes, no matter how such a middle range is defined and are supposed to be one of the main engines of economic growth.

Needless to say, including all these characteristics, assuming they are all relevant, to determine who belongs to the middle class is an impossible task, among other reasons because of the scarcity of data sources that would encompass all the potentially relevant variables mentioned previously. In addition there is no agreement, among those who have attempted to define the middle class, about the most important features of the middle class, although the amount of income available is almost always mentioned.

But even when the main focus is on the income level, there is no consensus concerning the critical thresholds, those that distinguish the middle class from the poor and from the rich. There is hence a need to be very careful when attempting to assess the size of the middle class in a given country, to find out whether its importance grew over time, to detect its main characteristics or to determine whether the identity of those belonging to the middle class does not change or varies a lot over time.

The importance of the middle class is clearly related to the concept of bipolarization. Foster and Wolfson (1992; 2010) recommended making a distinction between four stages when attempting to measure the relative importance of the middle class:

- choose a “space” (individual/family/household income, salary, expenditure etc. in an income- or people-space)
- define the "middle" (e.g., the median or the mean income)
- fix a range around the middle (identify the middle class by determining a percentage interval above and below the median or the mean)
- aggregate the data.

Various definitions based on the “income space” have been proposed. Thurow (1984) assumed that the middle class includes the households whose income ranges from 75% to 125% of the median household income. Blackburn and Bloom (1985) recommended using a wider range (60% to 225% of the median). Other ranges have been proposed: 50% to 150% (Davis and Huston, 1992) and two-thirds to four-thirds of men’s median weekly earnings (Lawrence, 1984). Birdsall et al. (2007) suggested including in the middle class those individuals above the equivalent of $10 day in 2005 and at or below the 90th percentile of the income distribution in their own country. In all these cases one computes which *share of the total population* the middle class includes.

Others have preferred to use definitions based on the people space. For Levy (1987), for example, the middle class ranges from the 20th to the 80th percentile. Whatever the definition adopted when using such an approach, one computes here the *share in total income* of those belonging to the selected population deciles. Graphical representations for representations in both the income and the people space have been proposed by Foster and Wolfson (1992; 2010). The present paper proposes rather a graphical representation of the change in bi-polarization which is derived from the concept of inter-distributional change.

2. Measuring Changes in Bi-polarization

2.1. The Concept of Overall Distributional Change and Inter-distributional Inequality and Lorenz curves

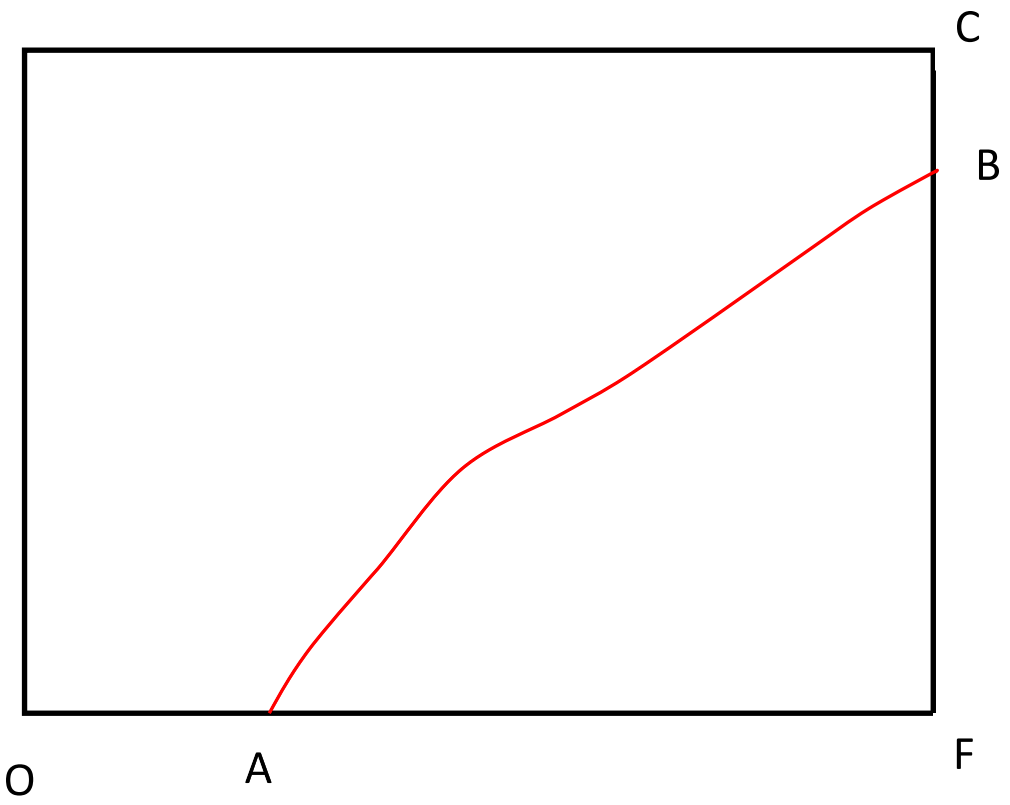
The concepts of Inter-distributional inequality and Lorenz Curves were introduced in the literature by Butler and McDonald (1987) and may be summarized as follows. Assume two different density functions $f(x)$ and $h(x)$ describing the distribution of income x in a given country at two different periods 0 and 1. Let $F(x)$ and $H(x)$ be the two distributions functions corresponding to the two density functions $f(x)$ and $h(x)$. These two distribution functions $F(x)$ and $H(x)$ will now be plotted respectively on the horizontal and vertical axis of a 1 by 1 square. In other words for each income x we plot the percentage of individuals with an income lower than or equal to x observed in the distributions $F(x)$ and $H(x)$. If the "distributional change curve" obtained happens to be completely below the diagonal, we can certainly conclude that the distribution $H(x)$ first order stochastically dominates the distribution $F(x)$. More generally if most of the curve lies below the diagonal we can conclude that the population with the income distribution $h(x)$ has an economic advantage over the population with an income distribution $f(x)$ (see, Bishop et al., 2011). If however most of the curve lies above the diagonal we would conclude that the population with the income distribution $f(x)$ has an economic advantage over the population with an income distribution $h(x)$.

2.2. Distributional Change in the Case of Pure Growth

Let us now call m_x^f and m_x^h the median incomes corresponding to the distributions $f(x)$ and $h(x)$ and let us, for example, assume that $m_x^h > m_x^f$. Let now $k(x)$ be the density function obtained when the density function $f(x)$ is horizontally translated by an amount $(m_x^h - m_x^f)$. Finally let $K(x)$ be the distribution functions corresponding to the density function $k(x)$. A plot of $K(x)$ on the vertical axis versus that of $F(x)$ on the horizontal axis would then give us a "distributional change curve" that would only be affected by growth (assuming that $F(x)$refers to time t and $K(x)$ to time $t+1$) since $K(x)$ was derived from $F(x)$ by a translation. Assuming there was positive growth (since we postulated that $m_x^h > m_x^f$), the distribution change curve obtained will then start at some point A on the horizontal axis (see, Figure 1). The segment OA would then represent the proportion of individuals who at time *t* (corresponding to distribution $F(x)$) had an income lower than the lowest income at time *t*+1 (corresponding to distribution $K(x)$). Such a distributional curve would also end at point B and the segment BC would represent the share of the population who at time *t*+1 had an income higher than the highest income at time *t* (see Figure 1). In the particular and exceptional case where the lowest income at time t+1 would be higher than the highest income at time t, the distributional change curve would become identical to the broken curve OFC. In such a case we know that there would be no overlap between the distributions $f(x)$ and $k(x)$ and the index of distributional change $DC(x)$, defined as being equal to twice the area between the distributional curve and the diagonal, would evidently be equal to 1. The complement to one of the distributional change index may hence be considered as a measure of the degree of overlap between the distributions $h(x)$ and $k(x)$ in the case of positive growth over time.

Figure 1: Distributional Change Curve

in the Case of Pure Positive Growth

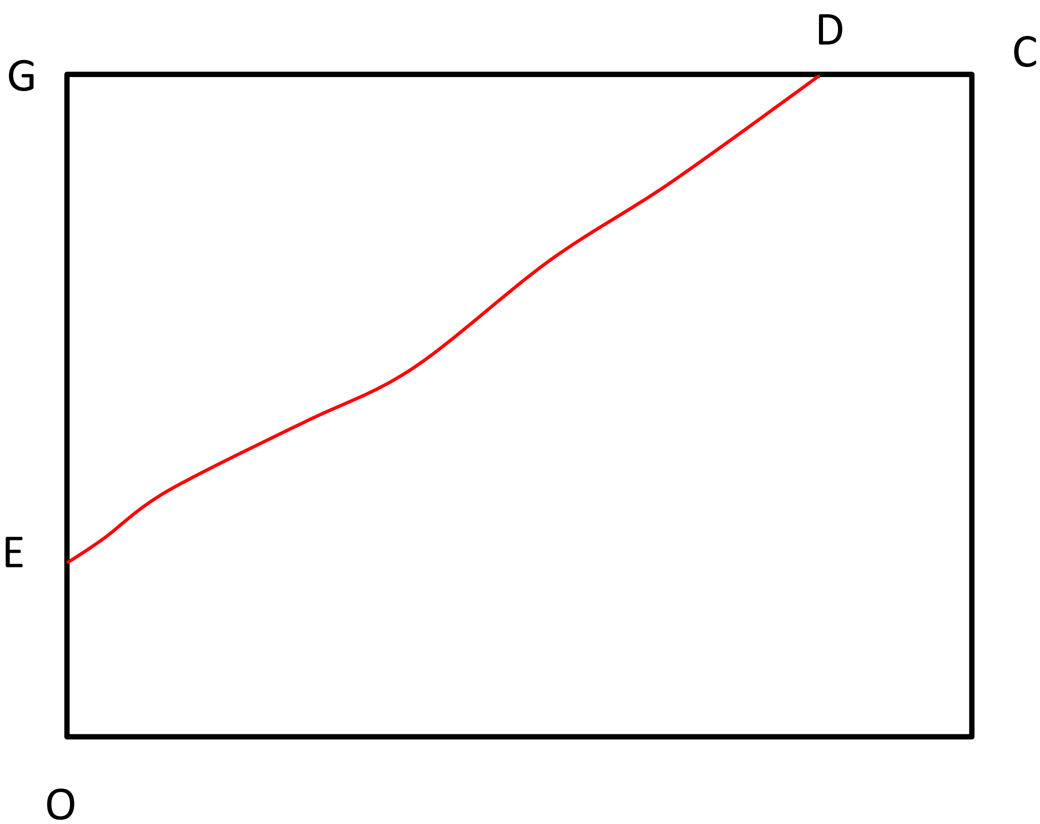


Conversely if there was negative growth so that the median of the distribution *K(x)* is smaller than that of the distribution *F(x),* we would get a curve that would generally start at some point E on the vertical axis. OE would then represent the proportion of individuals who at time 1 had an income smaller than the smallest income at time 0. The curve would end on the upper horizontal axis at some point D and DC would represent the proportion of individuals who at time 0 had an income higher than the highest income at time 1.

In the particular and exceptional case where the highest income at time t+1 would be lower than the lowest income at time t, the distributional change curve would become identical to the broken curve OGC. In such a case we know that there would be no overlap between the distributions f(x) and k(x) and the index of distributional change defined previously would evidently tend towards -1. The complement to -1 of such an index would hence be a measure of the degree of overlap in the case of negative growth.

Figure 2: Distributional Change Curve

in the Case of Pure Negative Growth



2.3. Distributional Change: The "Pure Shape Effect"

Assume now that we compare the cumulative distributions $H(x)$ (on the vertical axis) and $K(x)$ (on the horizontal axis). By construction (see section 2.2. above) these two distributions have the same median incomes. If for each income x we now plot the percentage of individuals with an income lower than or equal to x observed in the distributions $H(x)$ and $K(x)$, we will find out that, as expected, the distributional change curve obtained will pass through the point (0.5,0.5) since these two distributions have the same median. We also know that, by definition, the slope of this curve is positive. As a consequence up to the point (0.5,0.5) the curve will be located in the lower left square of size 0.5 by 0.5 while beyond the point (0.5, 0.5) the curve will be located in the upper right square of size 0.5 by 0.5.

The signed sum of the areas lying between such a "pure shape related" distributional change curve and the diagonal would hence be a good measure of such a distributional change. Note however that here again in computing this sum we have to give a positive sign to any area below the diagonal and a negative sign to any area lying above the diagonal.

2.4. Measuring the change in bi-polarization

We can also derive from the plot of $H(x)$ (on the vertical axis) versus $K(x)$ (on the horizontal axis) a measure of change in bi-polarization, as will now be shown.

If for any income x below the median we know that the proportion of individuals with an income lower than or equal to x is higher for distribution $H(x)$ than for distribution $K(x)$, then, up to the median, the curve obtained will not only be in the lower left square of size 0.5 by 0.5 but it will also lie above the diagonal. Conversely if for any income x above the median the proportion of individuals with an income higher than or equal to x is lower for distribution

$H(x)$ than for distribution $K(x)$, then, beyond the median, the curve obtained will not only be in the upper right square of size 0.5 by 0.5 but it will also lie below the diagonal. In fact the more distant the curve obtained is from the diagonal in these two 0.5 by 0.5 squares, the more bi-polarization there is in the distribution $H(x)$ in comparison to the distribution $K(x)$.

We can therefore measure the relative bi-polarization of the distribution $H(x)$ in comparison to the distribution $K(x)$ by the sum of the areas lying between the curve and the diagonal, in the two 0.5 by 0.5 squares previously defined. These areas will however be given here a positive value if in the lower left square the curve lies above the diagonal and if in the upper right square it lies below the diagonal.

Since the curve may cross once or more the diagonal, we will more generally define as follows the sign given to the areas lying between the diagonal and the curve. In the lower left square of size 0.5 by 0.5, any area lying between the curve and the diagonal which is located below the diagonal will be given a negative sign while any area lying between the diagonal and the curve which is located above the diagonal will be given a positive sign. Conversely in the upper right square of size 0.5 by 0.5, any area lying between the curve and the diagonal which is located below the diagonal will be given a positive sign while any area lying between the curve and the diagonal which is located above the diagonal will be given a negative sign. The sum of all these areas will then be considered as a measure of the relative bi-polarization of the distribution $H(x)$ when compared to the distribution $K(x)$.

Since the higher this relative bipolarization, the lower the relative importance of the middle class at time 1 when compared to time 0, we have here a measure of the change in the relative importance of the middle class that took place between two time periods (ignoring the impact of economic growth).

3. An Empirical Illustration: On Changes in Bi-Polarization in Latin America between 2000 and 2009:

3.1. Estimating the standards of living in Latin American countries:

The 2000 and 2009 Latinobarómetro surveys do not provide any data on the actual income of individuals. But these surveys provide information on the durables goods at the disposition of the individuals as well as on their access to a certain number of services. Eleven durables goods or types of access to basic services were taken into account: television, refrigerator, home, personal computer, washing machine, phone, car, second home, access to drinking water, access to hot water and sewage facilities. To estimate individual standards of living we use correspondence analysis. This correspondence analysis was implemented *separately* for each country.

Correspondence analysis

Correspondence Analysis (CA) was first developed by Benzécri and Benzécri (1980). It aims at analyzing simple two-way tables where some measure of correspondence is assumed to exist between the rows and columns. It allows one to obtain a graphical display of row and column points in biplots, which helps discovering some structural relationships that may exist between the variables and the observations.

Correspondence analysis (CA) is in a certain way similar to principal components analysis (PCA) but principal components analysis should be used when the variables are continuous, whereas correspondence analysis is mainly applied to the case of contingency tables, that is, when the variables take only the values of 0 or 1.

If we asume a contingency table with, for example, I rows and J columns, correspondence analysis will give us a plot of $(I+J)$ points, I points corresponding to the rows and J points to the columns. It can be proven that if two row points are close on the bi-plot, then their conditional distributions across the columns are similar whereas if two column points are close this implies that their conditional distributions across the rows are similar.

The main outputs of correspondence analysis are principal components which are orthogonal and each component turns out to be a linear combination of the variables on one hand, the observations on the other.

Like PCA, CA is a data exploration technique that uncovers correlation patterns across sets of variables described by single components named principal components.

The main difference between such approaches and standard econometric approaches is that the dependent variable is unobserved. We therefore assume that welfare is a multidimensional latent variable.

There are at least two advantages in using CA in addition to its suitability for categorical data.

The first is that CA gives more weight to indicators with a smaller number of "hits". In other words if, for example, for a dimension of the standard of living like having a refrigerator, we observe that only a few individuals have a refrigerator, then these individuals will be given a higher weight.

The second property is reciprocal bi-additivity. This property states that the composite "standard of living" score of an individual is the simple average of the factorial weights of the "standard of living" categories for this individual and that the weight of a given dimension of "standard of living" is the simple average of the composite "standard of living" scores of the population units that belong to the given dimension.

Deriving the distributional change curves:

By comparing the standards of living at times 0 and 1, we can now derive the various distributional change curves.

The Overall Distributional Change

The results of the computation of the measures of the overall, "pure growth related" and "shape related" distributional change are summarized in Table 1 below.

Table 1: Values of the overall, "pure growth based" and "shape related" distributional change measures for the period 2000-2009 in various Latin American countries.

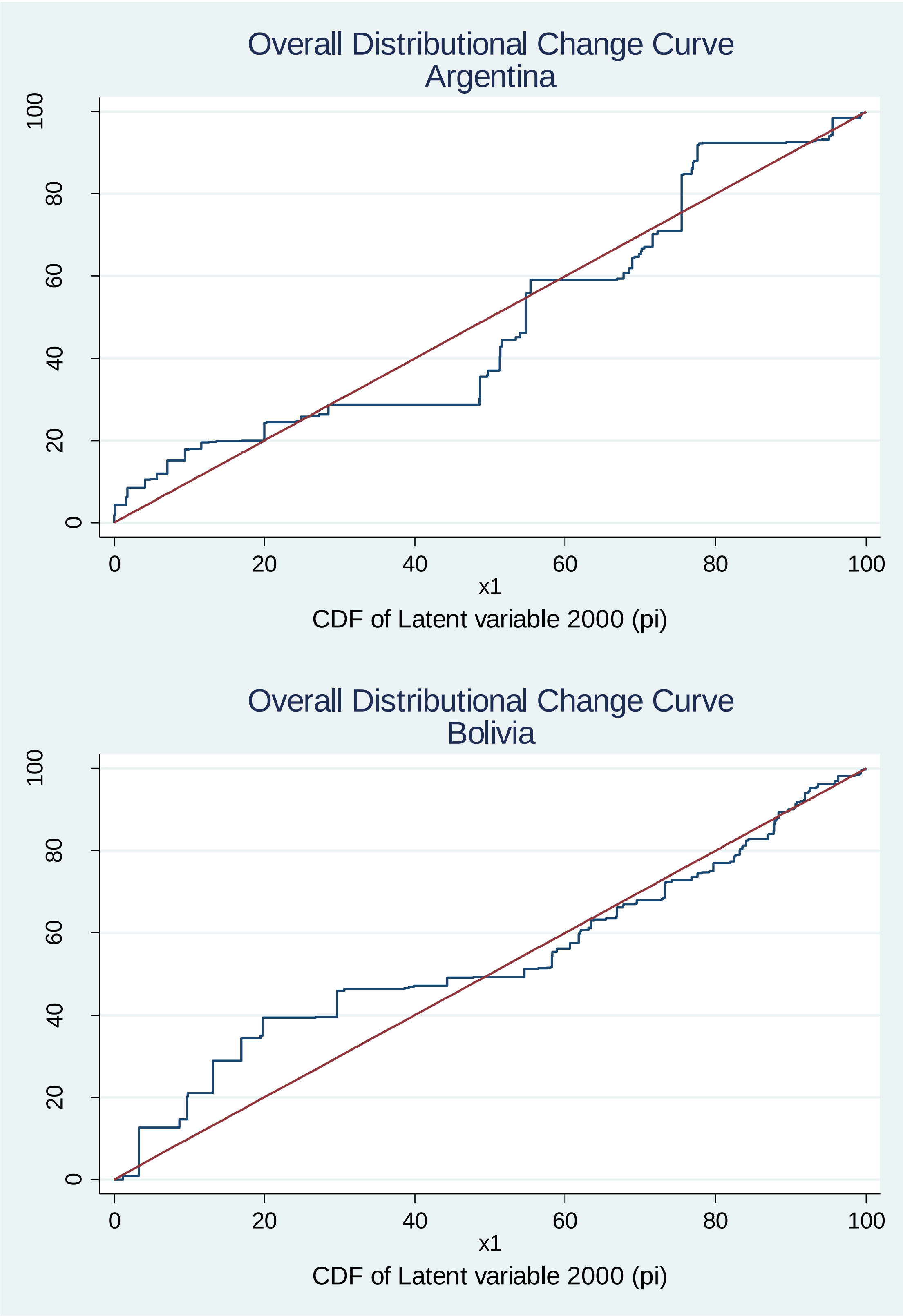
Country	Overall distributional change	"Pure Growth based" distributional change	"Shape related" distributional change for standards of living below the median	"Shape related" distributional change for standards of living above the median	Total "Shape related" distributional change
Argentina	0.0083	0.1515	-0.0626	-0.0485	-0.1111
Bolivia	-0.0360	0.0110	-0.0487	0.0097	-0.0390
Brazil	-0.0169	0.0648	-0.0432	-0.0061	-0.0493
Colombia	-0.0254	0.0435	-0.0374	-0.0098	-0.0472
Costa_Rica	0.0077	0.0761	-0.0404	-0.0250	-0.0654
Chile	-0.0076	0.1597	-0.0747	-0.0269	-0.1016
Ecuador	-0.0575	0.0739	-0.0891	-0.0084	-0.0975
El_Salvador	0.0116	0.1091	-0.0747	0.0042	-0.0705
Guatemala	-0.0616	-0.2177	0.0554	0.0758	0.1312
Honduras	-0.0715	-0.0852	0.0037	0.0122	0.0159
Mexico	-0.0007	0.0509	-0.0307	-0.0017	0.0324
Nicaragua	-0.0308	-0.0706	0.0277	0.0237	0.0514
Panama	-0.0491	0.0545	-0.0757	0.0007	-0.0750
Paraguay	-0.0397	-0.0301	-0.0382	0.0159	-0.0223
Peru	-0.0306	0.0416	-0.0519	0.0000	-0.0519
Uruguay	0.0161	0.1043	-0.0560	-0.0229	0.0789
Venezuela	0.0260	-0.0366	0.0494	0.0009	0.0503

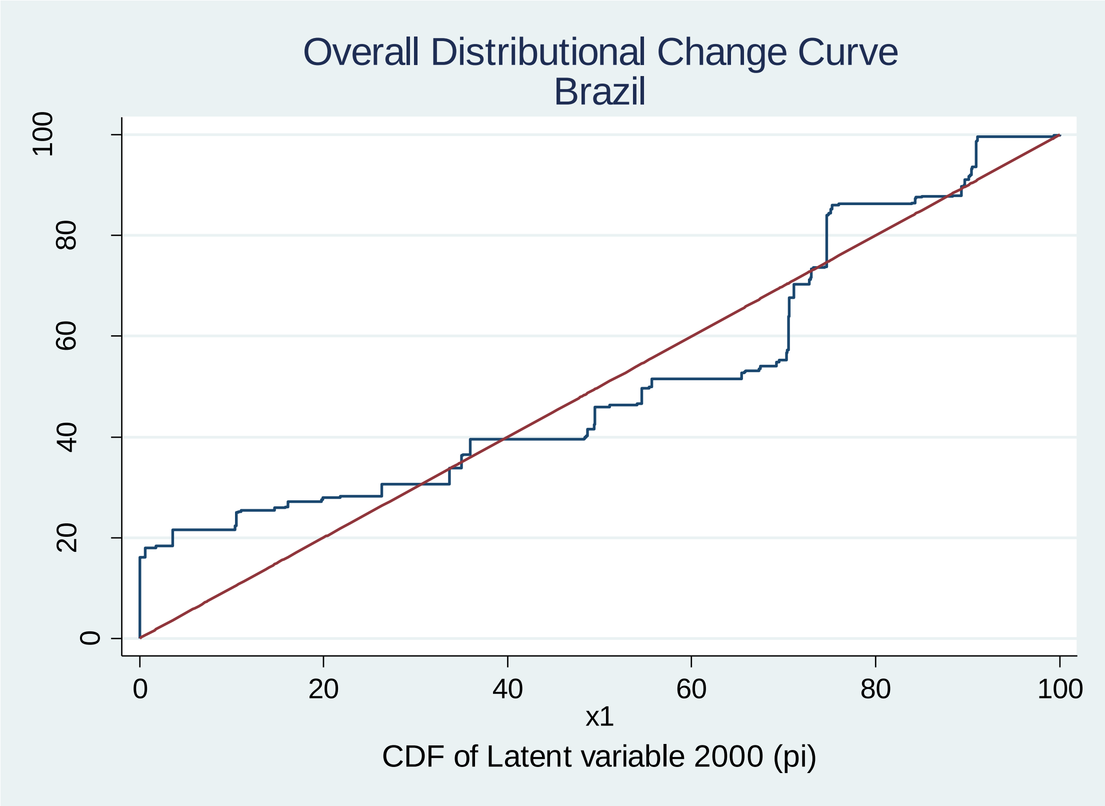
It appears that the overall distributional change was positive only in Venezuela, Uruguay, El Salvador, Argentina and Costa Rica, the change being very small in the two last countries..

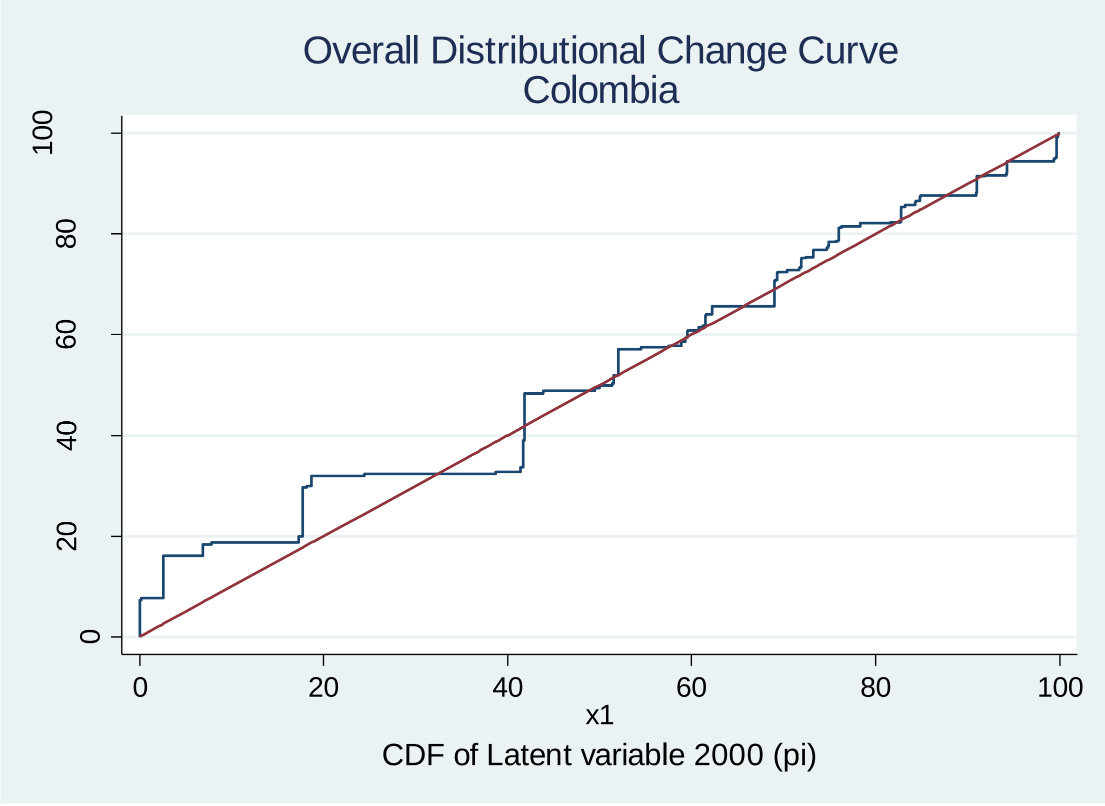
The highest negative values were observed in Honduras, Guatemala, Ecuador, Panama, Paraguay, Bolivia, Nicaragua and Peru. These negative values imply evidently that in these countries, as a whole, the standard of living was lower in 2009 than in 2000, although this was not necessarily true for all the segments of the population.

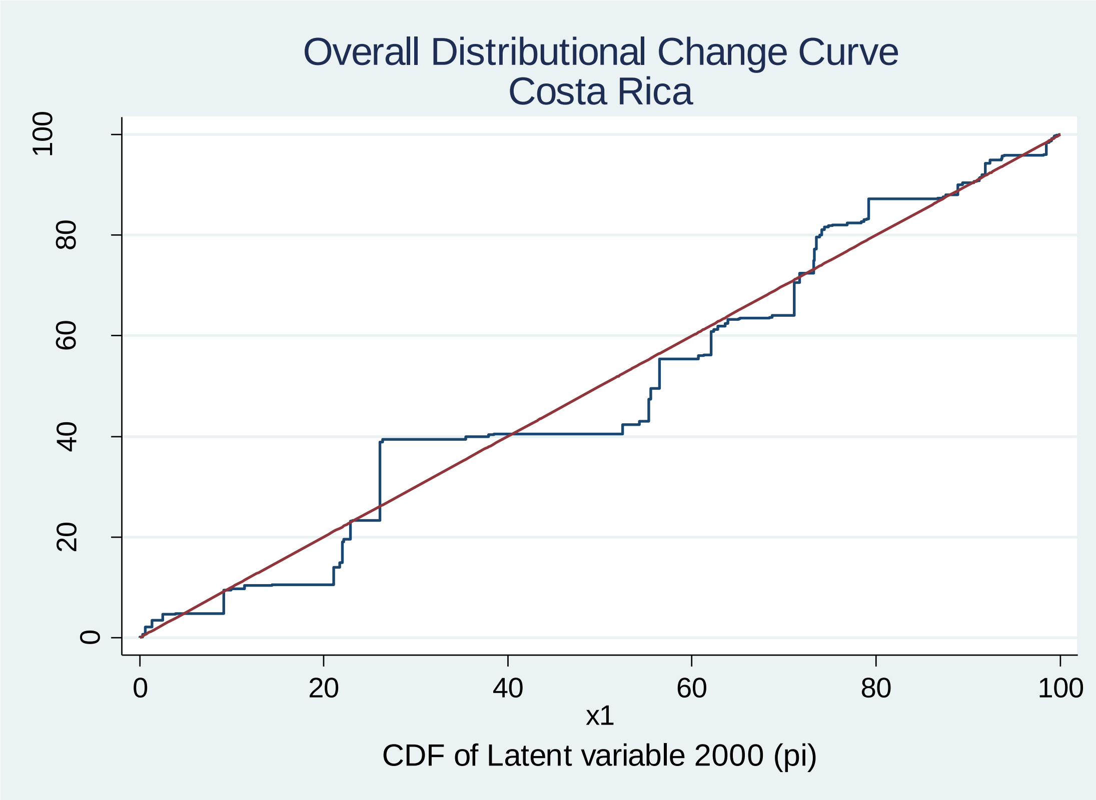
The graphical illustrations given below in Figure 3 allow one to find out which segments of the population seem to have improved their living conditions and for which segments the situation worsened between 2000 and 2009. Note however that if one observes a given area lying below the diagonal one can say that this segment of the population as a whole improved its standard of living. Nevertheless we cannot conclude that every centile in this segment improved its situation. Such a conclusion is true only for those centiles for which the slope of the distributional curve is smaller than one which is the slope of the diagonal. As a consequence there are centiles located in these areas lying below the diagonal whose situation worsened (when the slope of the curve is greater than one). Conversely if one observes an area lying above the diagonal, as a whole the subpopulation located in this area saw its situation becoming worse between 2000 and 2009 but for those centiles for which the slope is smaller than one, the situation in fact improved.

Figure 3: Overall distributional change, by country.

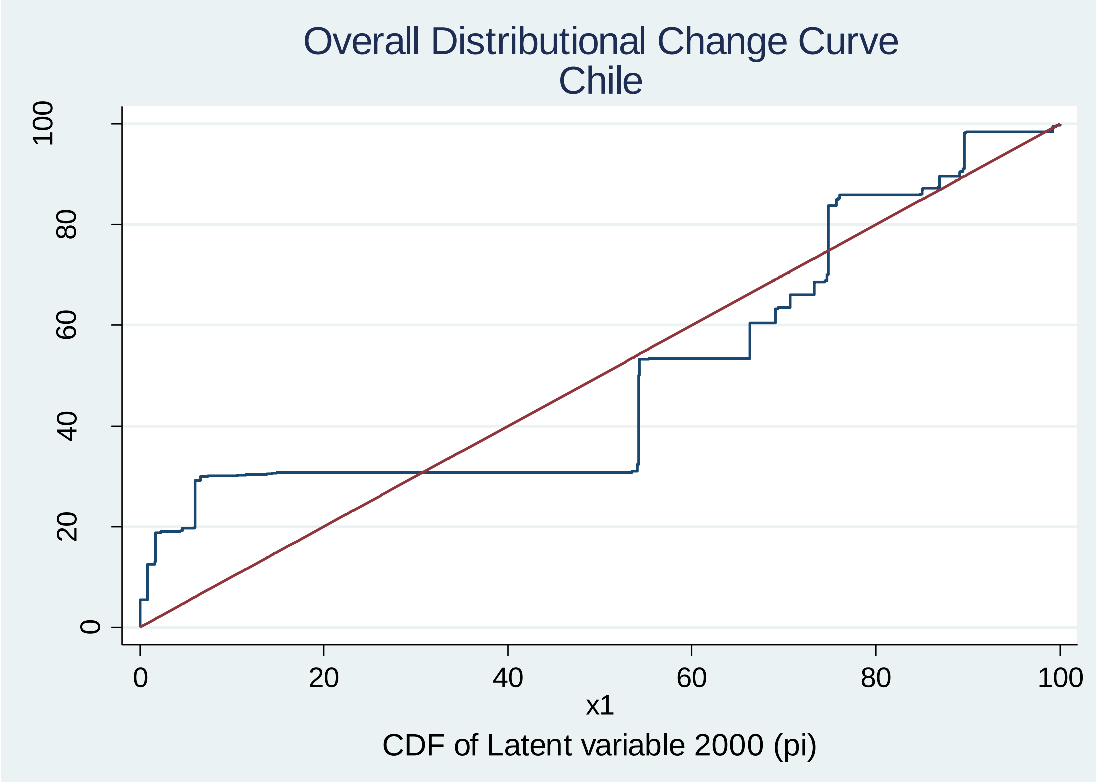



Overall Distributional Change Curve
Brazil
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

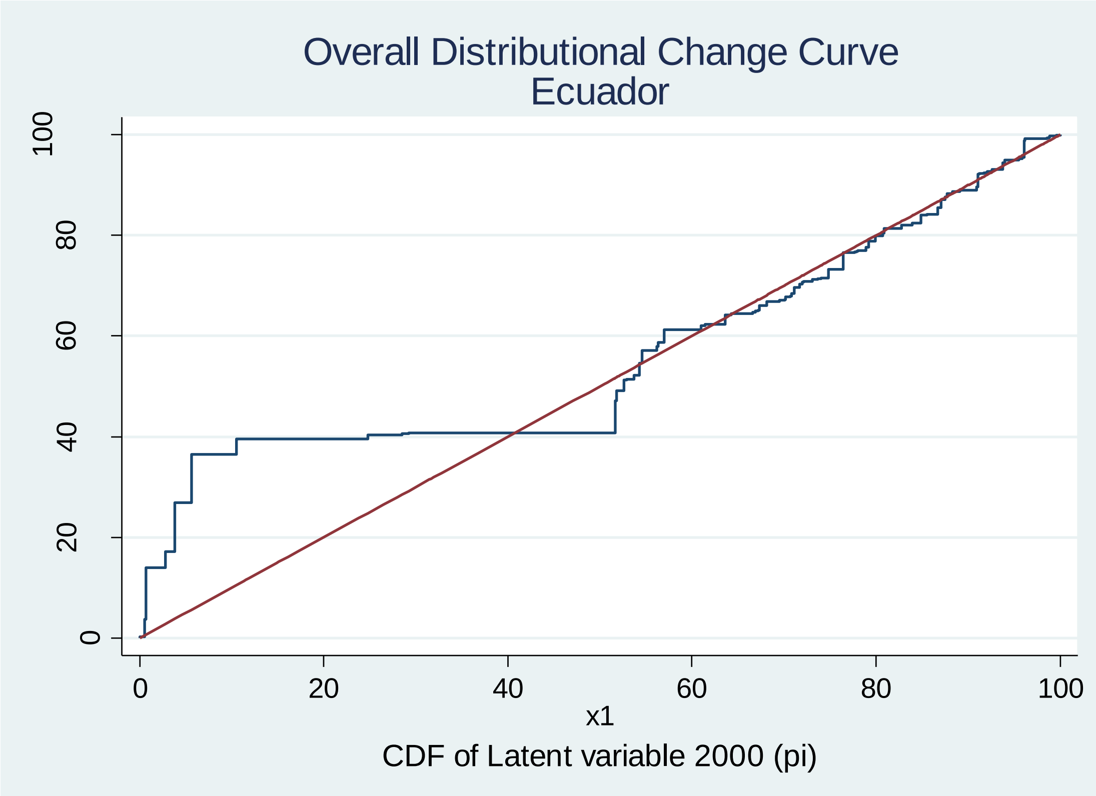

Overall Distributional Change Curve
Colombia
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

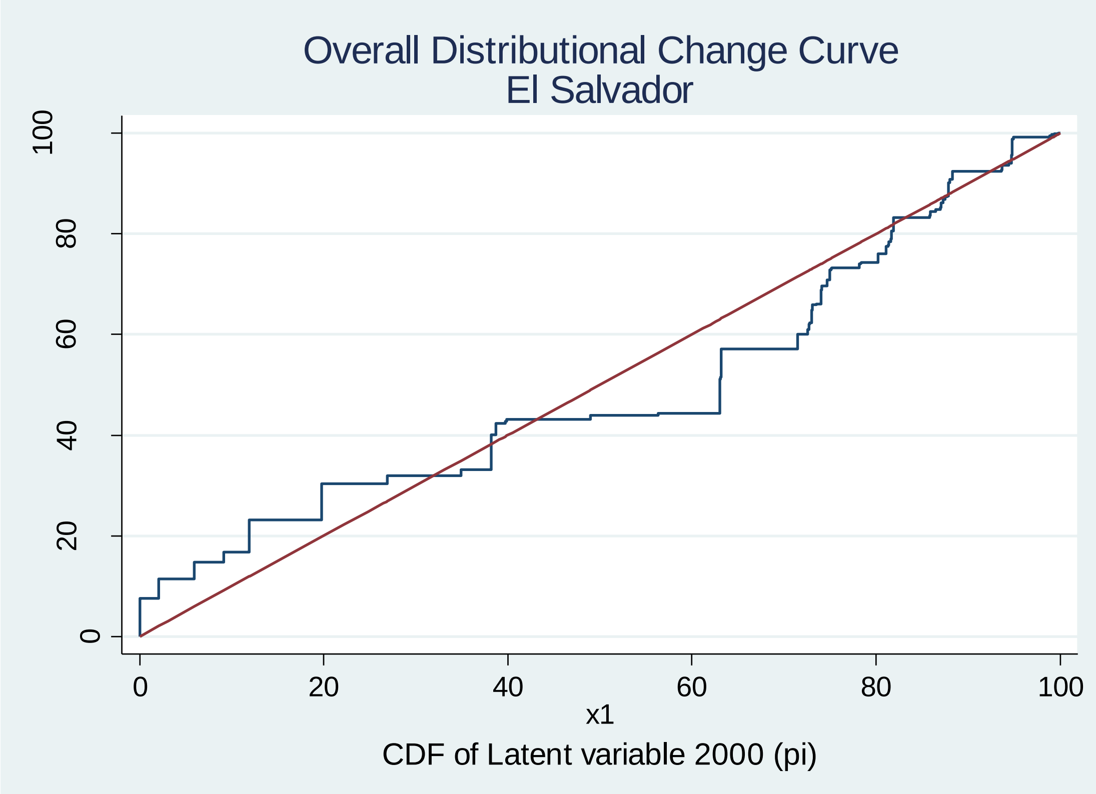


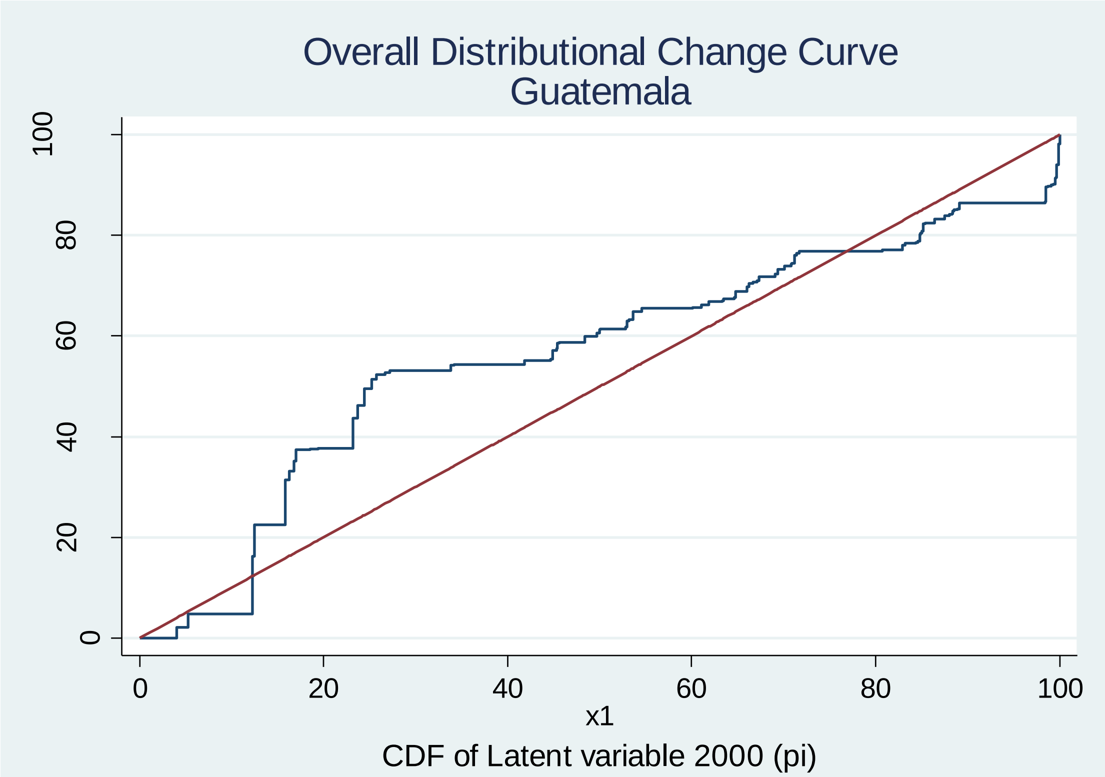
Overall Distributional Change Curve
Costa Rica
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



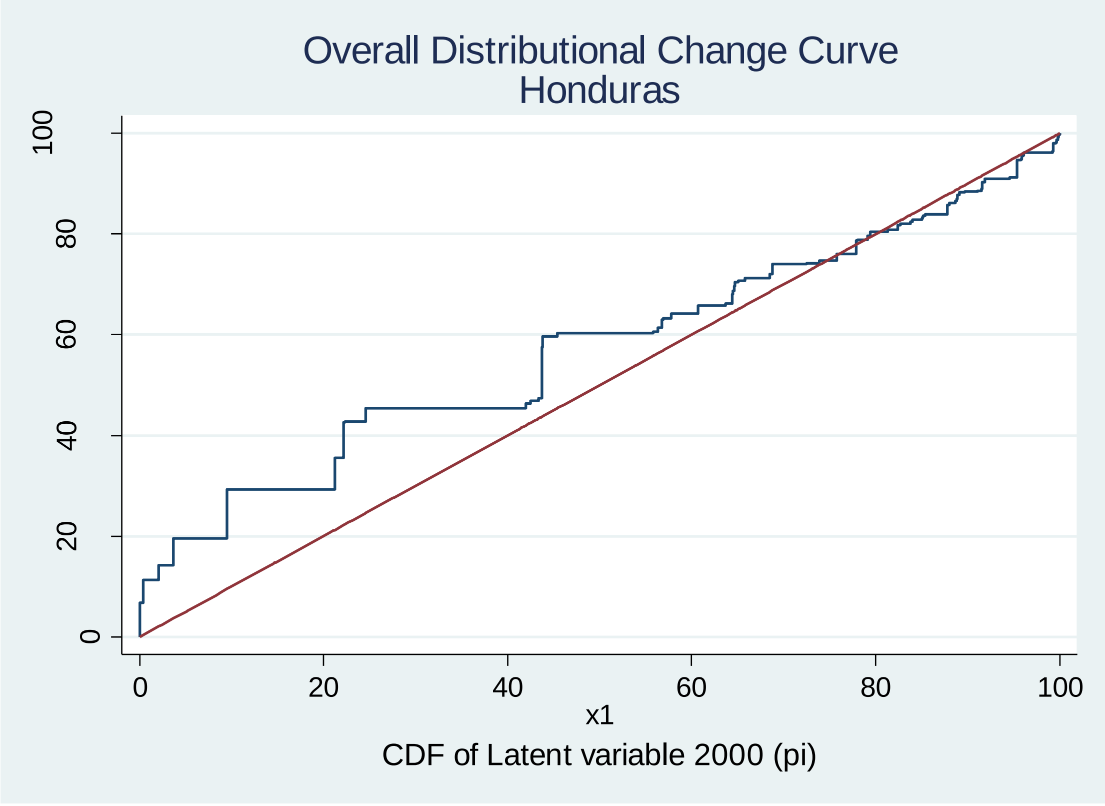
Overall Distributional Change Curve
Chile
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Ecuador
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

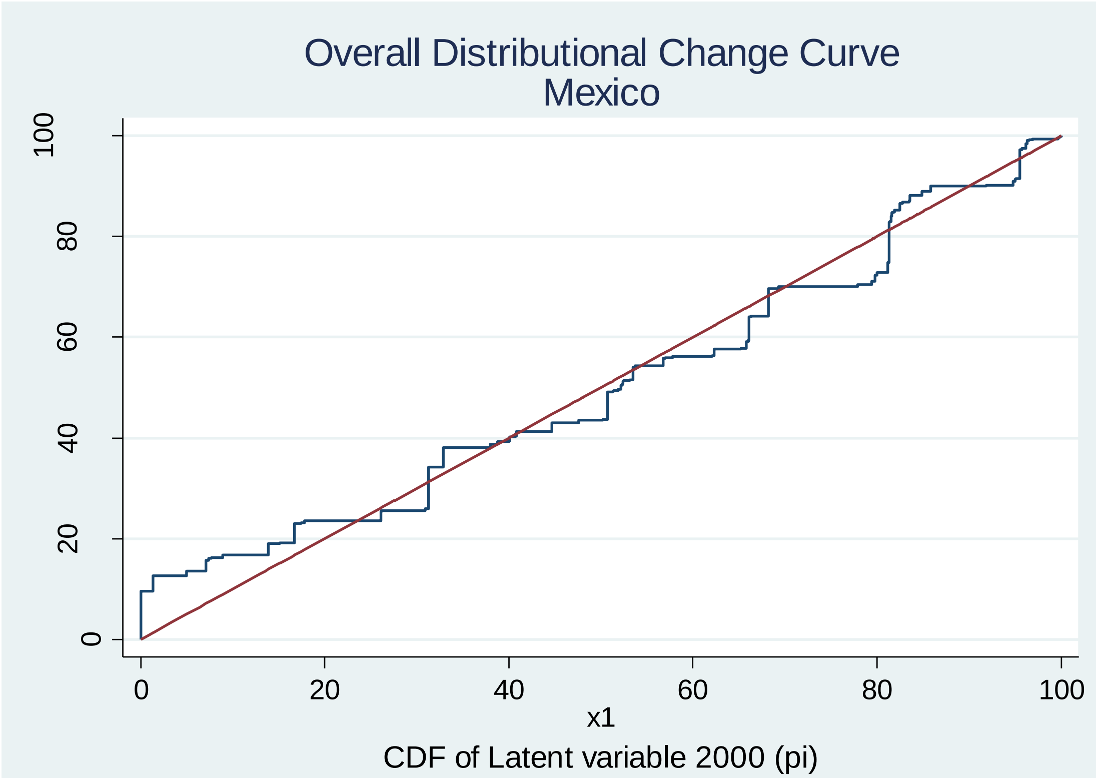

Overall Distributional Change Curve
El Salvador
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

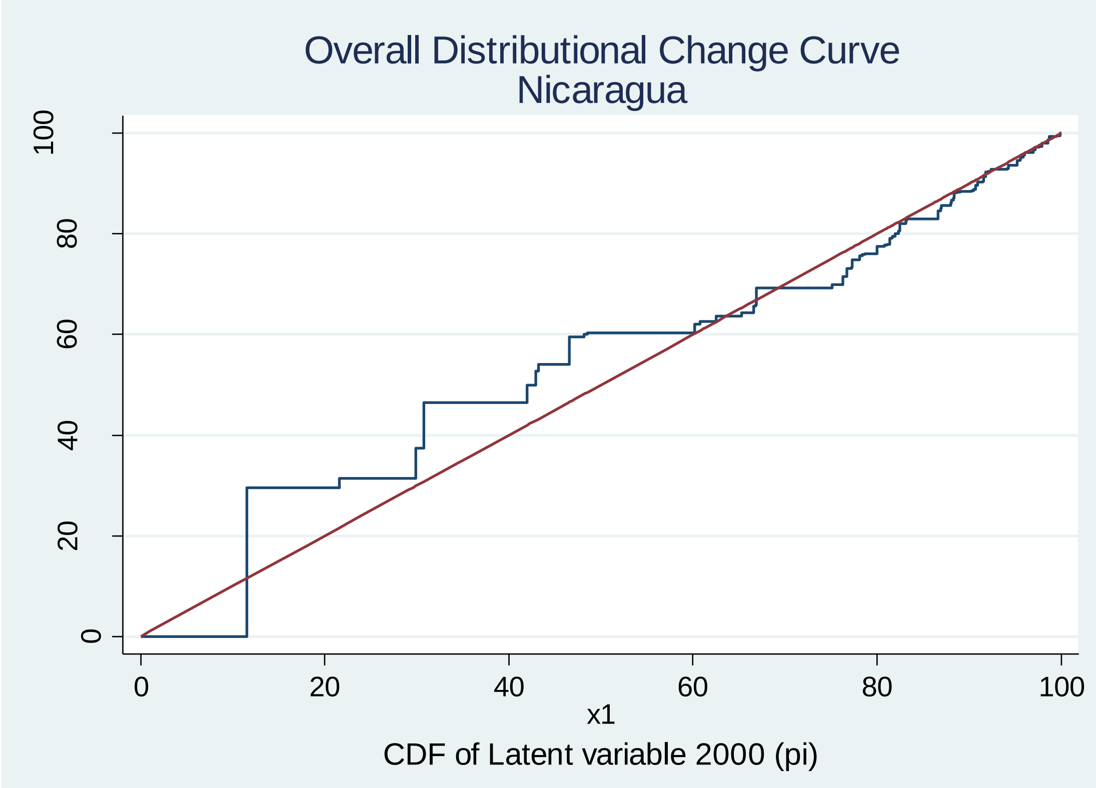


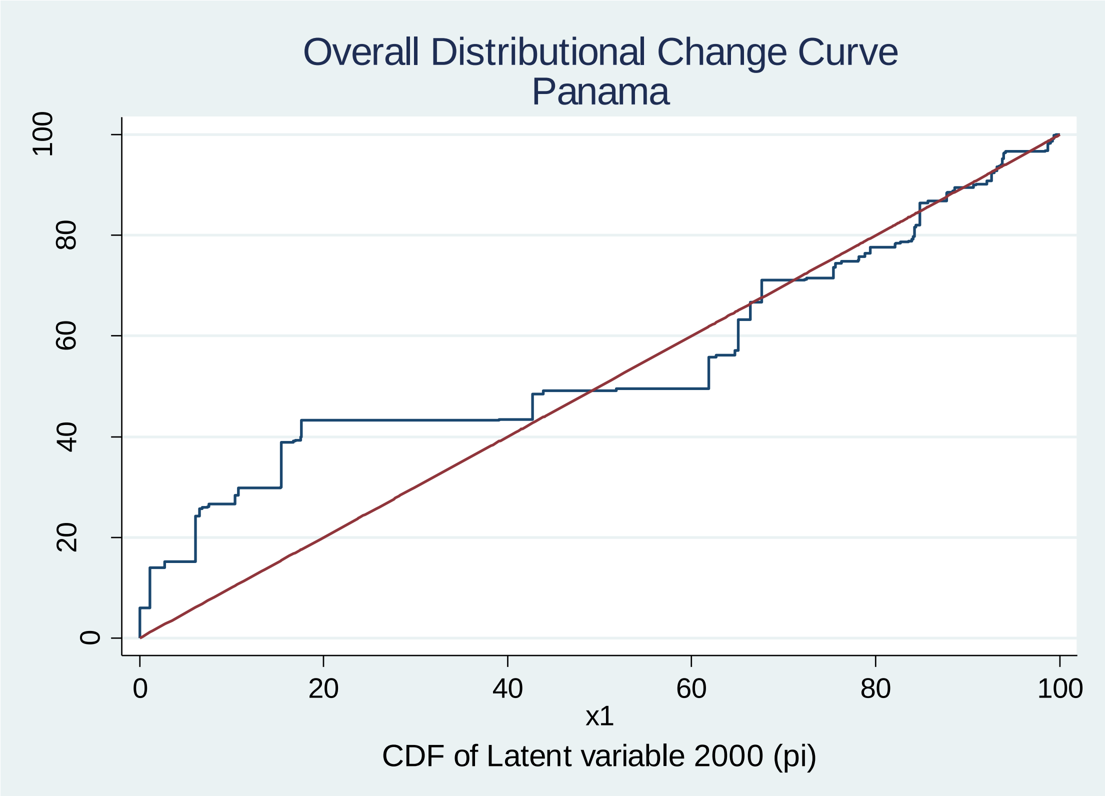
Overall Distributional Change Curve
Guatemala
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

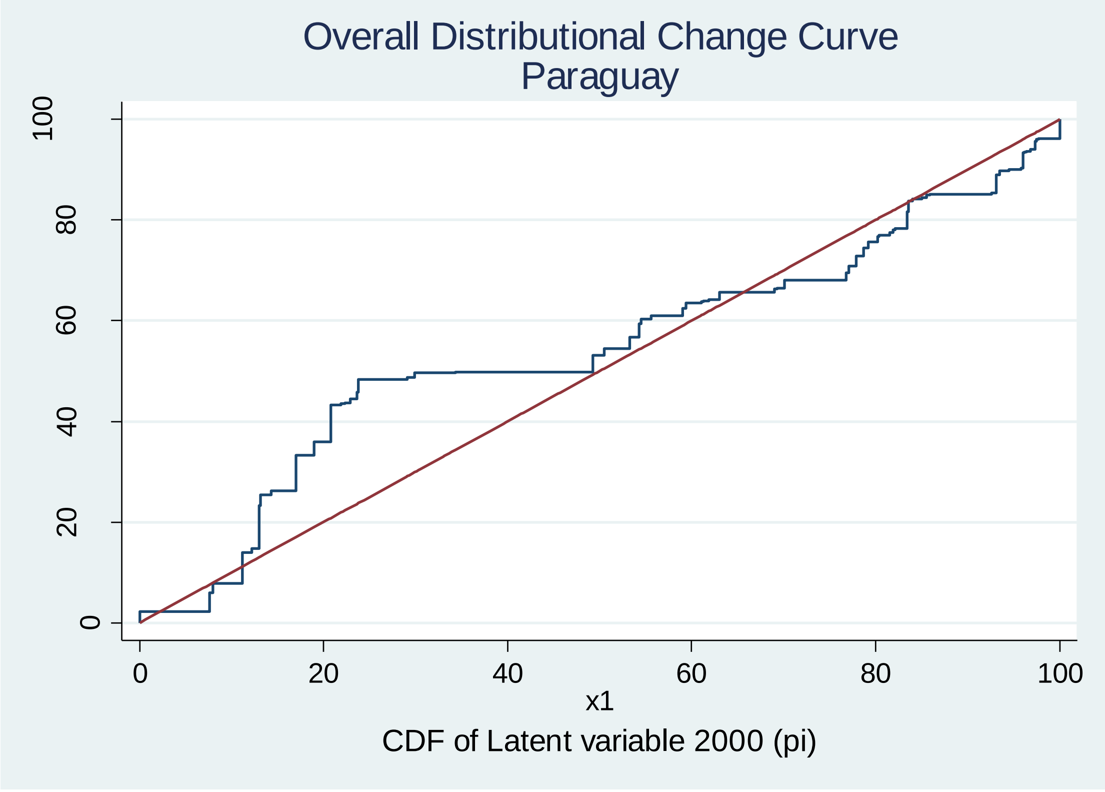


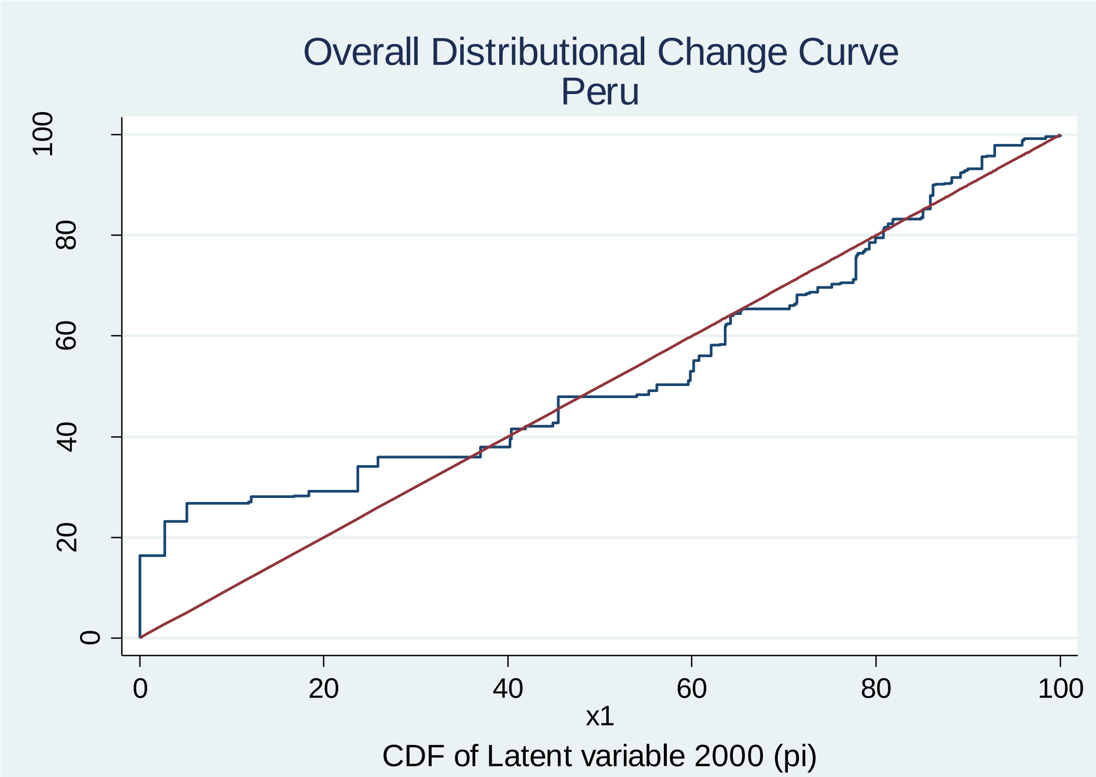
Overall Distributional Change Curve
Honduras
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

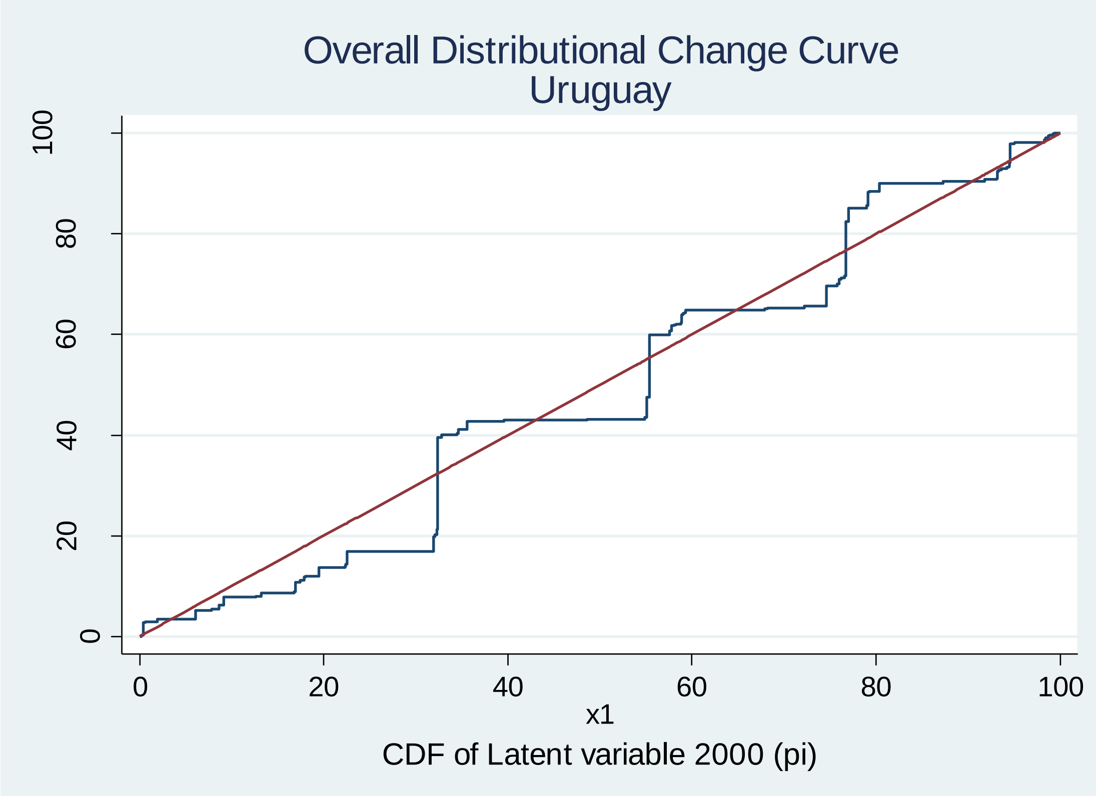

Overall Distributional Change Curve
Mexico
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

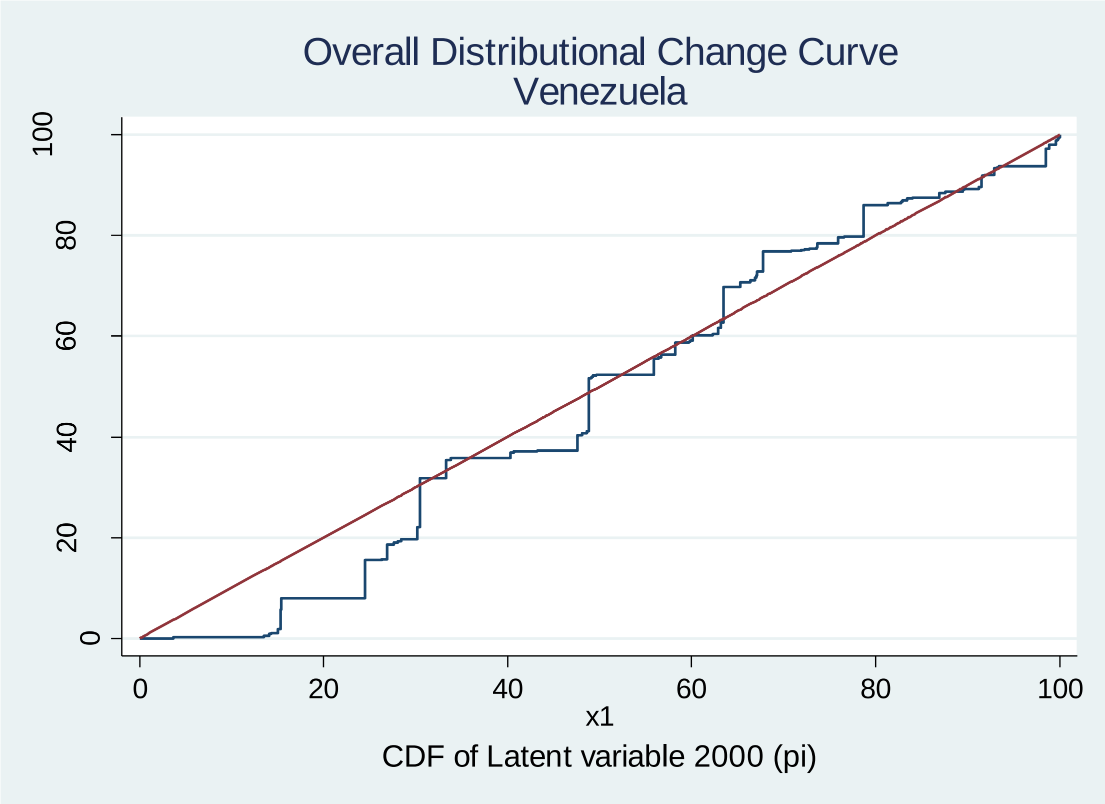

Overall Distributional Change Curve
Nicaragua
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Panama
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Paraguay
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Peru
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Uruguay
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Venezuela
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

"Pure growth related" distributional change

The results concerning the "pure growth related" distributional change are also given in Table 1. It appears that as a whole growth was highest in Chile, Argentina, El Salvador, Uruguay, Costa Rica and Brazil. On the other side the countries where as a whole growth was negative were Guatemala, Honduras, Nicaragua, Venezuela and Paraguay.

Table 2 gives information on the length of the "non-overlapping" segments while Table 3a gives the variation between 2000 and 2009 in the median standard of living of the different countries. Table 3b compares the ranking of the countries according to the average growth rate during the period 2001-2010 and to the value of the "pure growth related" index of distributional change. The index of rank correlation between these two measures turns out to be equal to 0.47.

We then show in Figure 4 the "pure growth related" distributional change curves for the various countries.

Table 2: Non overlapping segments of the "pure growth related" distributional change curves.

	Overall positive growth. Horizontal segment OA (in percentage)	Overall positive growth. Vertical segment CD (in percentage)	Overall negative growth. Horizontal segment FD (in percentage)	Overall negative growth. Vertical segment OE (in percentage)
Argentina	7.1	0.6		
Bolivia	1.2	0.3		
Brazil	1.8	0.6		
Colombia	2.6	0.1		
Costa_Rica	0.3	0.1		
Chile	7.4	0.4		
Ecuador	0.5	0.1		
El_Salvador	11.9	0.1		
Guatemala			16.6	11.6
Honduras			2.1	0.1
Mexico	6.4	0.2		
Nicaragua			11.5	0.1
Panama	0.3	0.1		
Paraguay			7.6	2.3
Peru	9.7	0.1		
Uruguay	1.9	0.2		
Venezuela			1.1	0.1

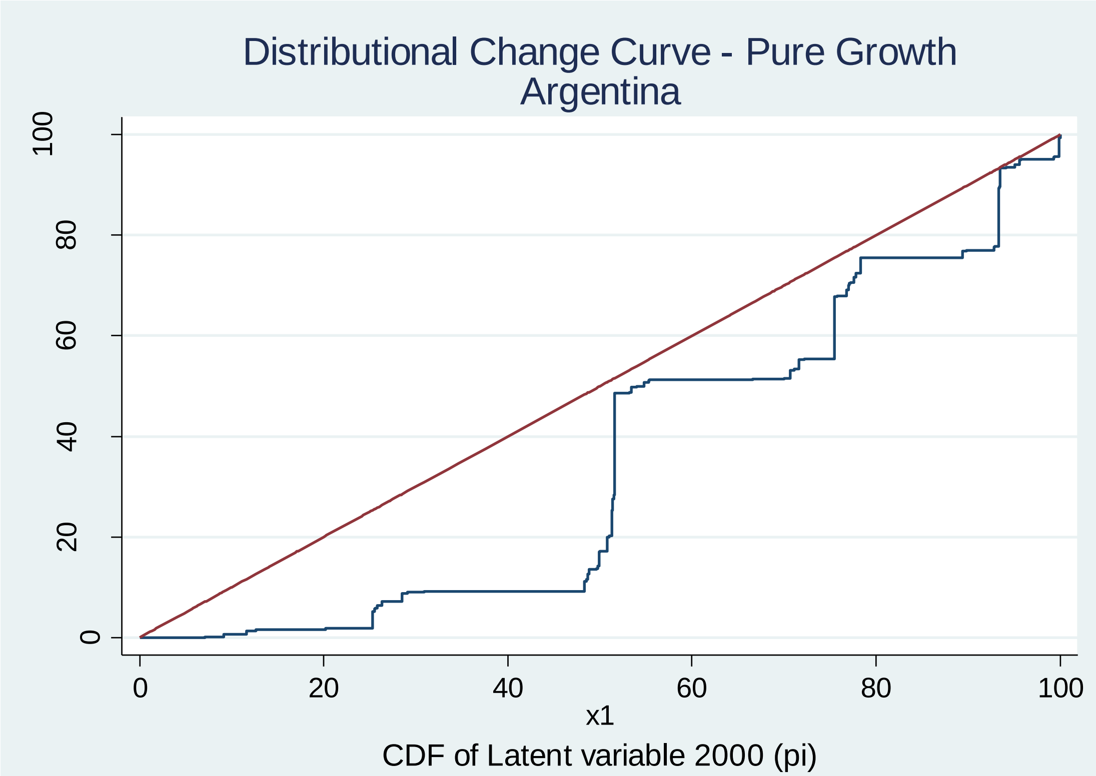
Table 3a: Variation in the Median Standard of Living in Various Latin American Countries between 2000 and 2009.

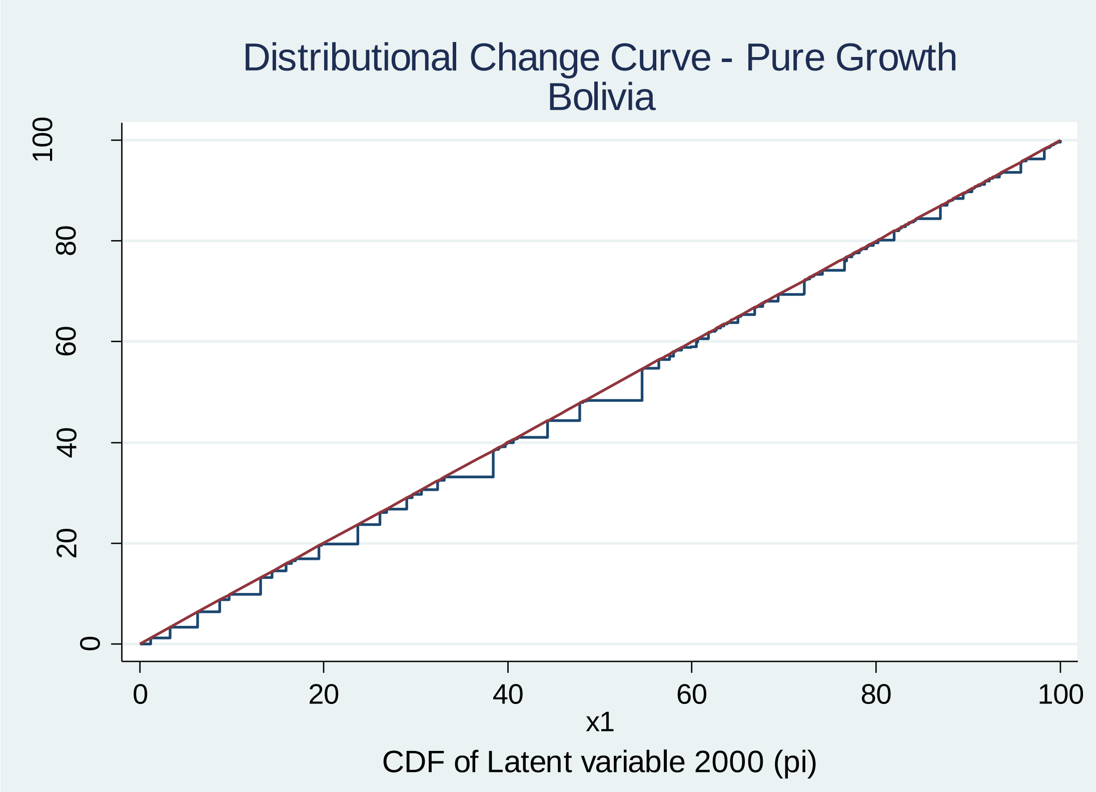
	Median Standard of Living		
	2000	**2009**	**Difference**
Argentina	-0.2118	-0.0687	0.1431
Bolivia	-0.2145	-0.2116	0.0029
Brazil	-0.1181	-0.0628	0.0553
Colombia	-0.2157	-0.1891	0.0266
Costa_Rica	-0.1755	-0.0891	0.0864
Chile	-0.2881	-0.1601	0.1280
Ecuador	-0.3496	-0.2961	0.0535
El_Salvador	-0.3677	-0.2551	0.1126
Guatemala	-0.0871	-0.4341	-0.3471
Honduras	-0.2133	-0.3040	-0.0906
Mexico	-0.1813	-0.1318	0.0496
Nicaragua	-0.2098	-0.3095	-0.0998
Panama	-0.2277	-0.1948	0.0328
Paraguay	-0.1409	-0.1719	-0.0310
Peru	-0.2934	-0.2541	0.0394
Uruguay	-0.2035	-0.1000	0.1035
Venezuela	-0.1052	-0.1368	-0.0317

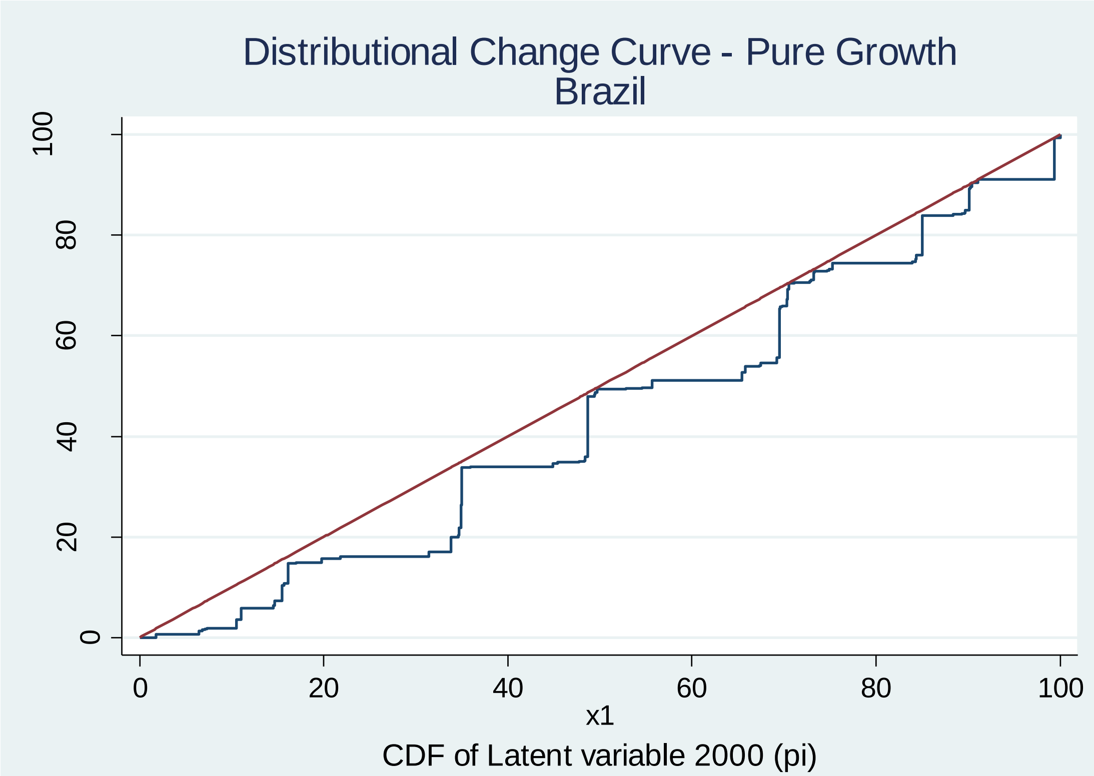
Table 3b: Ranking of per capita GDP growth rates versus ranking of "pure growth related" index of distributional change.

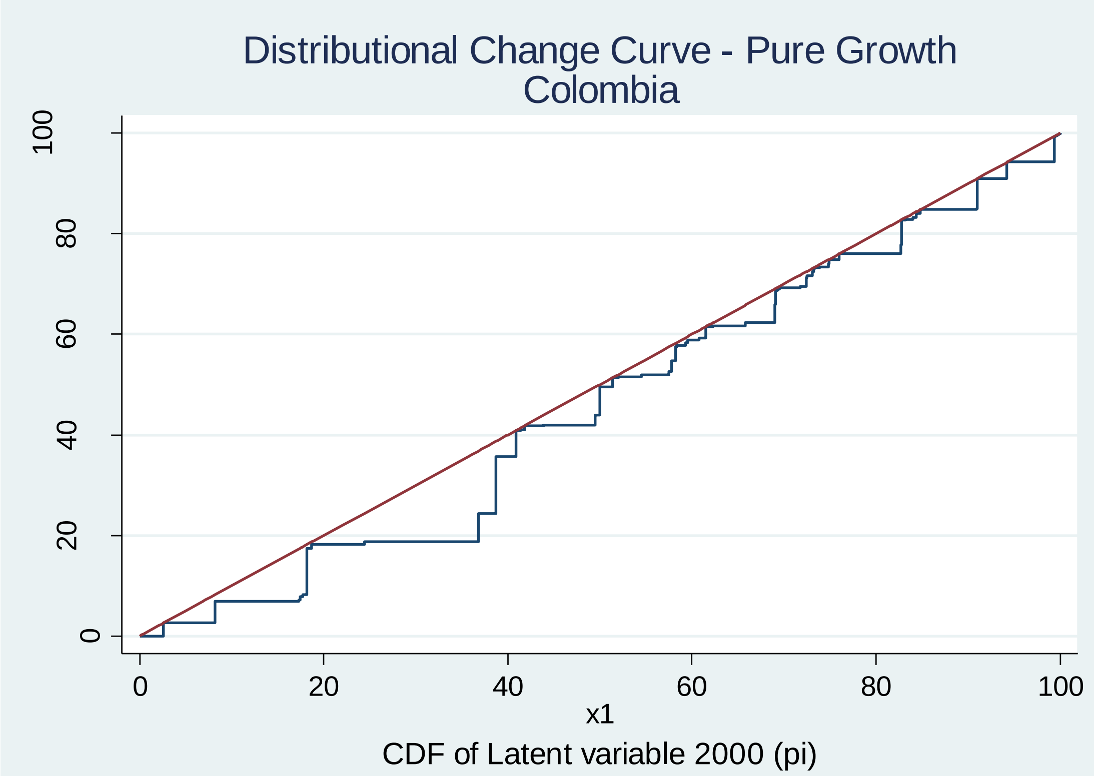
	Mean growth rate 2001-2010	Rank of mean growth rate	"Pure growth related" distributional change index	Rank of "Pure growth related" distributional change index	Rank difference
Argentina	3.61%	3	0.1515	2	1
Bolivia	2.10%	11	0.0110	12	-1
Brazil	2.46%	8	0.0648	7	1
Colombia	2.53%	7	0.0435	10	-3
Costa_Rica	2.38%	9	0.0761	5	4
Chile	2.66%	6	0.1597	1	5
Ecuador	2.98%	5	0.0739	6	-1
El_Salvador	1.57%	14	0.1091	3	11
Guatemala	0.80%	16	-0.2177	17	-1
Honduras	2.01%	12	-0.0852	16	-4
Mexico	0.58%	17	0.0509	8	9
Nicaragua	1.59%	13	-0.0706	15	-2
Panama	4.47%	2	0.0545	9	-7
Paraguay	2.16%	10	-0.0301	13	-3
Peru	4.48%	1	0.0416	11	-10
Uruguay	3.09%	4	0.1043	4	0
Venezuela	1.55%	15	-0.0366	14	1

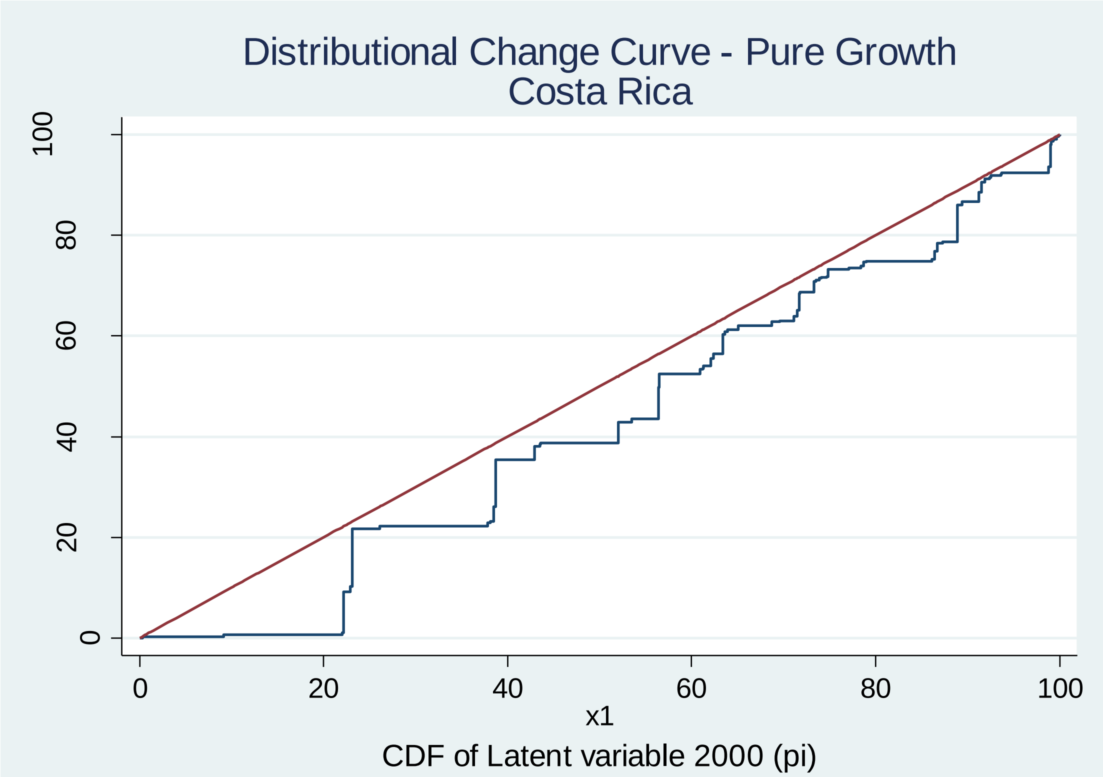
Figure 4: "Pure growth based" distributional change curves.



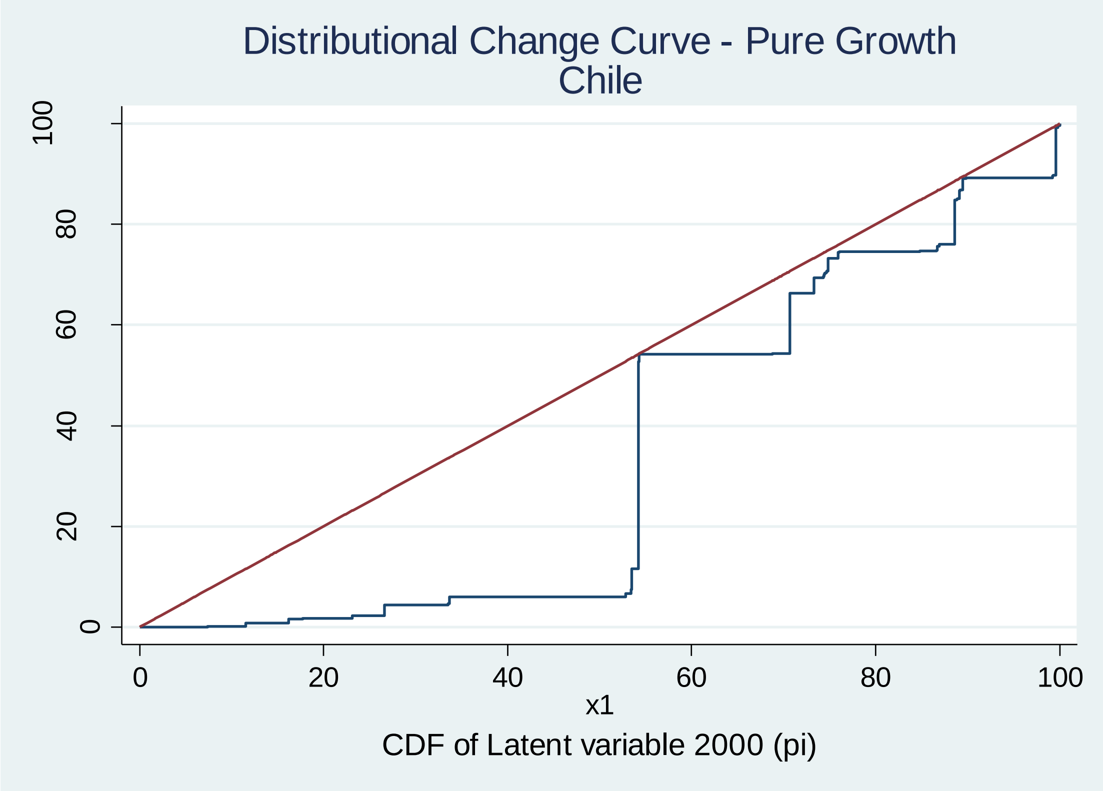



Distributional Change Curve - Pure Growth
Brazil
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

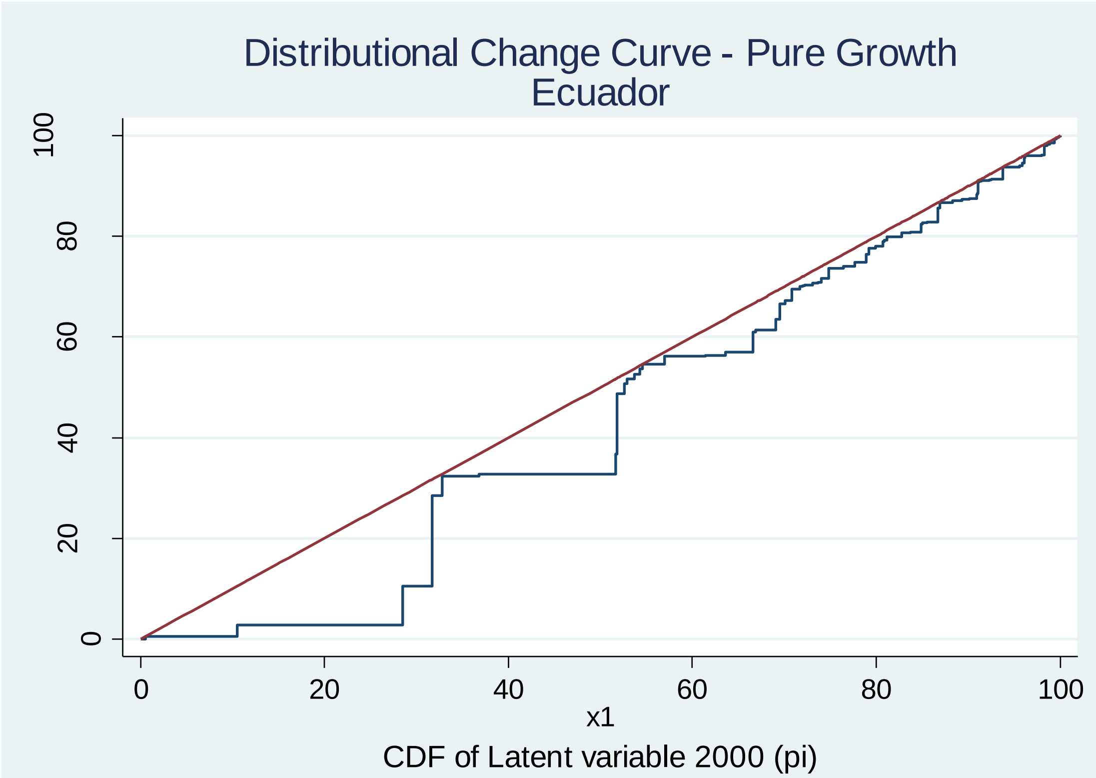

Distributional Change Curve - Pure Growth
Colombia
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

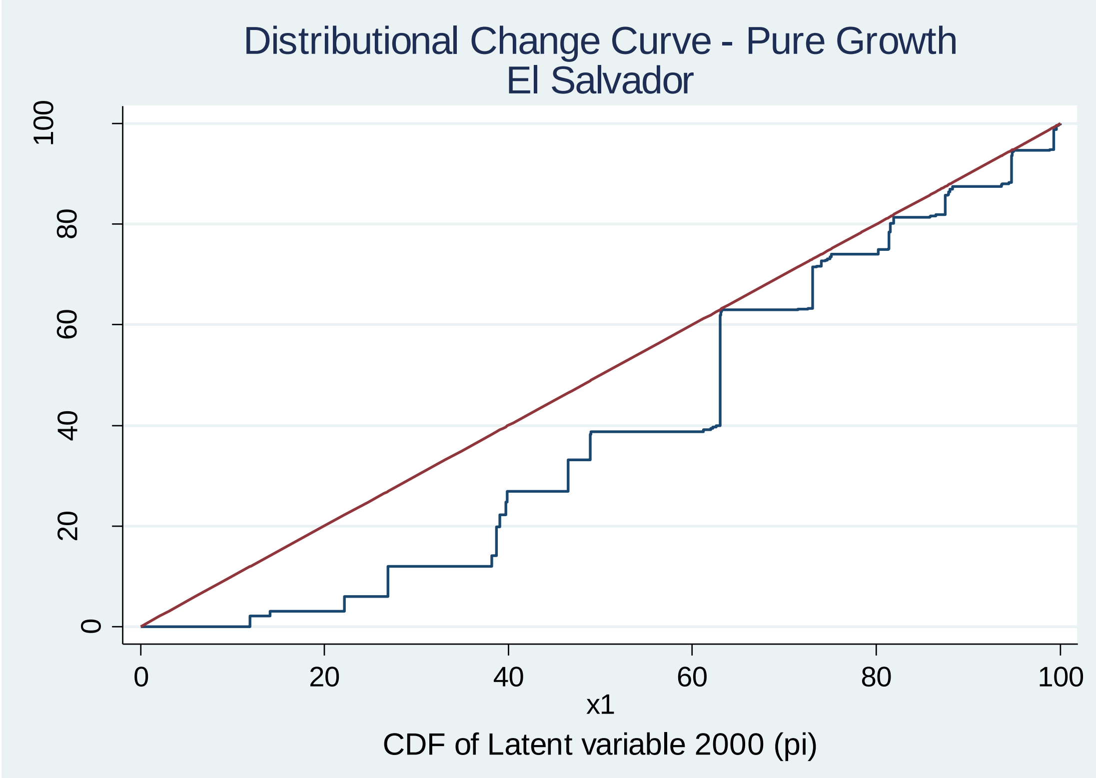


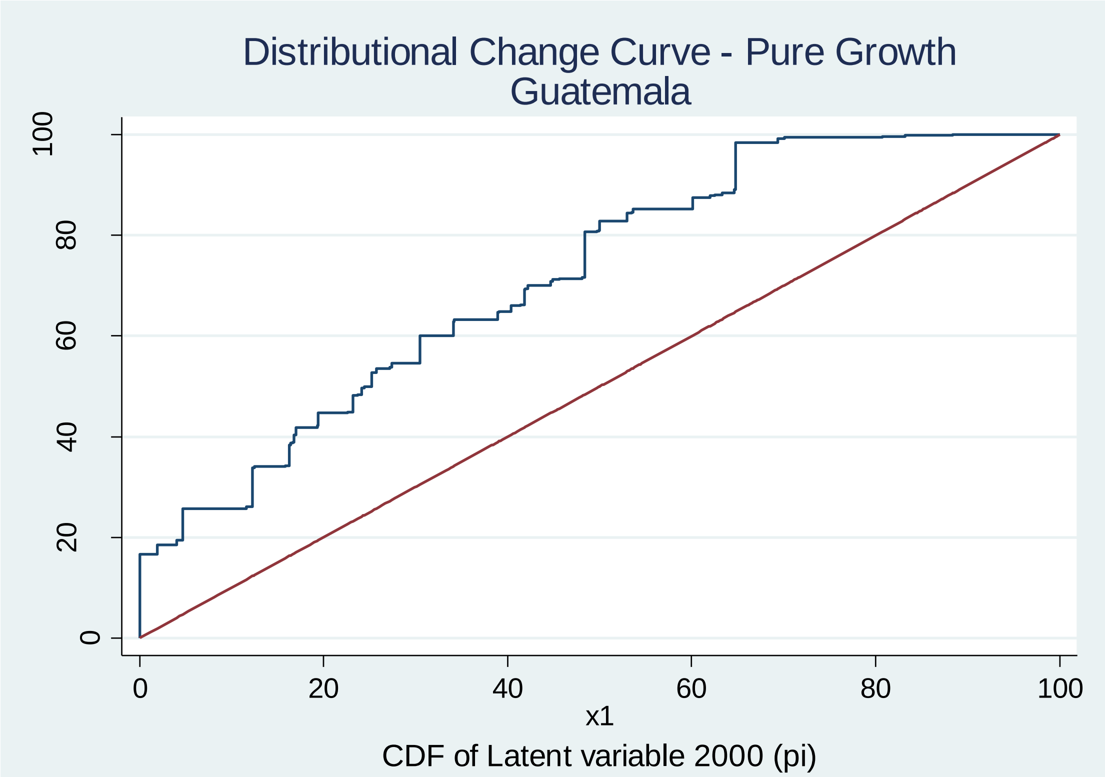
Distributional Change Curve - Pure Growth
Costa Rica
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



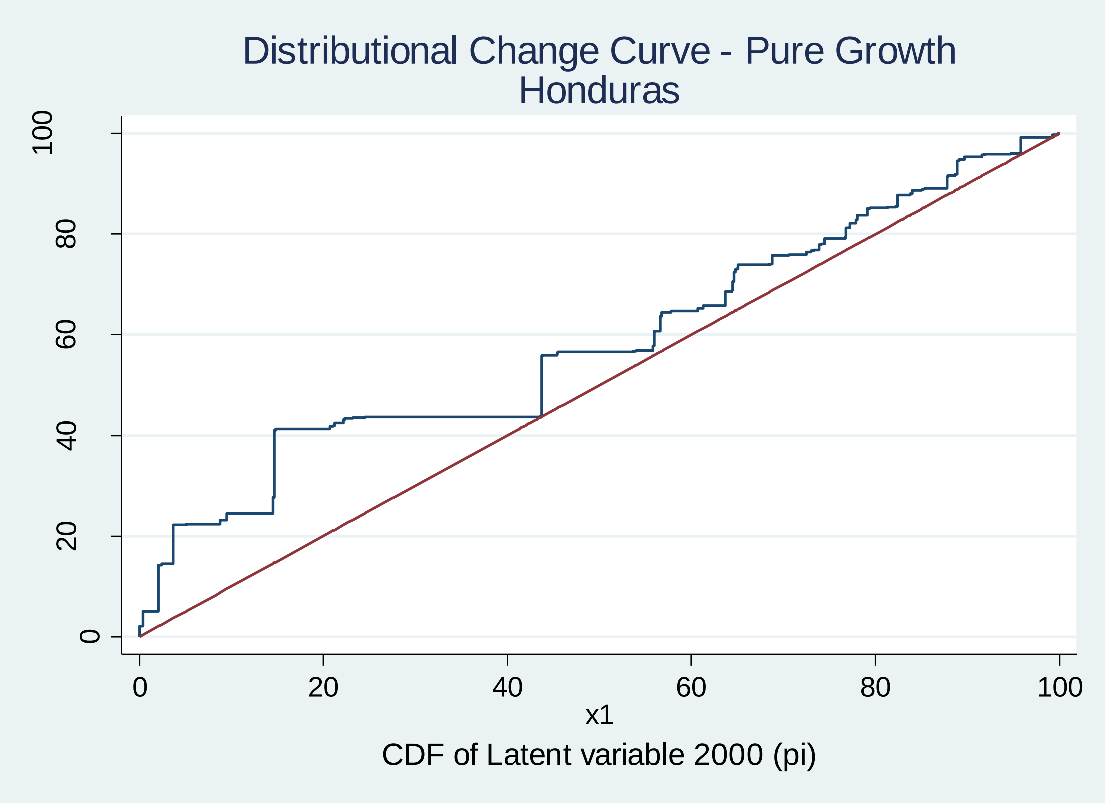
Distributional Change Curve - Pure Growth
Chile
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Distributional Change Curve - Pure Growth
Ecuador
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

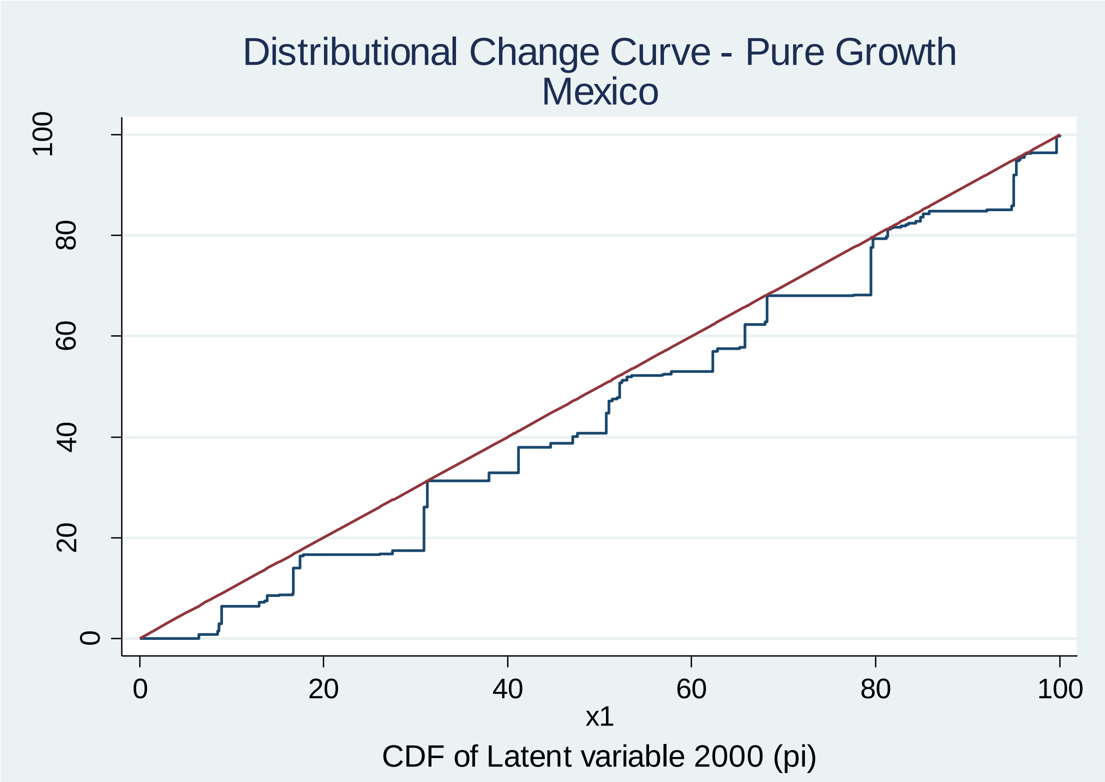

Distributional Change Curve - Pure Growth
El Salvador
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



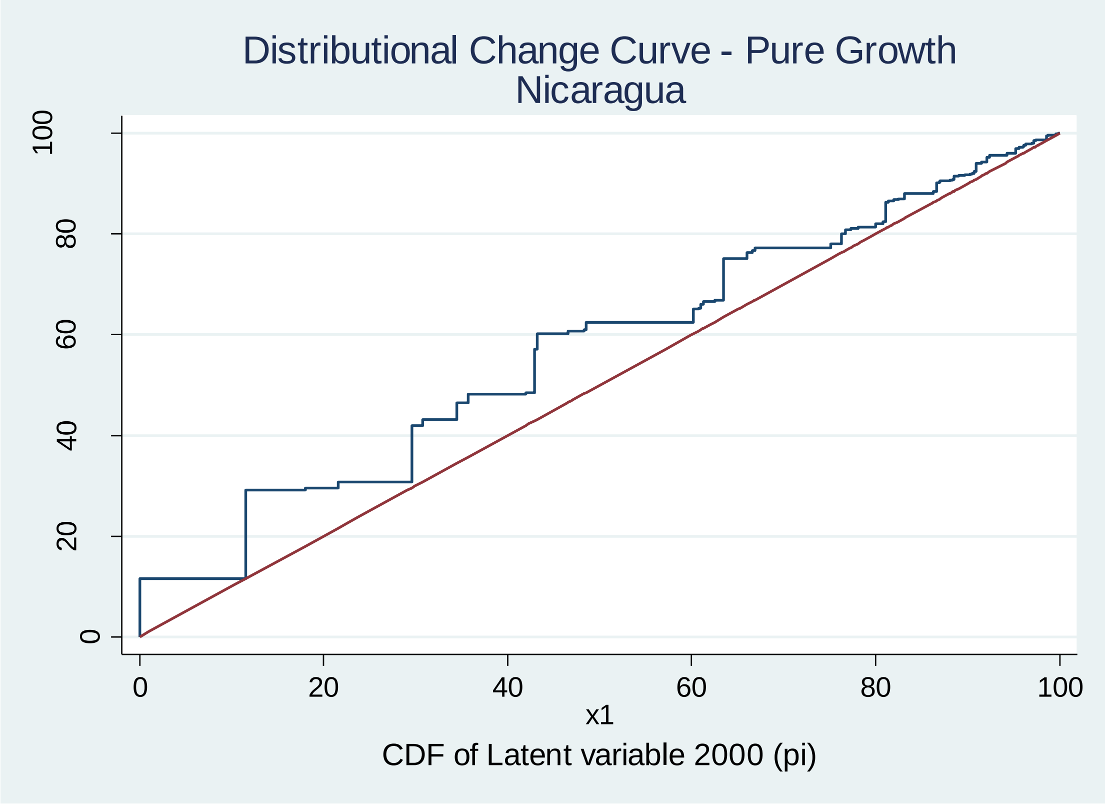
Distributional Change Curve - Pure Growth
Guatemala
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



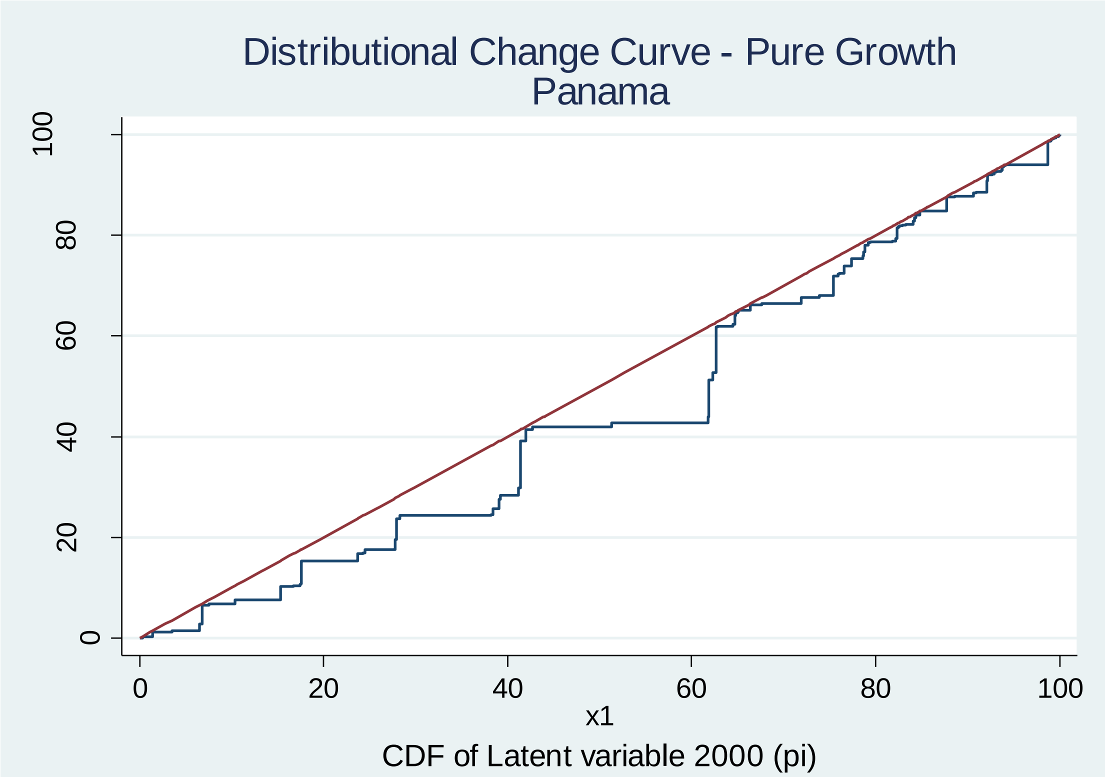
Distributional Change Curve - Pure Growth
Honduras
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

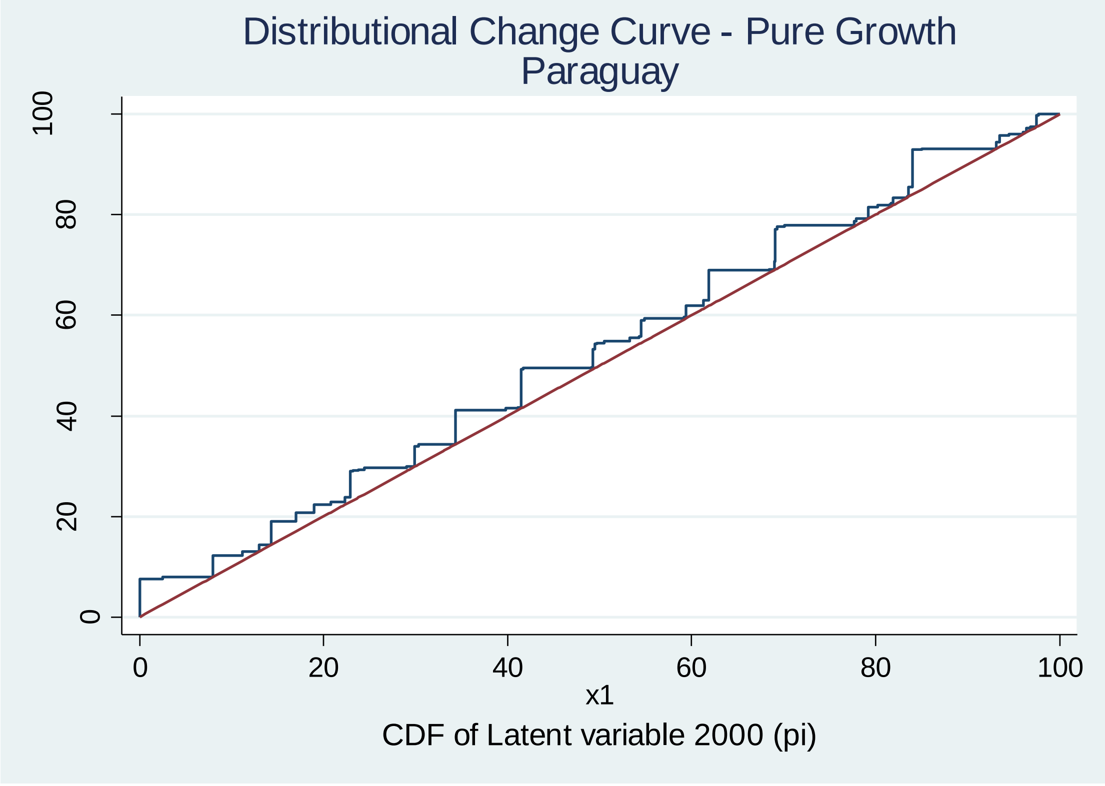


Distributional Change Curve - Pure Growth
Mexico
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

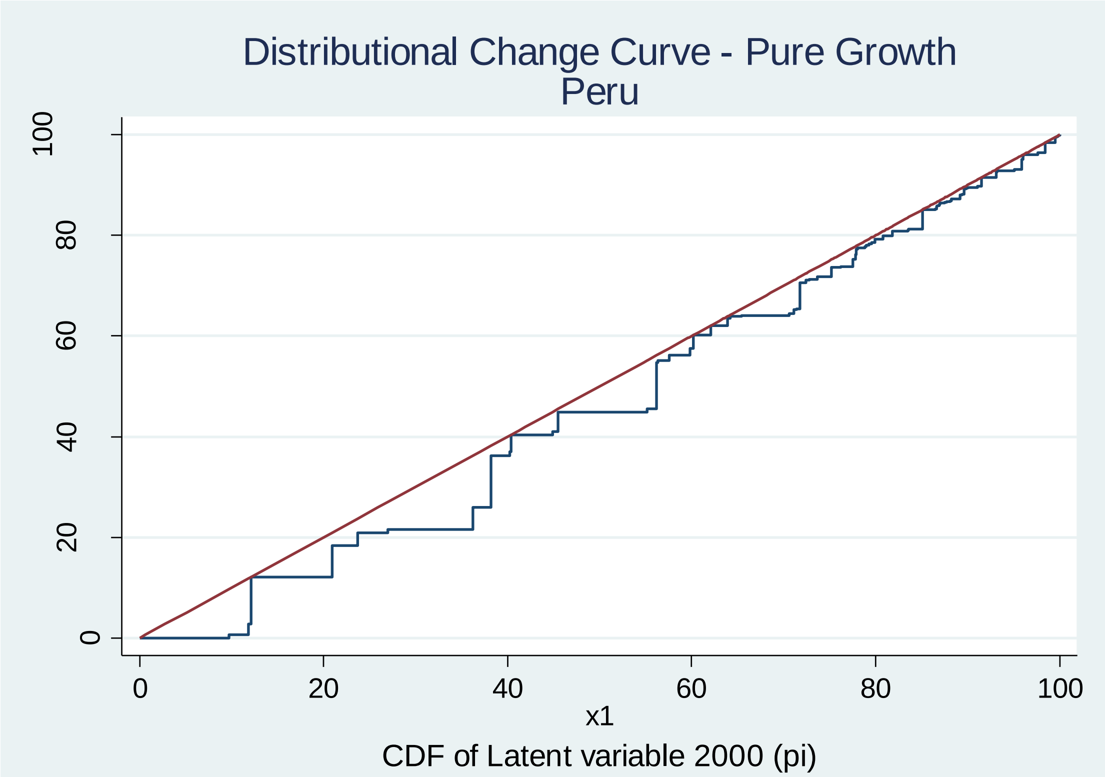


Distributional Change Curve - Pure Growth
Nicaragua
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

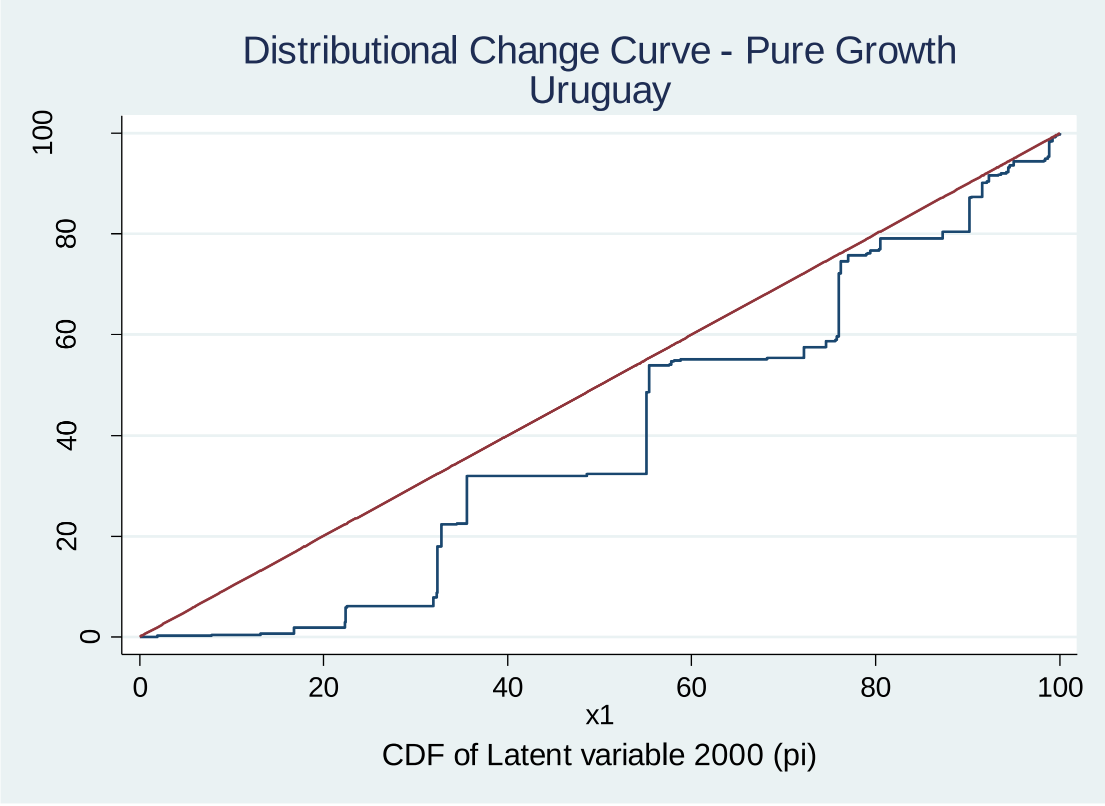

Distributional Change Curve - Pure Growth
Panama
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

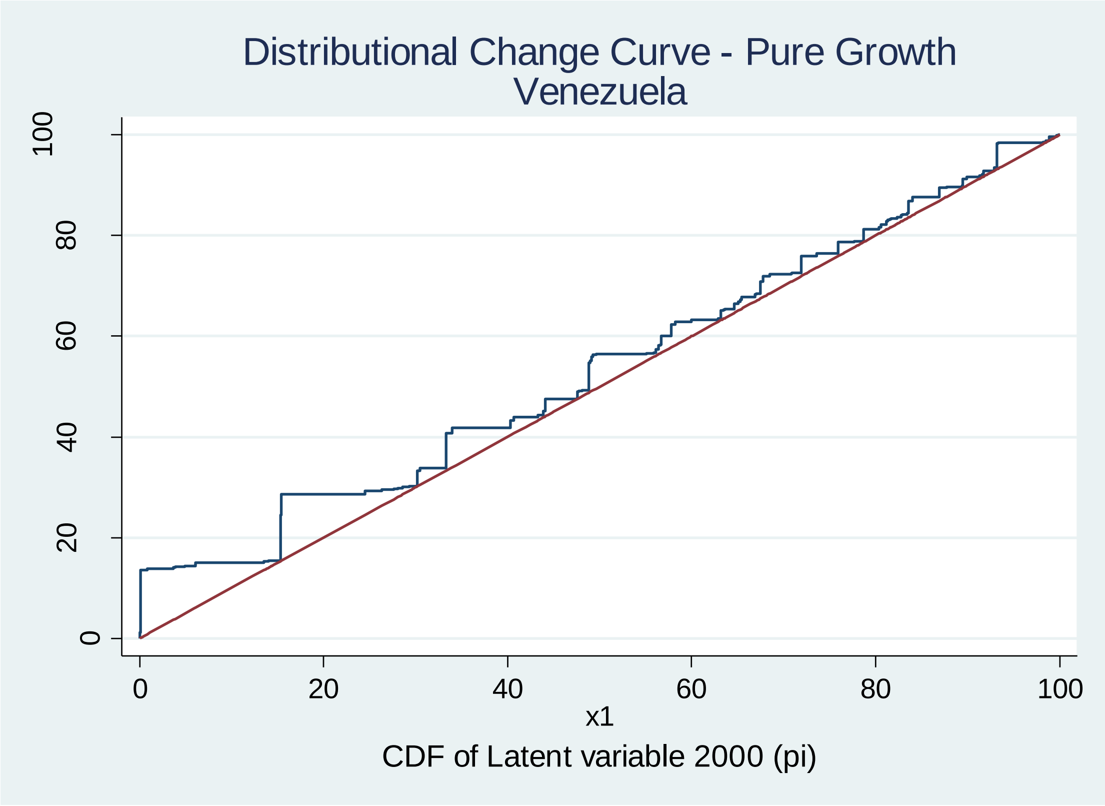

Distributional Change Curve - Pure Growth
Paraguay
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

Distributional Change Curve - Pure Growth
Peru
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



Distributional Change Curve - Pure Growth
Uruguay
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)





Distributional Change Curve - Pure Growth
Venezuela
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

"Shape related" distributional change

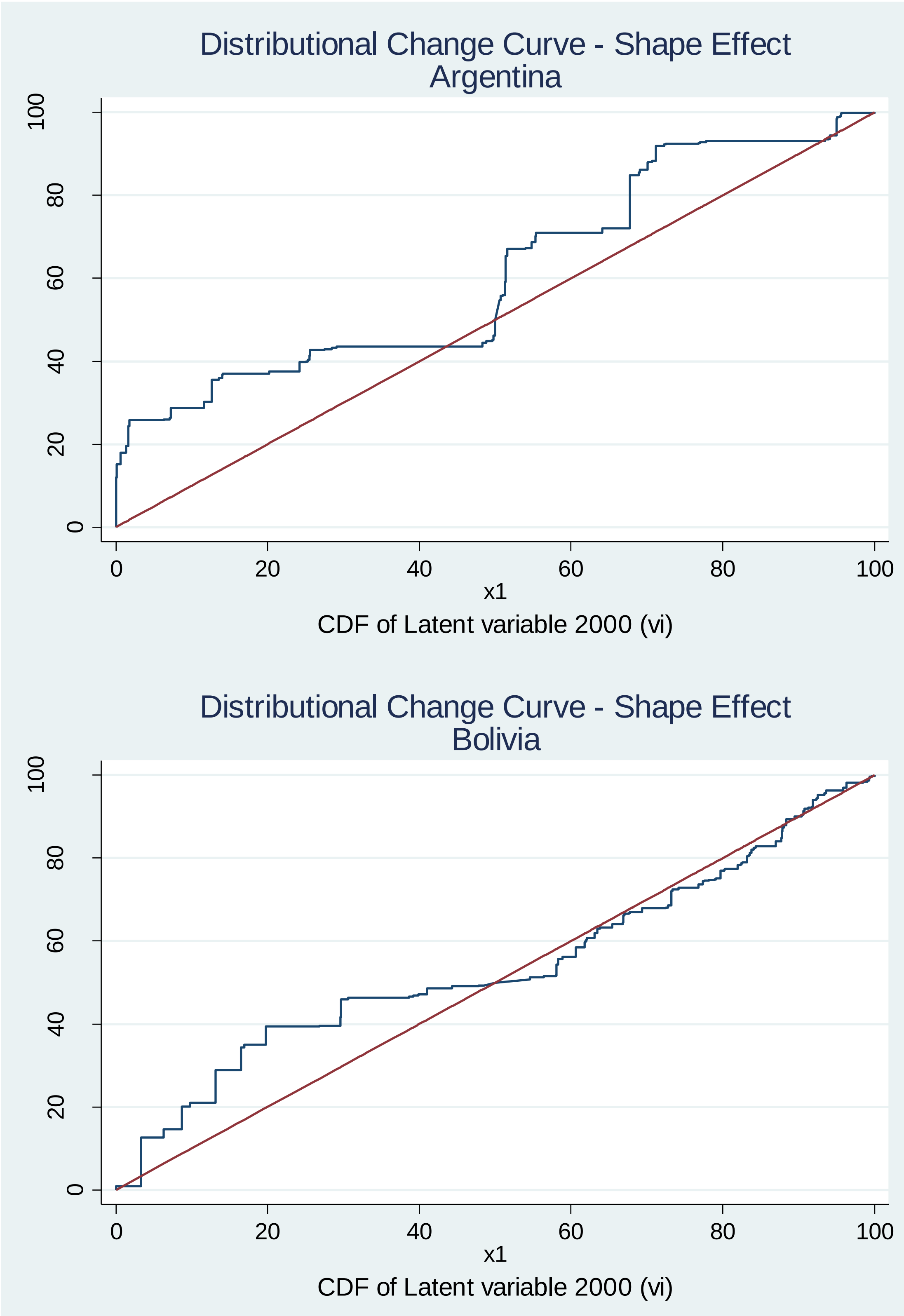
We now turn to the analysis of "shape related" distributional change. Table 1 indicates that this type of distributional change was highest (and positive) for Guatemala, Uruguay, Nicaragua and Venezuela and lowest (and negative) for Argentina, Chile, Ecuador, Panama, El Salvador and Costa Rica.

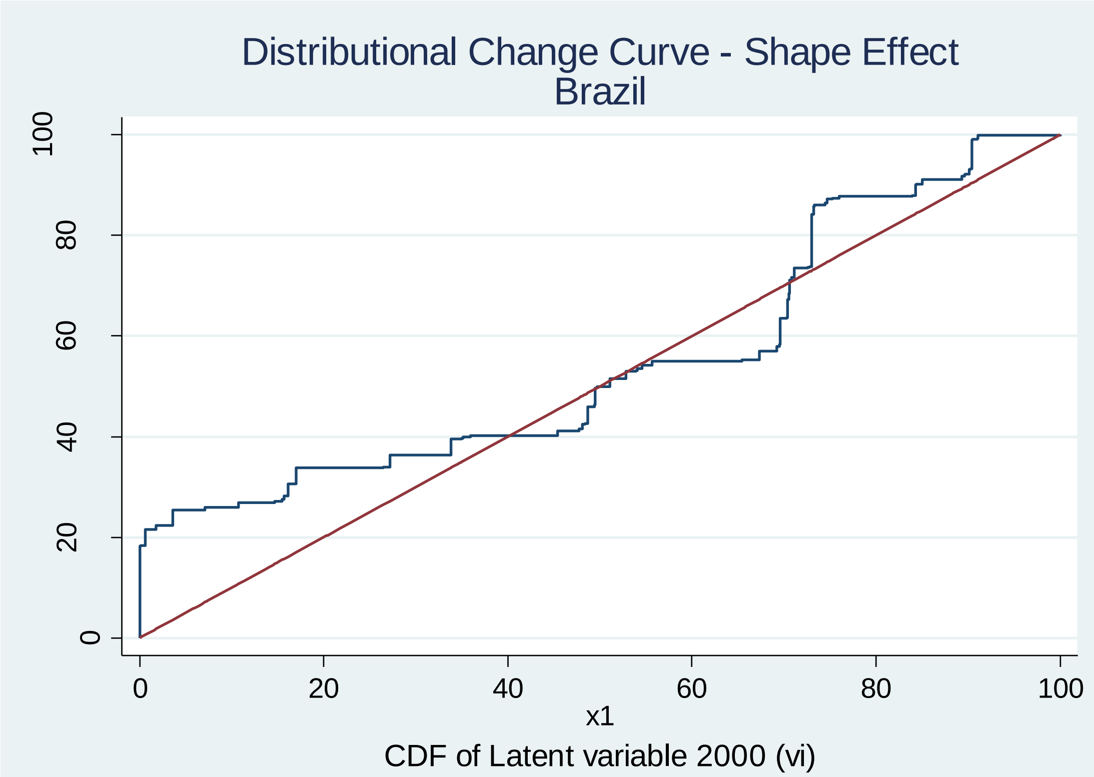
Figure 5 presents these "shape related" distributional change curves for the various countries and allows one to make a more detailed analysis of the observed change.

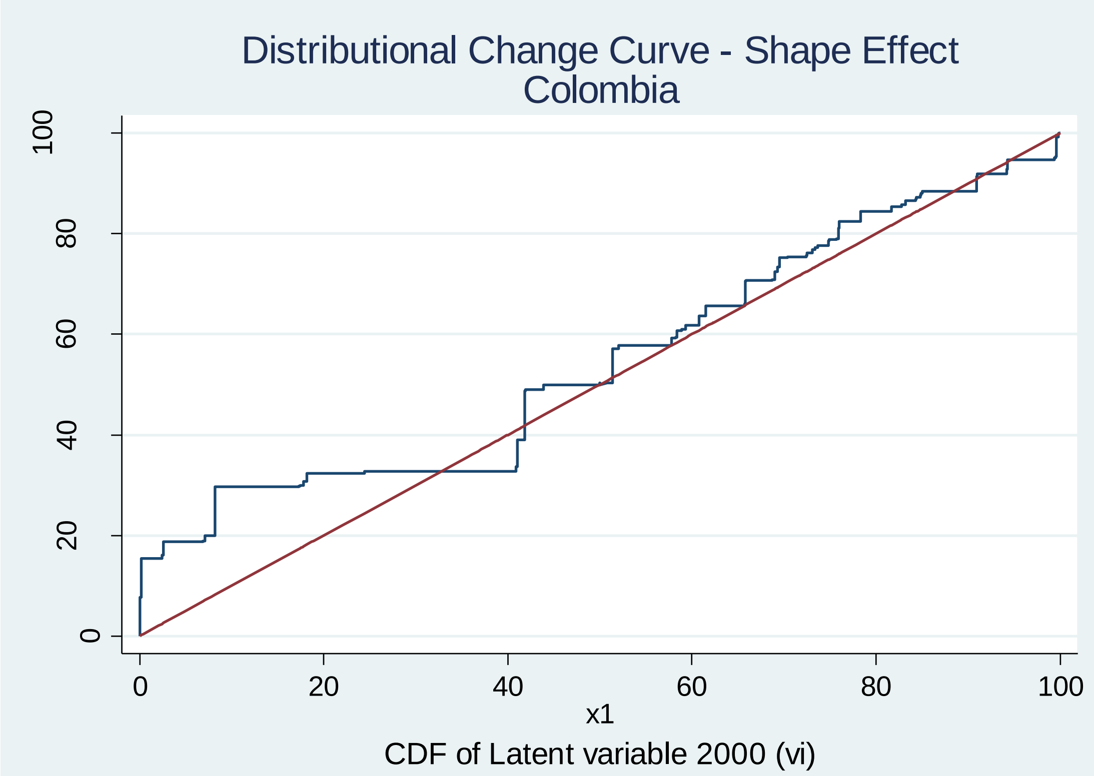
Let us take a look for example at the case of Guatemala. One observes that from a pure change in shape point of view almost everyone in 2009 would have had a higher standard of living than in 2000. There was thus between 2000 and 2009 among the "poor" a shift of the observations towards the median. We also observe that among the "rich"there was a rightward shift of the observations toward higher standards of living.

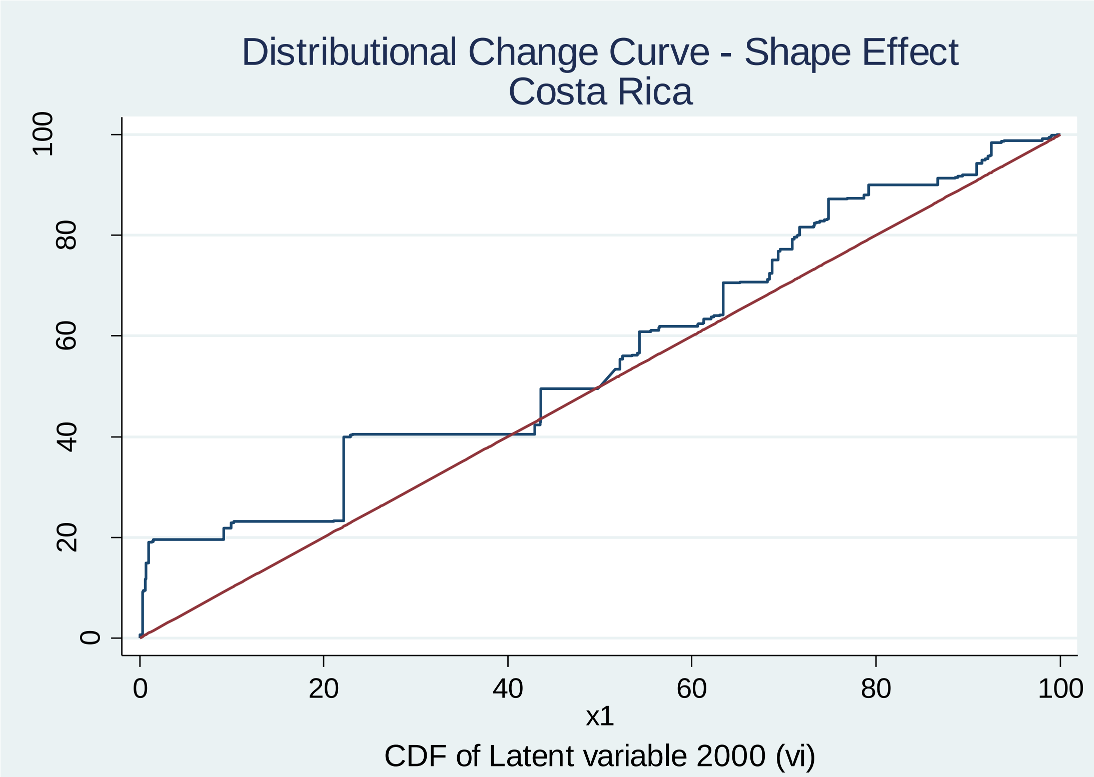
Another interesting case is Ecuador where the "shape related" distributional change was almost nil for those having a standard of living above the median whereas for those with in 2000 a standard of living below the median, there was a downard shift towards the lowest levels of standard of living.

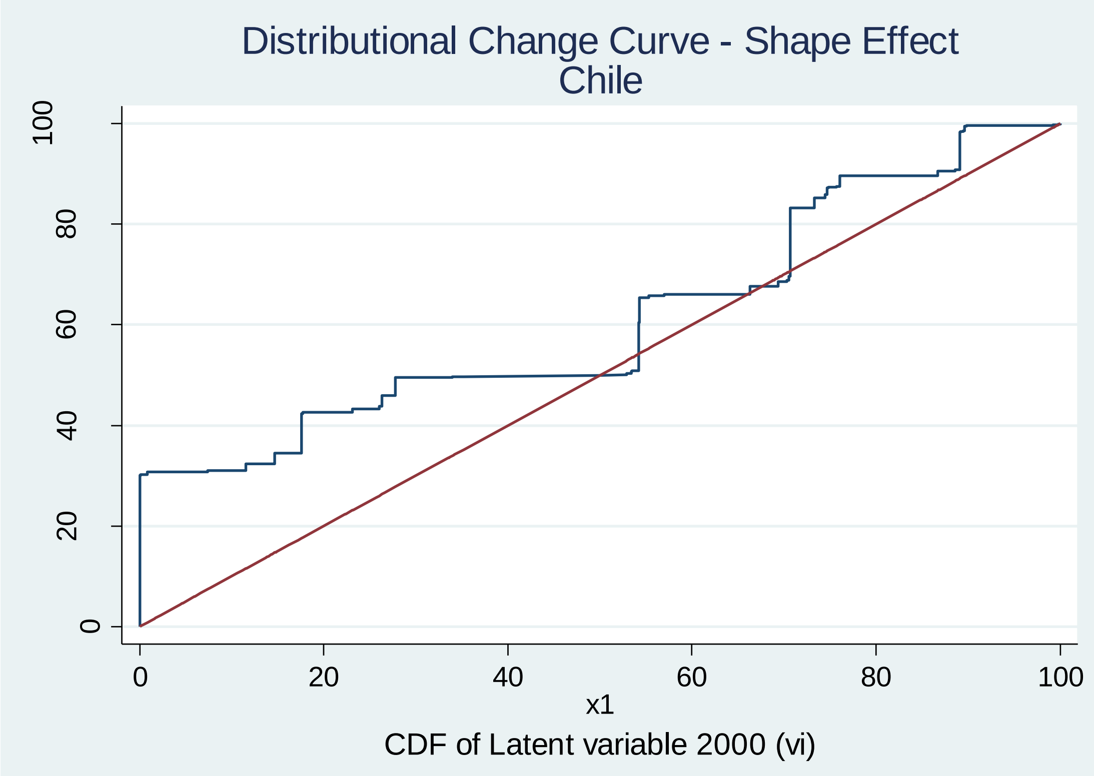
Figure 5: “Shape Effect” Distributional Change Curves

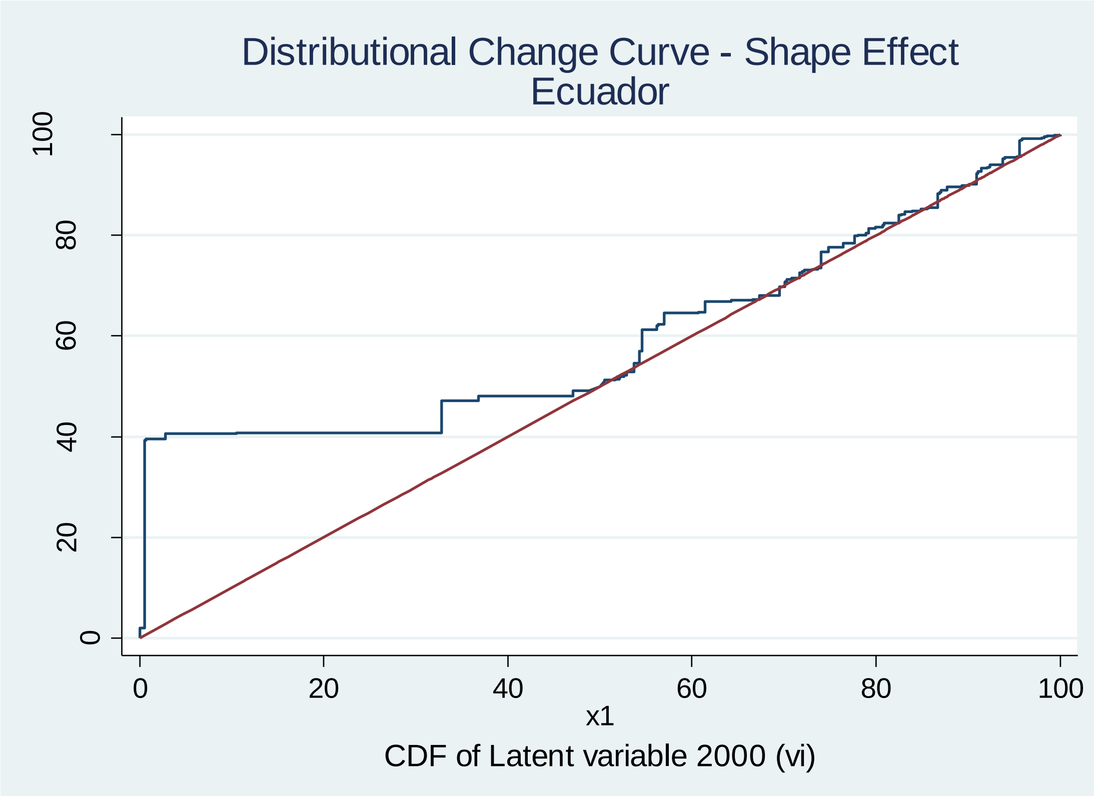


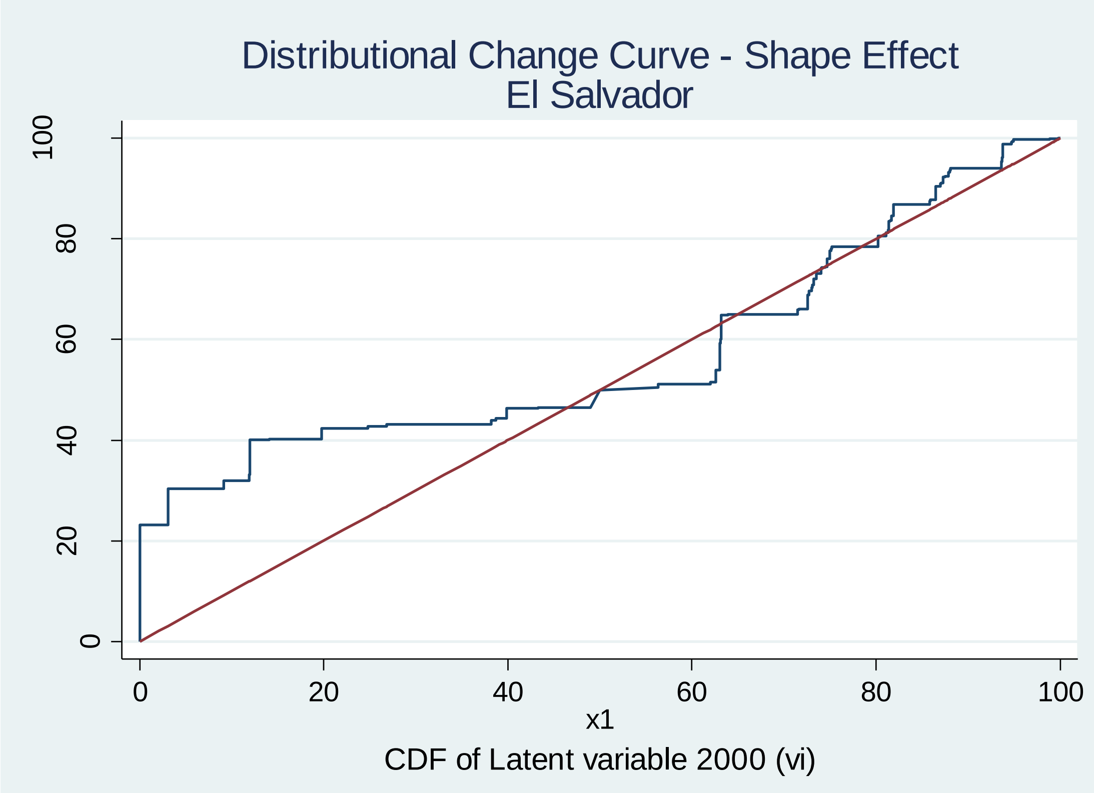

Distributional Change Curve - Shape Effect
Brazil
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)

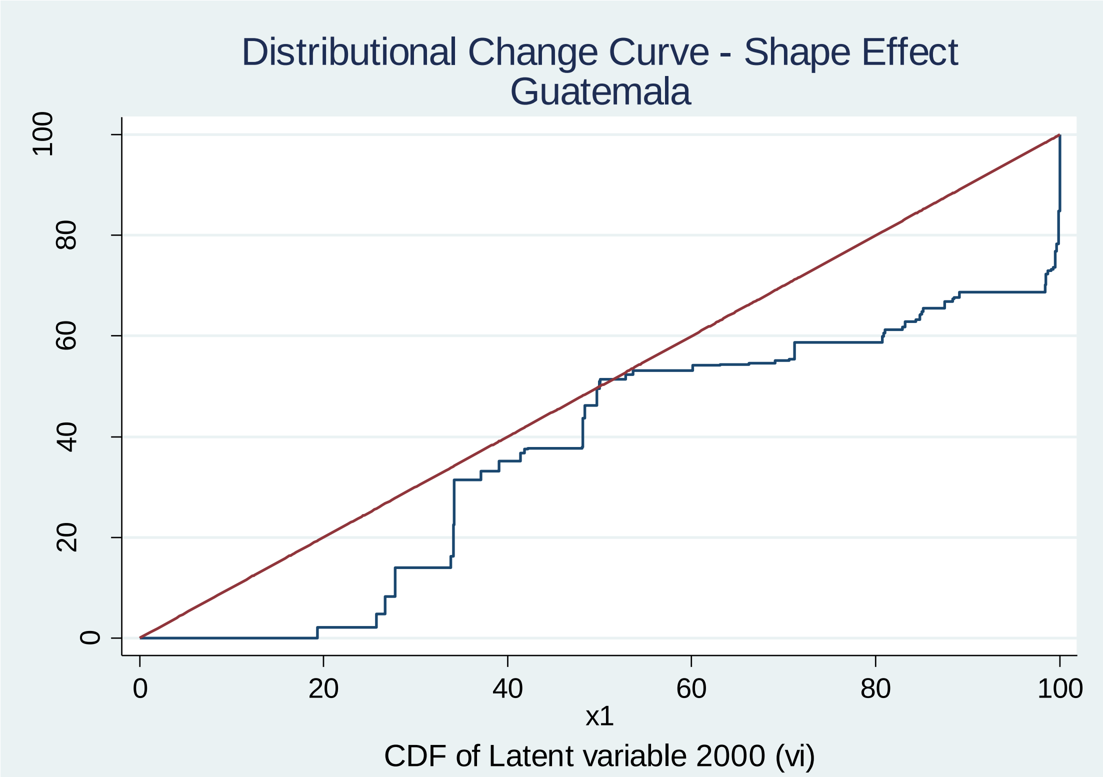

Distributional Change Curve - Shape Effect
Colombia
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)

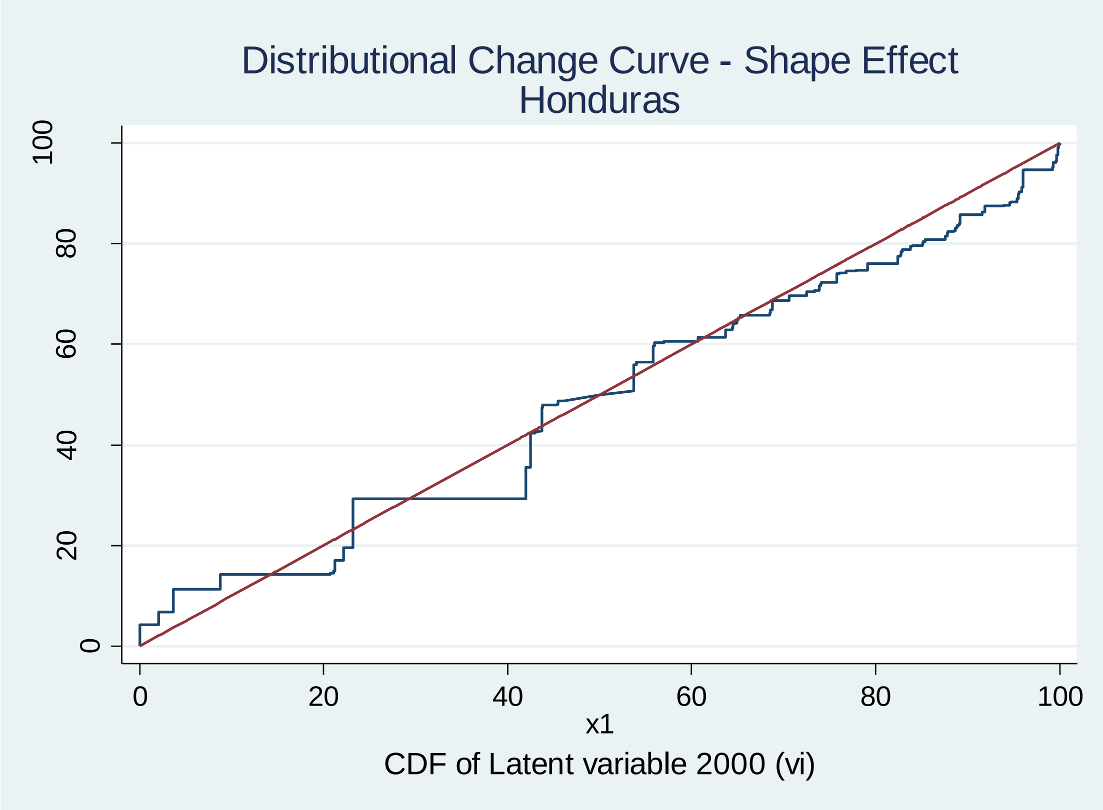

Distributional Change Curve - Shape Effect
Costa Rica
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)

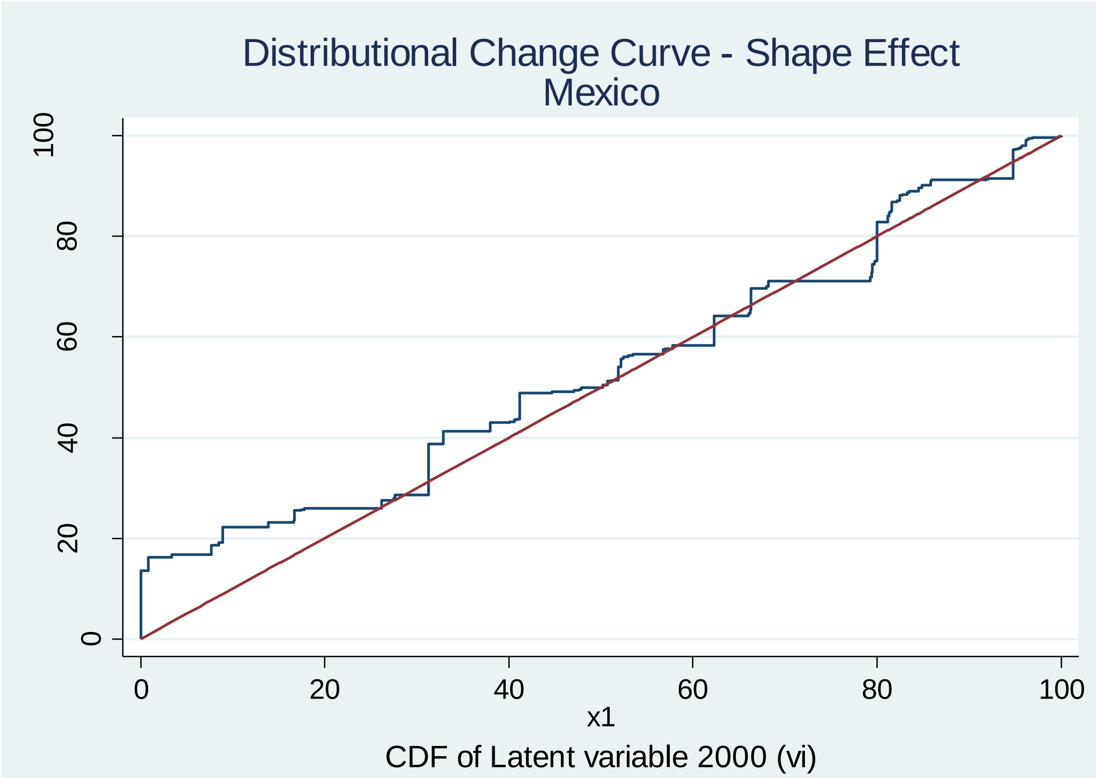

Distributional Change Curve - Shape Effect
Chile
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)

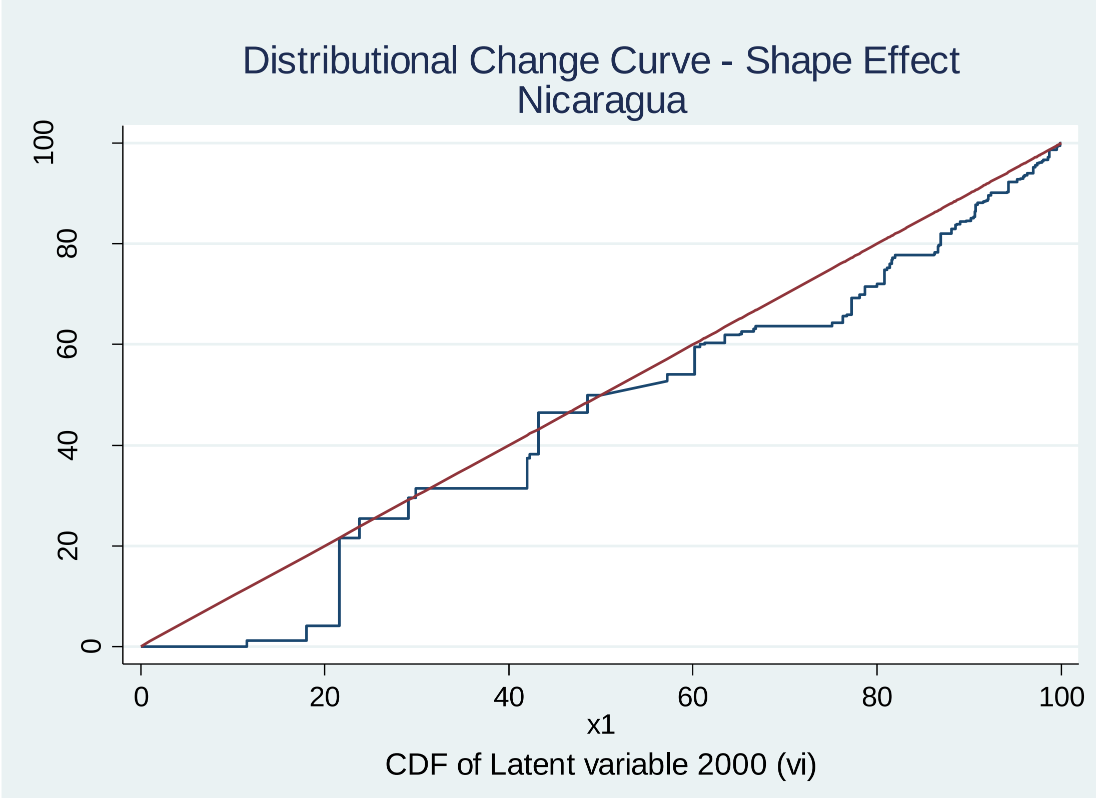

Distributional Change Curve - Shape Effect
Ecuador
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
El Salvador
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Guatemala
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Honduras
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Mexico
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Nicaragua
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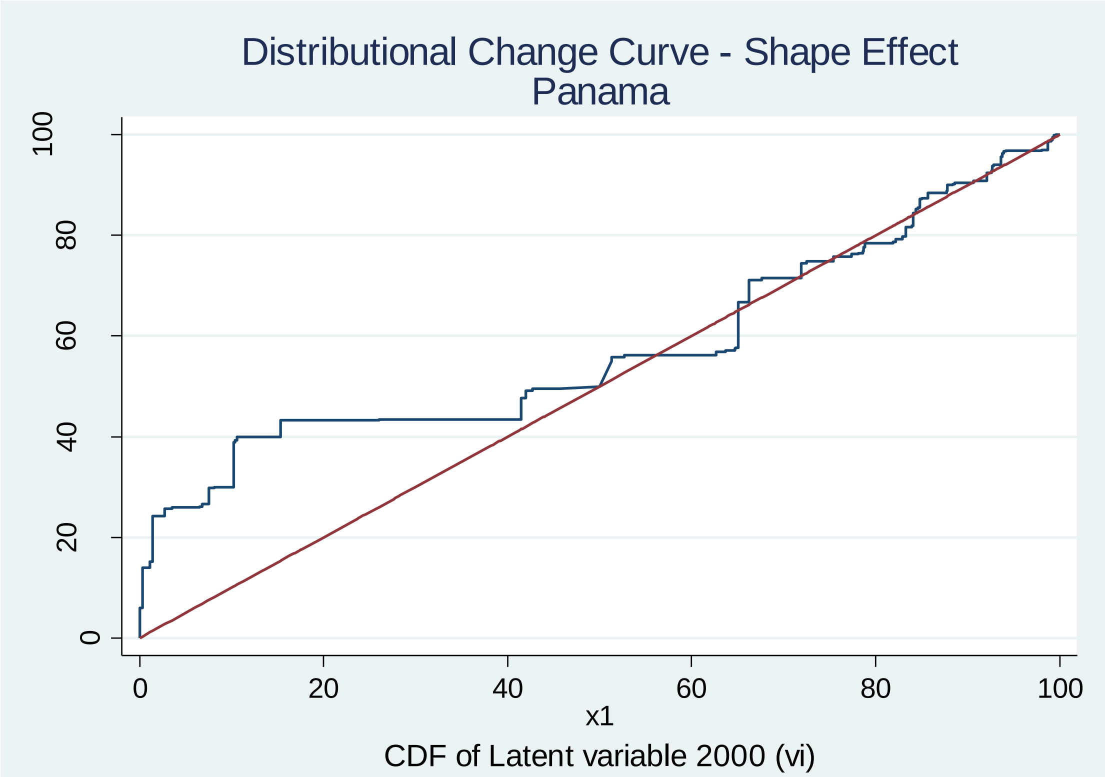
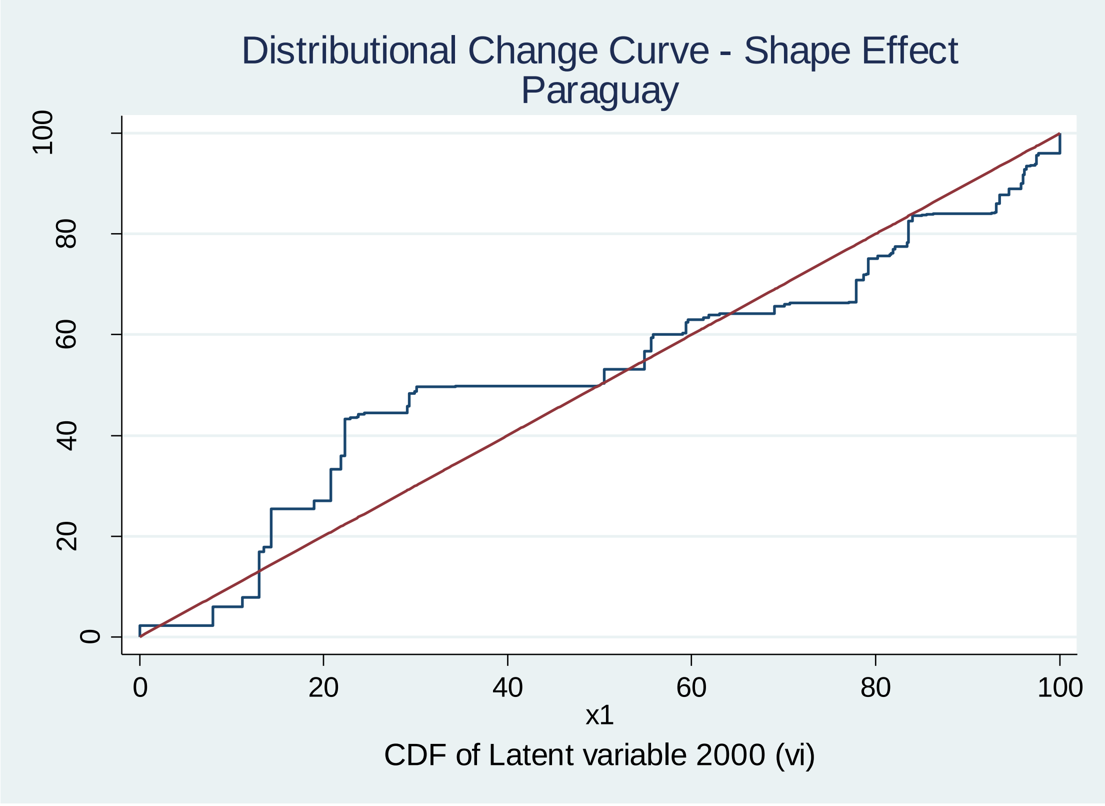
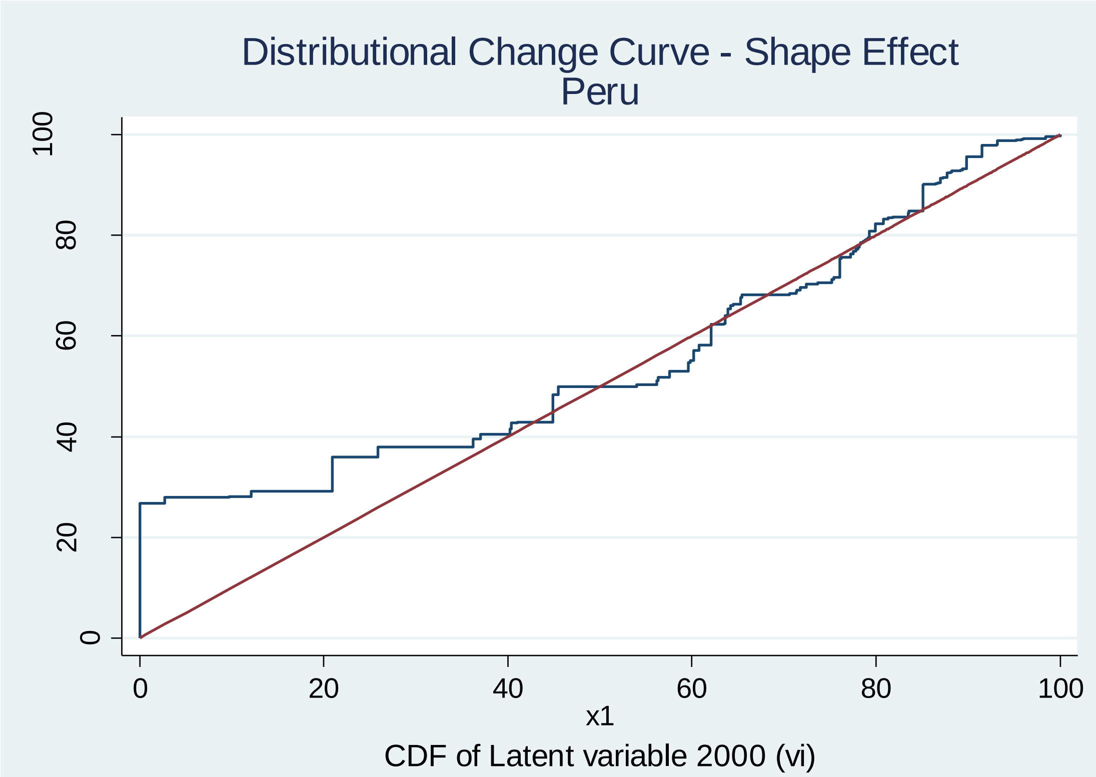
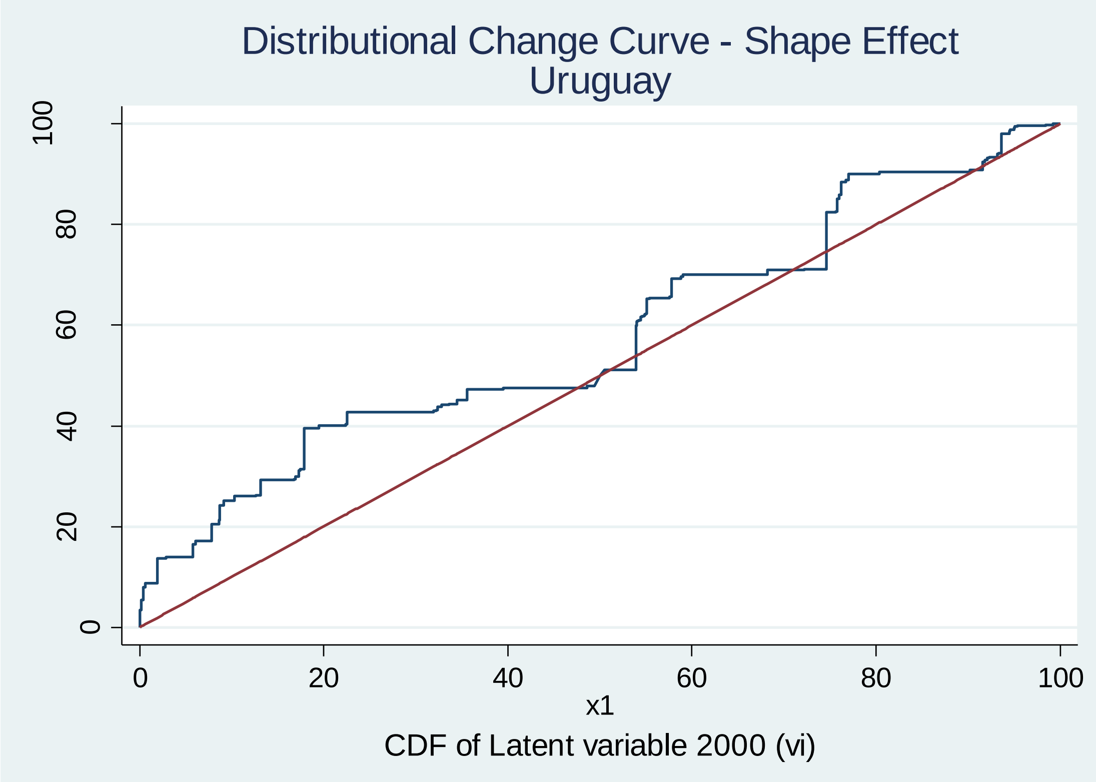
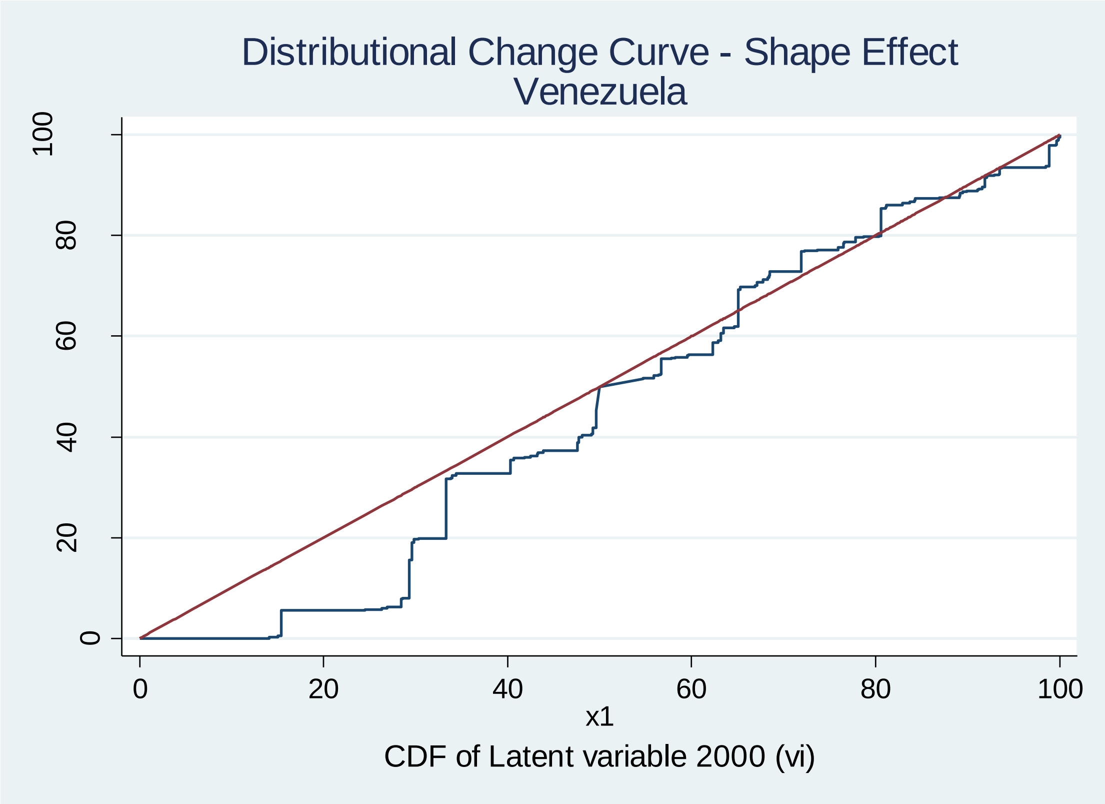
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Panama
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Paraguay
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Peru
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)


Distributional Change Curve - Shape Effect
Uruguay
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (vi)



Variations in Bi-Polarization

The observations that were just made concerning the "shape related distributional change" have evidently implications concerning the variation between 2000 and 2009 in the degree of bi-polarization of the distribution of standards of living. A new measure of change in bi-polarization was previously proposed and Table 4 gives the value of this index for the various Latin American countries. We recall that a distinction has to be made between the "poor", those whose standard of living is below the median, and the "rich", those with a standard of living higher than the median. If the curve for the poor is mostly above the diagonal, then the poor have become "poorer" (assuming the two distributions compared have the same standard of living) so that bipolarization increases. On the other hand bipolarization will increase if the "shape related" distributional curve corresponding to the rich is mostly below the diagonal, since this implies that the rich have become richer.

Table 4: Value of the "Change in Bi-Polarization" Index for the Various Countries.

	"Change in Bi-Polarization" Index	Rank
Argentina	0.0142	4
Bolivia	0.0584	14
Brazil	0.0370	10
Colombia	0.0276	7
Costa_Rica	0.0155	5
Chile	0.0371	11
Ecuador	0.0806	17
El_Salvador	0.0801	16
Guatemala	0.0204	6
Honduras	0.0080	3
Mexico	0.0288	8
Nicaragua	-0.0042	2
Panama	0.0746	15
Paraguay	0.0541	13
Peru	0.0509	12
Uruguay	0.0332	9
Venezuela	-0.0473	1

Table 4 then indicates that bipolarization increased the most in Ecuador and El Salvador whereas it decreased only in Venezuela and Nicaragua. Figure 4 indicates that in the case of Venezuela what happened was essentially an improvement in the standards of living of the poor, the same being true for Nicaragua though the changes were less important there.

In Ecuador and El Salvador, on the contrary, Figure 4 shows that there was a clear deterioration in the standards of living of the poor.

The Case of the Middle Class

To have a better view of what happened to the middle class we will now truncate the distributional change curve and ignore the lowest 20% and highest 10% of the distributions of standard of living in 2000 and 2009. The corresponding curves are given in Figure C-1 in Appendix C. We will limit our analysis to some countries for which the overall distributional change curve covering the whole population was partly above and partly below the diagonal.

Let us take the case of Brazil, for example. The corresponding curve shows now clearly that the lower middle class (more or less the lower 30% of what we defined as middle class) were in a worse situation in 2009 than in 2000. This was also true of the very upper middle class (the upper 25% of what was defined as middle class). But for those located in the middle of this truncated distribution the standard of living was clearly higher in 2009 than in 2000.

Similar conclusions may be drawn when looking at the graph for Costa Rica in Figure C-1.

For Chile the picture is less clear-cut. The lower middle class (lower 15% of the middle class) and the upper middle class (upper 30% of the middle class) were worse off in 2009 so that as a whole one cannot say that those we defined as belonging to the middle class were better off in 2009.

Finally in Argentina we observe that everyone in the middle class, but the upper 22-23%, was better off in 2009.

4. The link with traditional bi-polarization curves

Let A and B be two distributions of a continuous variable Y and we define $F_A(y) \equiv \Pr[Y \leq y]$ for distribution A and, likewise, $y(q_A) \equiv F_A^{-1}(q)$ for $0 \leq q \leq 1$. If m stands for the median then we can standardize the distributions by *dividing* each value by the median. Such division yields the variable z and we will have, e.g. $z(0.5) = 1$, $z(q) < 1\ \forall 0 \leq q < 0.5$ and $z(q) > 1\ \forall 0.5 < q \leq 1$.

Define now the spread from the median: $S_A(q) \equiv |z(q_A) - 1|$. Following Foster and Wolfson (2009) let us compare the following two (first-order) change-in-polarization indices. The one based on the first-order polarization curve appears in the middle graph in Figure 6 and is defined as:

$$P_1 = \int_0^1 [S_A(q) - S_B(q)]dq = \int_0^{\max\{S_A(0),S_B(0)\}} [q_A(S) - q_B(S)]\mathbb{I}(q_A \leq 0.5 \wedge q_B \leq 0.5)dS$$

$$+ \int_0^{\max\{S_A(0),S_B(0)\}} [q_B(S) - q_A(S)]\mathbb{I}(q_A \geq 0.5 \wedge q_B \geq 0.5)dS$$

The one based on distributional change curves is given in the right graph in Figure 6 and is defined as:

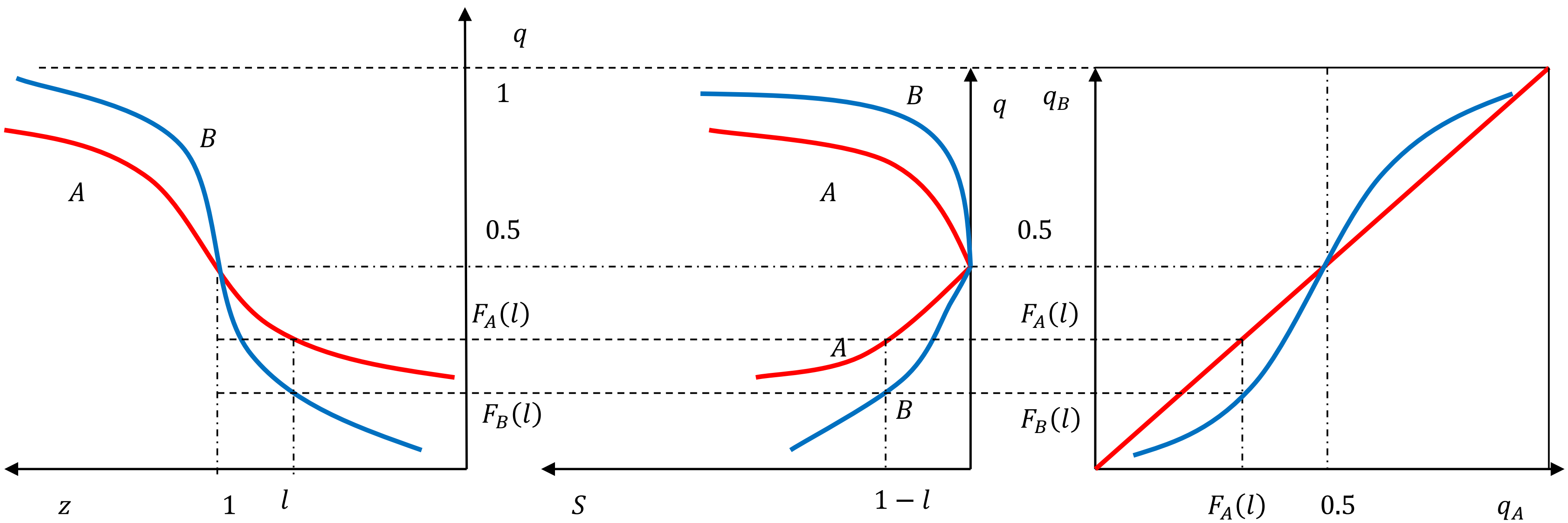
$$R_1 = 2\int_0^{0.5} [q_A - q_B(q_A)]dq_A + 2\int_{0.5}^{1} [q_B(q_A) - q_A]dq_A$$

Net positive values mean that B exhibits relatively less polarization than A (although there may be compensation effects operating at different percentiles of the distributions). Net negative values mean that A exhibits relatively more polarization.

We may note the proportionality relationship between P_1 and R_1. When the spread difference $[S_A(q) - S_B(q)]$ increases, the right-hand side of P_1 has to increase as well and that can only be accomplished by the widening of some of the percentile gaps, $q_A(S) - q_B(S)$ below the median and/or $q_B(S) - q_A(S)$ above the median. Hence R_1, which is a function of both sets of gaps, also increases when a spread difference increases. Both P_1 and R_1 can be expressed as functions of the percentile gaps, but while the first index is a weighted sum of these gaps in which the weights are dS, the second index is a weighted sum of percentile gaps in which the weights are dq.

Note also that a Pigou-Dalton transfer *across the median* should reduce polarization and increase P_1 and R_1 if A represents the pre-transfer distribution and B represents the post-transfer distribution.

Figure 6: First-order polarization curves and distributional change curves



Foster and Wolfson (2009) discuss also what happens if a Pigou-Dalton transfer takes place *on one side of the median*. If the transfer preserves ranks, then it is easy to show that P_1 is insensitive to it, as it is only measuring polarization with respect to the median. However such transfer should increase bi-polarization as it concentrates the distribution on the side of the median where the transfer took place. An index that is sensitive to these transfers and thus measures changes in bipolarization can be constructed using second-order polarization curves.

Let us define the cumulative spread from the median as

$$C_A(q) \equiv \left|\int_q^{0.5} S_A(q)dq\right| = \left|\int_q^{0.5} |z(q_A) - 1| dq\right|.$$

Let us now compare the following two (first-order) change-in-polarization indices.

The first one is based on the second-order polarization curve (middle graph in Figure 2)and is defined as:

$$P_2 = \int_0^1 [C_A(q) - C_B(q)]dq = \int_0^{\max\{C_A(0),C_B(0)\}} [q_A(C) - q_B(C)]\mathbb{I}(q_A \leq 0.5 \wedge q_B \leq 0.5)dC$$

$$+ \int_0^{\max\{C_A(0),C_B(0)\}} [q_B(C) - q_A(C)]\mathbb{I}(q_A \geq 0.5 \wedge q_B \geq 0.5)dC$$

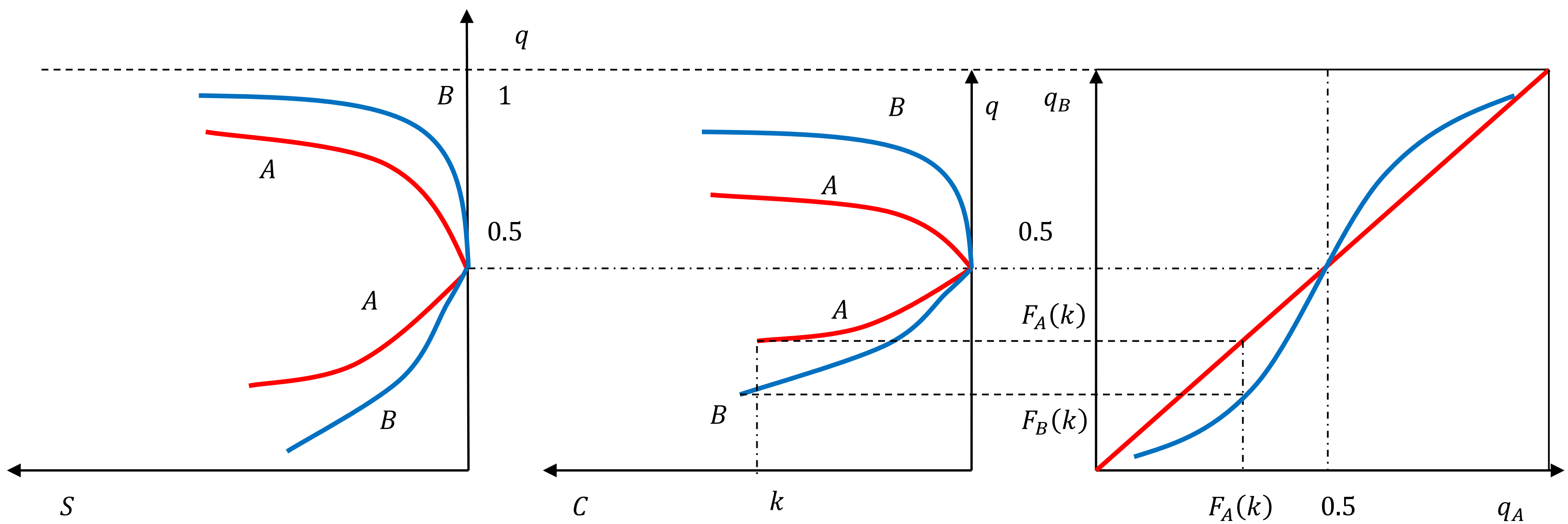
The second one is based on the concept of cumulative relative distribution (right graph in Figure 1) and may be defined as:

$$R_2 = 2\int_0^{0.5} [q_A - q_B(q_A)]dq_A + 2\int_{0.5}^1 [q_B(q_A) - q_A]dq_A$$

Here also we may note the proportionality relationship between P_2 and R_2. It should also be stressed that the indices attach more weight to spreads closer to the median. Therefore if a Pigou-Dalton transfer occurs on one side of the median, A representing the pre-transfer distribution and B the post-transfer distribution, then the indices will take a negative value thereby showing an increase in bipolarization.

The empirical illustration given in Section 3 dealt only with a "change in first order bipolarization". We plan in the near future to complete this illustration by looking also at "second order changes in bi-polarization".

Figure 2: Second-order polarization curves and cumulative relative distributions



5. Concluding Comments

This paper proposed a new index and graphical representation of the change in bi-polarization and in the relative importance of the middle class that took place in a given country during a given period. These tools extend in fact the concepts of inter-distribution income inequality and Lorenz curves by making a distinction between overall, “pure growth based” and “shape related” distributional changes.

The empirical illustration was based on data covering 17 Latin American countries in 2000 and 2009, obtained from the Latinobarómetro surveys for these years. The standard of living of individuals was derived on the basis of correspondence analysis. It seems that the new tools proposed in this paper help understanding the changes that took place in the distribution of standards of living in Latin America during the period analysed. They also suggest a new way of determining what happened there to the middle class between 2000 and 2009.

This empirical analysis was limited to the case of a “first order change in bi-polarization”. In future work we plan to extend this analysis to the case of a “second order change in bi-polarization”.

Bibliography

Benzécri, J.-P. and F. Benzécri, Pratique de L'Analyse des Données, I, Analyse des Correspondances, Exposé Elémentaire, Paris: Dunod Bordas, 1980.

Birdsall, N. (2007a) "Do No Harm: Aid, Weak Institutions, and the Missing Middle in Africa," Working Paper No 113, Center for Global Development.

Birdsall, N. (2007b) "Reflections on the Macro Foundations of the Middle Class in the Developing World," Working Paper 130, Center for Global Development.

Blackburn, M. and D. Bloom (1985) "What is Happening to the Middle Class?" *American Demographics* 7(1): 18-25.

Butler, R. J. and J. B. McDonald (1987) "Interdistributional Income Inequality," *Journal of Business & Economic Statistics* 5(1): 13-18

Davis, J.C. and J.H. Huston (1992) "The Shrinking Middle-income Class: a Multivariate Analysis," *Eastern Economic Journal* 18(3): 277-285.

Easterly, W. (2001) "The Middle Class Consensus and Economic Development," *Journal of Economic Growth* 6(4): 317-335.

Foster, J. E. and M. C.Wolfson (1992) "Polarization and the Decline of the Middle Class: Canada and the U.S.," mimeo.

Foster, J. E. and M. C.Wolfson (forthcoming) "Polarization and the Decline of the Middle Class: Canada and the U.S.," *Journal of Economic Inequality*.

Handcock, M. S. and M. Morris, 1999, *Relative Distribution Models in the Social Sciences*, Springer Verlag, New York.

Landes, D. (1998) *The Wealth and Poverty of Nations*: *Why Some Are So Rich and Some So Poor*. Norton (New York NY).

Lawrence, R.Z. (1984) "Sectoral Shifts and the Size of the Middle Class," *The Brookings Review* Fall: 3-11.

Levy, F. (1987) "The Middle Class: Is It Really Vanishing?" *The Brookings Review* 5: 17-21.

Massari, R., M. G. Pittau and R. Zelli (2009) "A dwindling middle class? Italian evidence in the 2000s," *Journal of Economic Inequality* 7(4): 333-350.

Pressman, S. (2007) "The Decline of the Middle Class: An International Perspective," *Journal of Economic Issues* 41: 181-201.

Roemer, J. E., 1998, *Equality of Opportunity*, Cambridge, MA: Harvard University Press.

Card, editors, *Handbook in Labor Economics*, Volume 3A, North Holland.

Thurow, L.C. (1984, February 05) "The Disappearance of the Middle Class," *The New-York Times*, E2.

Wolfson, M.C. (1994) "When Inequalities Diverge," *American Economic Review* 84(2): 353-358.

Wolfson, M.C. (1997) "Divergent Inequalities: Theory and Empirical Results," *Review of Income and Wealth* 43(4): 401-421.

Appendix A: On Correspondence Analysis

Correspondence analysis (CA) was originally introduced by Benzecri and Benzecri (1980). It is strongly related to principal components analysis (PCA) but while PCA assumes that the variables are quantitative, CA has been designed to deal with categorical variables. More precisely CA offers a multidimensional representation of the association between the row and column categories of a two-way contingency table. In short the goal of CA is to find scores for both the row and column categories on a small number of dimensions (axes) that will account for the greatest proportion of the *chi²* measuring the association between the row and column categories. There is thus a clear parallelism between CA and PCA, the main difference being that PCA[1] accounts for the maximum *variance*. A clear presentation of CA is given in Asselin and Vu Tuan Anh (2008), chapter 5 in Kakwani and Silber (2008).

Let us first recall what the main features of PCA. It is in fact a data reduction technique that consists of building a sequence of orthogonal and normalized linear combinations of the K primary indicators that will exhaust the variability of the set primary indicators. These orthogonal linear combinations are evidently latent variables and usually called "components". In PCA the first component has the greatest variance and all subsequent components have decreasing variances.

Let *N* be the size of the population, *K* the number of indicators I_k . The first component F^1 may be expressed for observation *i* as $F_i^1 = \sum_{k=1}^{K} \omega_k^1 I_i^{*k}$. where I^{*k} refers to the standardized primary indicator I^k. Note that ω_k^1 is the (first) factor score coefficient for indicator *k*. It turns out that the scores ω_k^1 are in fact the multiple regression coefficients between the component F^1 and the standardized primary indicators I^{*k}. It is very important to understand that PCA has some limitations, of which the most important is probably the fact that PCA has been developed for *quantitative* variables.

It is therefore better not to use PCA when some of the variables are of a qualitative nature. (Multiple) Correspondence Analysis (MCA) is in fact the data reduction technique that should be used in the presence of categorical variables

Let us therefore assume now that the *K* primary indicators are categorical ordinal and that the indicator I^k has J^k categories. Note that if some of the variables of interest are quantitative, it is always possible to transform them into a finite number of categories. To each primary indicator I^k we therefore associate the set of J^k binary variables that can only take the value 0 or 1.

Let us now call $X(N,J)$ the matrix corresponding to the *N* observations on the *K* indicators which are now decomposed into J^k variables. Note that $J = \sum_{k=1}^{K} J^k$ represents now the total number of categories. Call $N_{.j}$ the absolute frequency of category *j*. Clearly $N_{.j}$ is equal to the sum of column *j* of the matrix *X*. Let $N_{..}$ refer to the

[1] For an illustration of the use of PCA, see, for example, Berrebi and Silber,1981.

sum of all the $(N \;\; by\, K)$ elements of the matrix *X*. Let also f_j be the relative frequency $(N_j / N_{..})$, f^i be the sum of the i^{th} line of matrix *X*, f_{ij} be the value of cell (*i*,*j*) and f_j^i be equal to the ratio (f_{ij} / f^i). Finally call $\{f_j^i\}$ the set of all f_j^i's for a given observation *i* (*j* = 1 to *J*). This set will be called the profile of observation *i*.

As stressed previously CA is a PCA process applied to the matrix *X*, but with the χ^2 - metric on row/column profiles, instead of the usual Euclidean metric. This χ^2 - metric is in fact a special case of the Mahalanobis distance developed in the 1930s. This metric defines the distance $d^2(f_j^i, f_j^{i'})$ between two profiles *i* and *i'* as

$$d^2(f_j^i, f_j^{i'}) = \sum_{j=1}^{J} (1/f_j)(f_j^i - f_j^{i'})^2$$

Note that the only difference with the Euclidean metric lies in the term $(1/f_j)$. This term indicates that categories which have a low frequency will receive a higher weight in the computation of distance. As a consequence CA will be overweighting the smaller categories within each primary indicator. It can be shown that

$$\omega_j^{1,k} = \frac{1}{(N_j^k / N)} Cov(F^{1*}, I_j^k)$$

where $\omega_j^{1,k}$ is the score of category j_k on the first (non-normalized) factorial axis, I_j^k is a binary variable taking the value 1 when the population unit belongs to the category j_k, F^{1*} is the normalized score on the first axis and N_j^k is the frequency of the category j_k of indicator *k*.

Ir is also very interesting to note that CA offers a unique duality property since it can be shown that

$$F_1^i = \frac{\sum_{k=1}^{K} \sum_{j=1}^{J_k} \frac{w_j^{1,k}}{\lambda_1} I_{i,j}^k}{K}$$

where *K* is the number of categorical indicators, J_k is the number of categories for indicator *k*, $w_j^{1,k}$ is the score of category j_k on the first (non normalized) factorial axis, $I_{i,j}^k$ is a binary variable taking the value 1 when unit *i* belongs to category j_k and F_1^i is the (non normalized) score of observation *i* on the first factorial axis[2].

Reciprocally it can be shown that

$$\omega_j^{1,k} = \frac{\sum_{i=1}^{N} \frac{F_1^i}{\lambda_1}}{N_j^k}$$

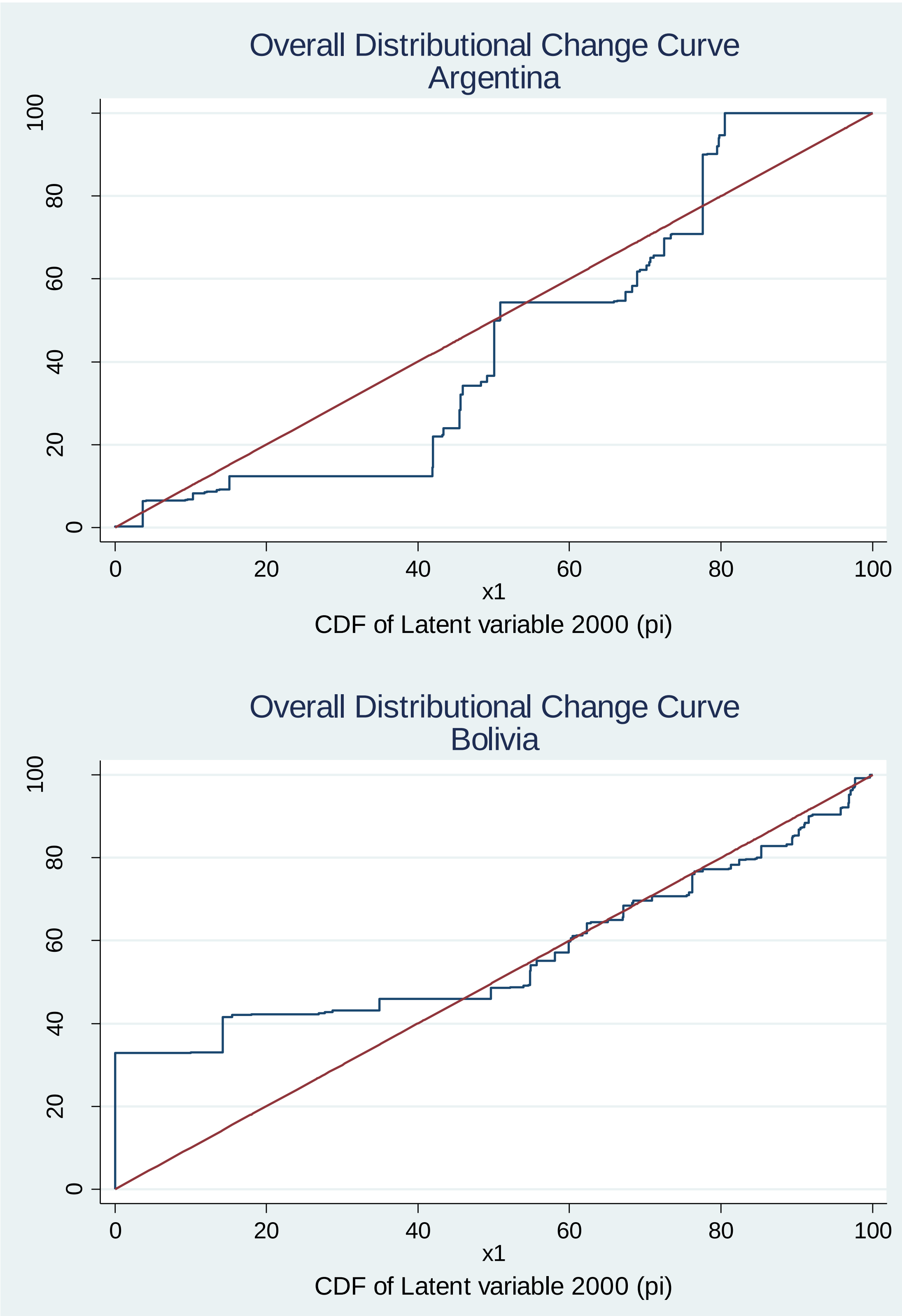
[2] Very similar results can be derived for the other factorial axes.

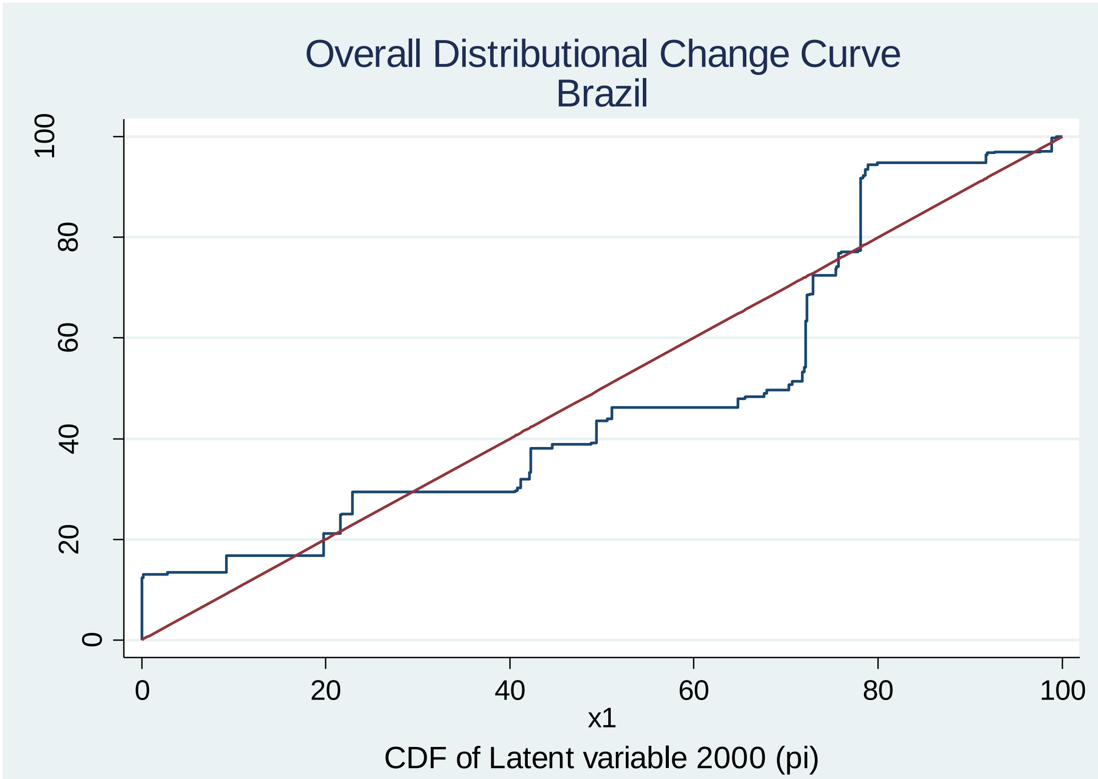
This duality relationship implies thus that the score of a population unit on the first factor is equal to the average of the standardized factorial weights of the K categories to which it belongs. Conversely the weight of a given category is equal to the average of the standardized scores of the population units belonging to the corresponding

Appendix B: List of Educational Levels (Parents of Respondent)

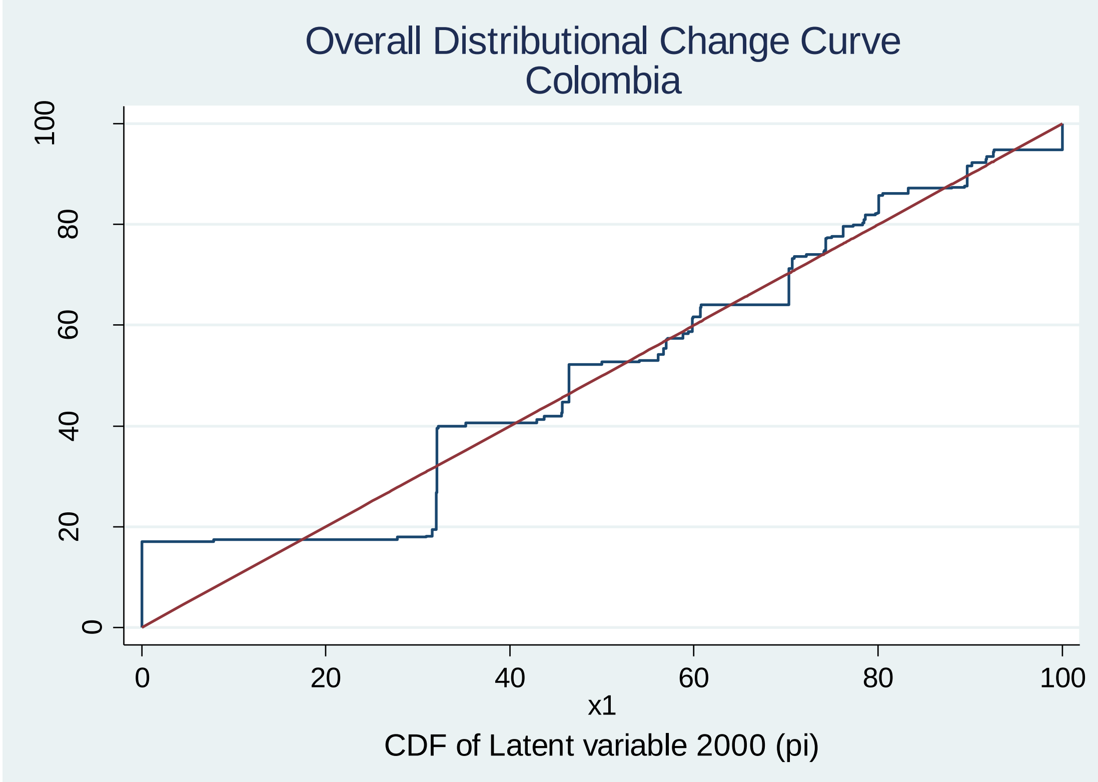
1: without education
2: 1 year of education
3: 2 years of education
4: 3 years of education
5: 4 years of education
6: 5 years of education
7: 6 years of education
8: 7 years of education
9: 8 years of education
10: 9 years of education
11: 10 years of education
12: 11 years of education
13: 12 years of education
14: High school/academies/incomplete technical training
15: High school/academies/complete technical training
16: Incomplete University
17: Completed University

Appendix C: Overall Distributional Change Curves for the Middle Class only.

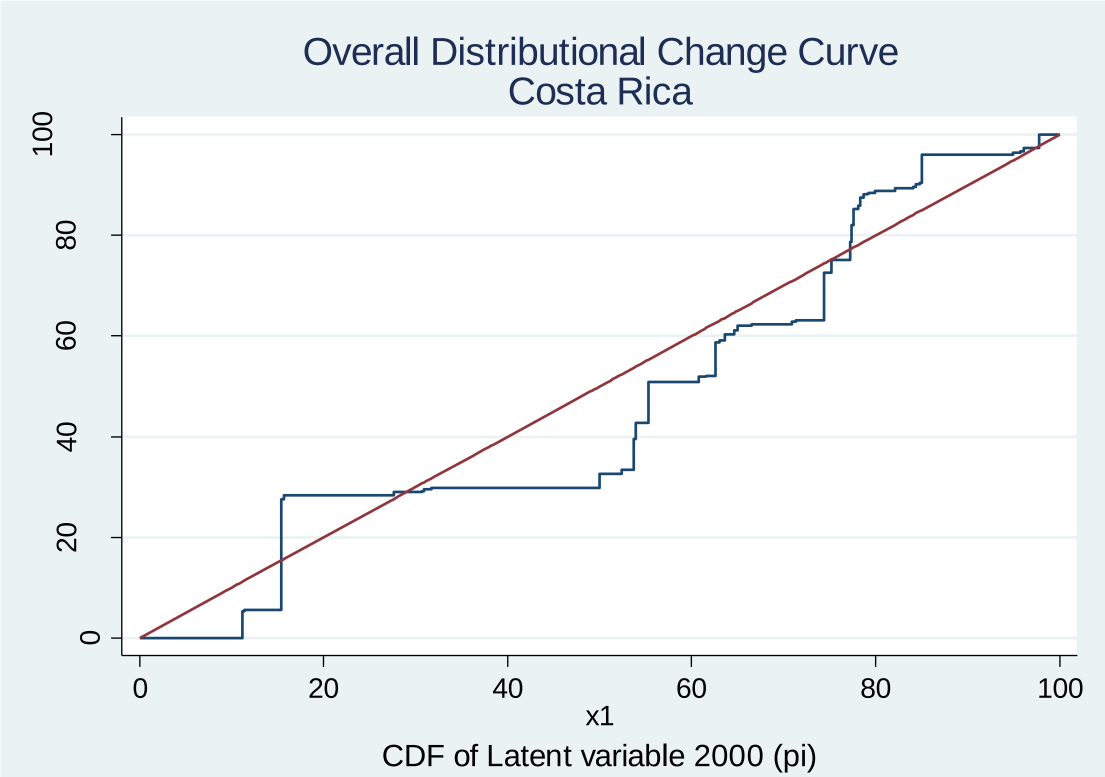




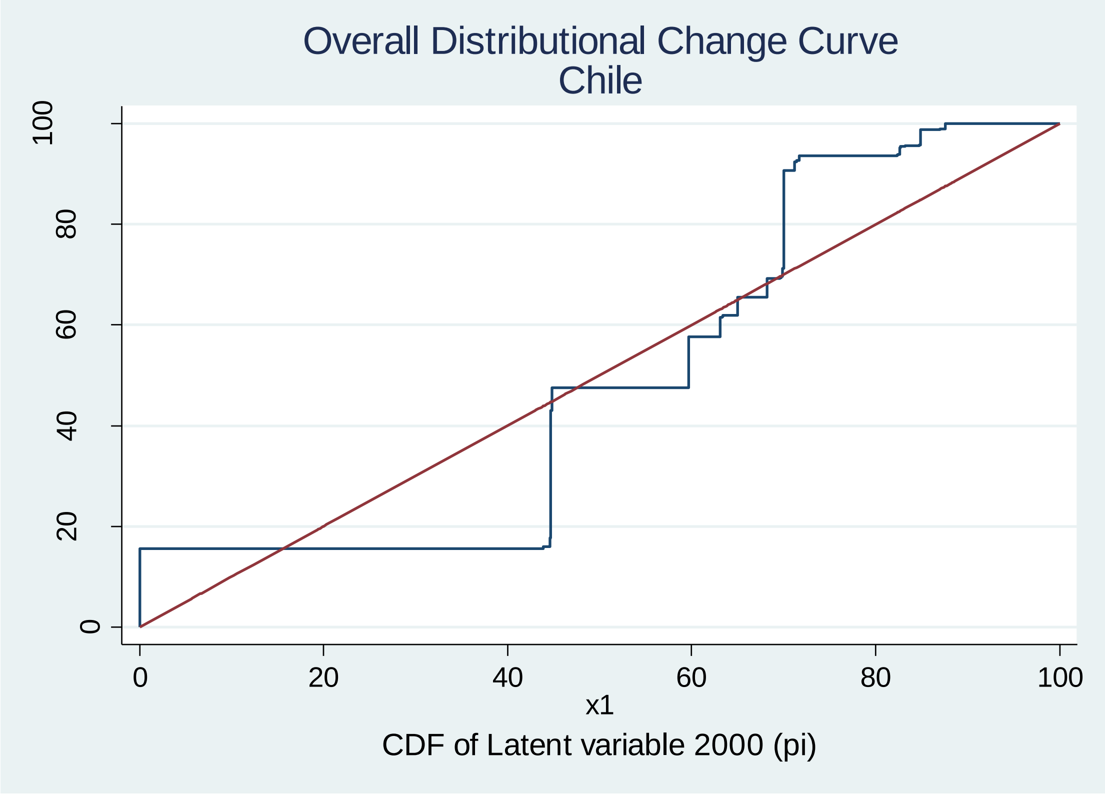
Overall Distributional Change Curve
Brazil
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



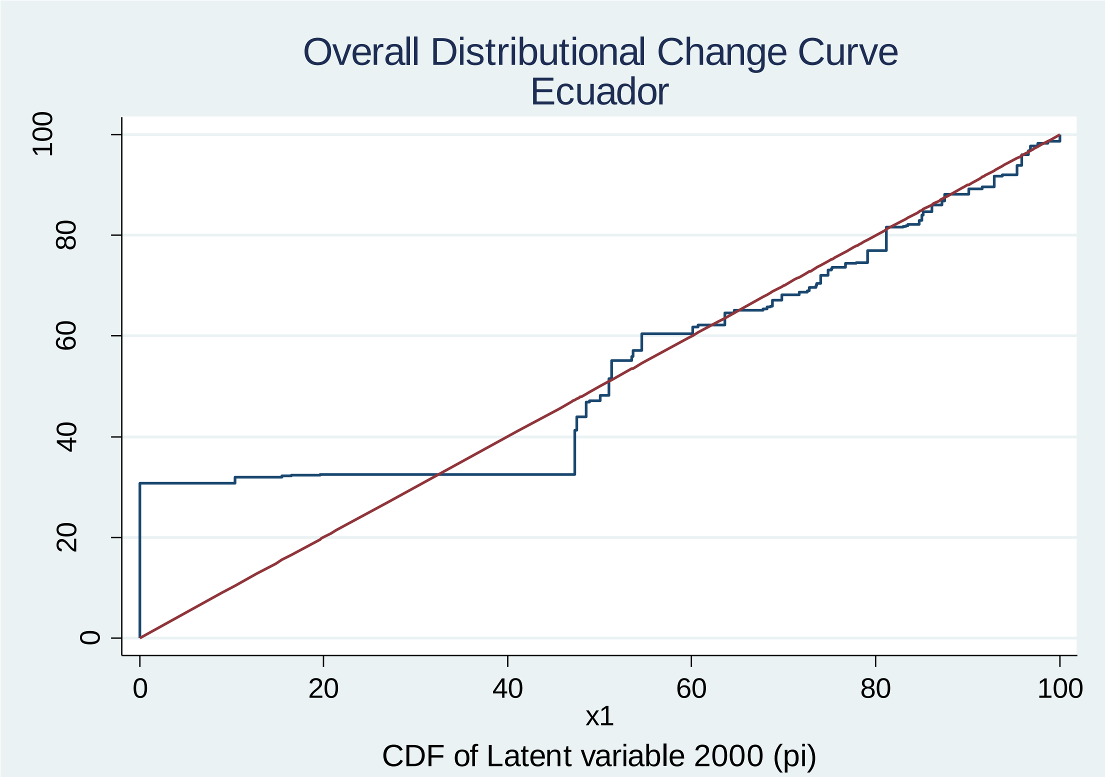
Overall Distributional Change Curve
Colombia
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



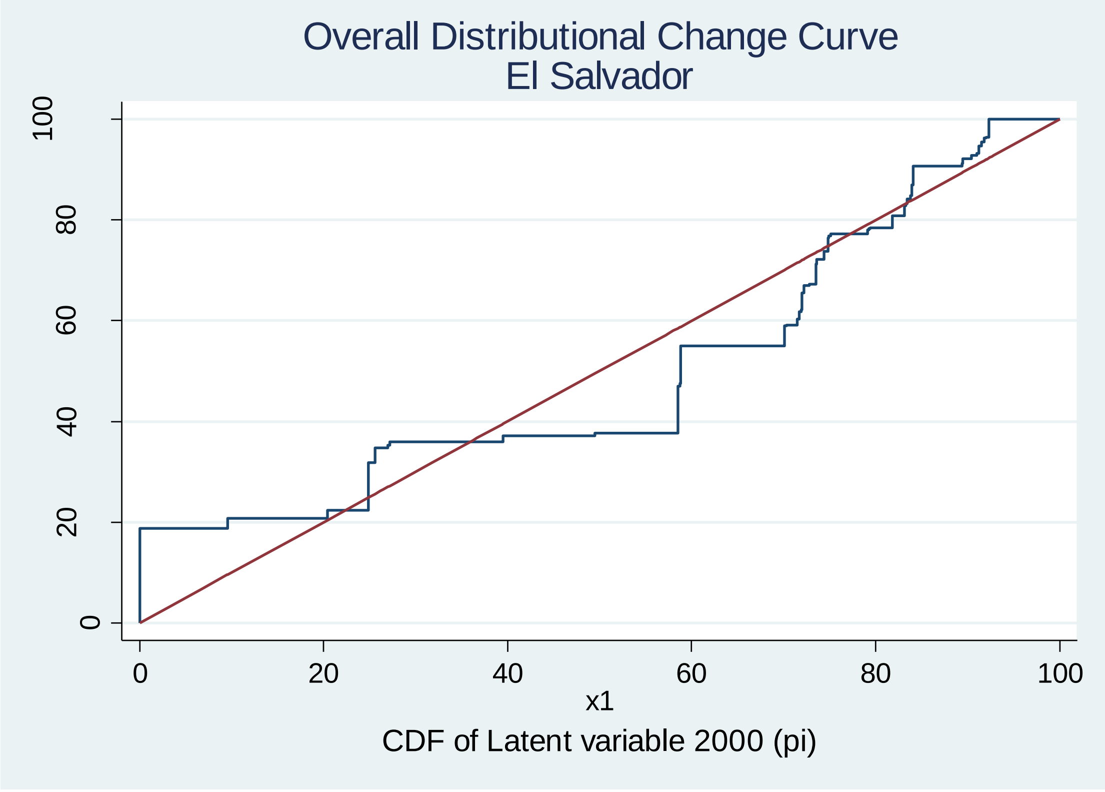
Overall Distributional Change Curve
Costa Rica
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



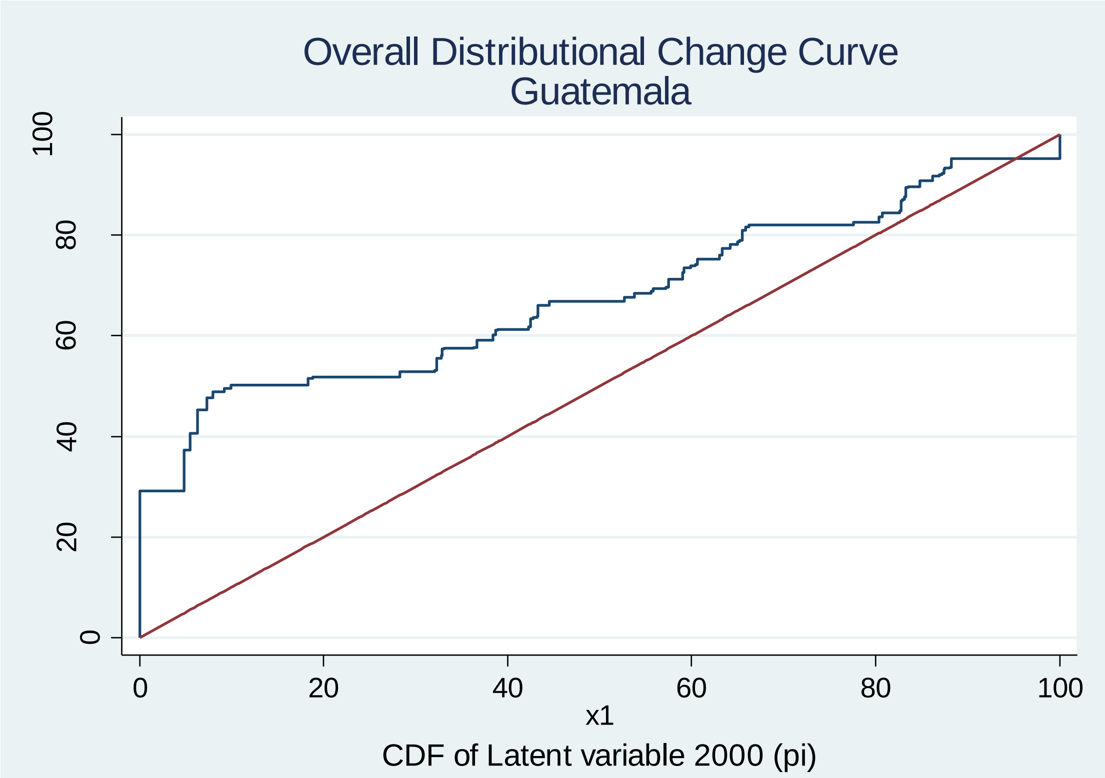
Overall Distributional Change Curve
Chile
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

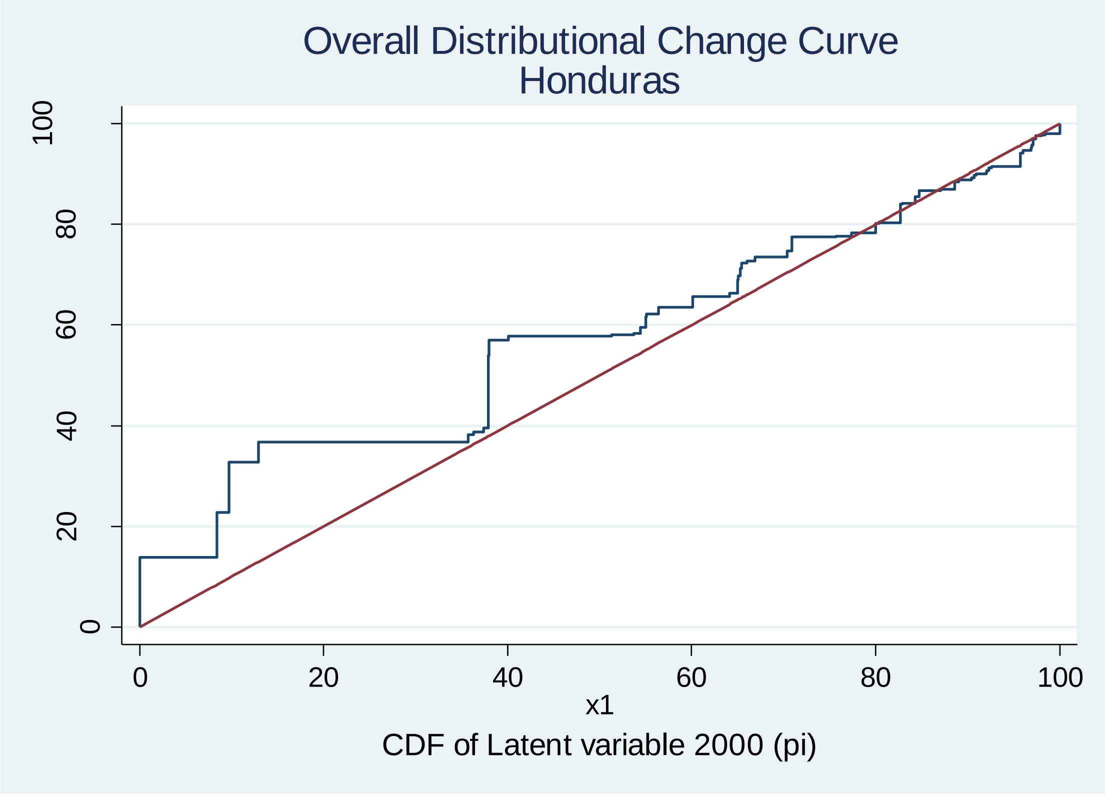


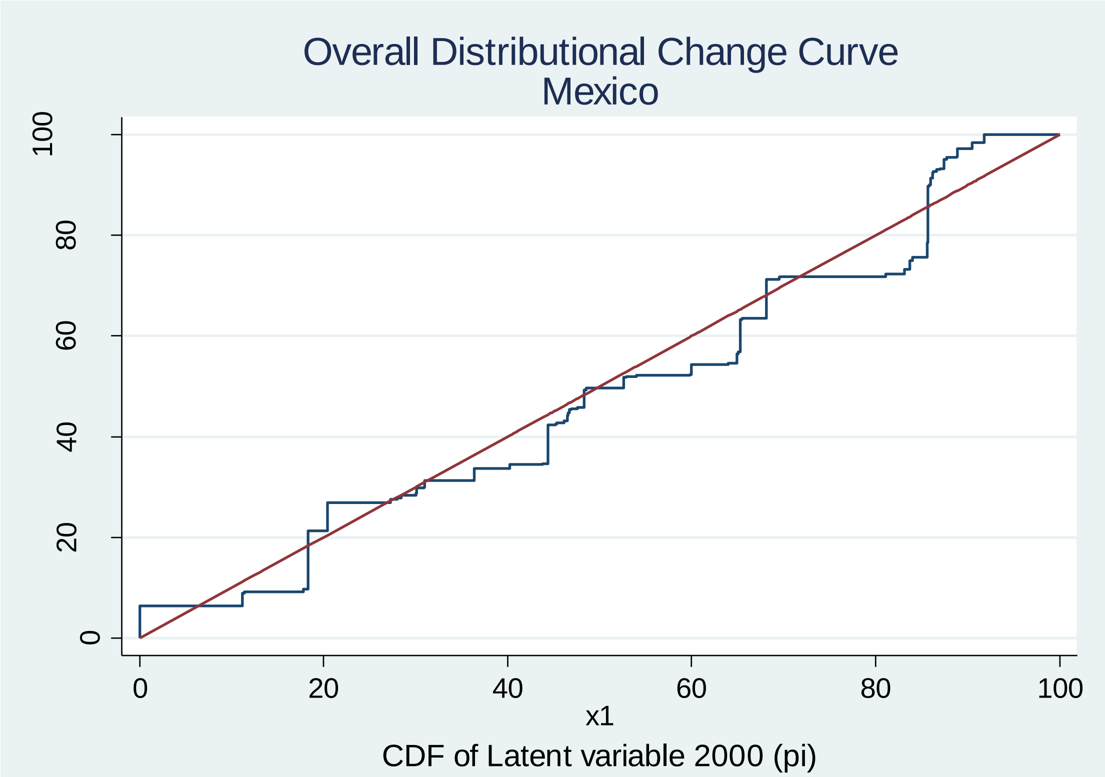
Overall Distributional Change Curve
Ecuador
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)



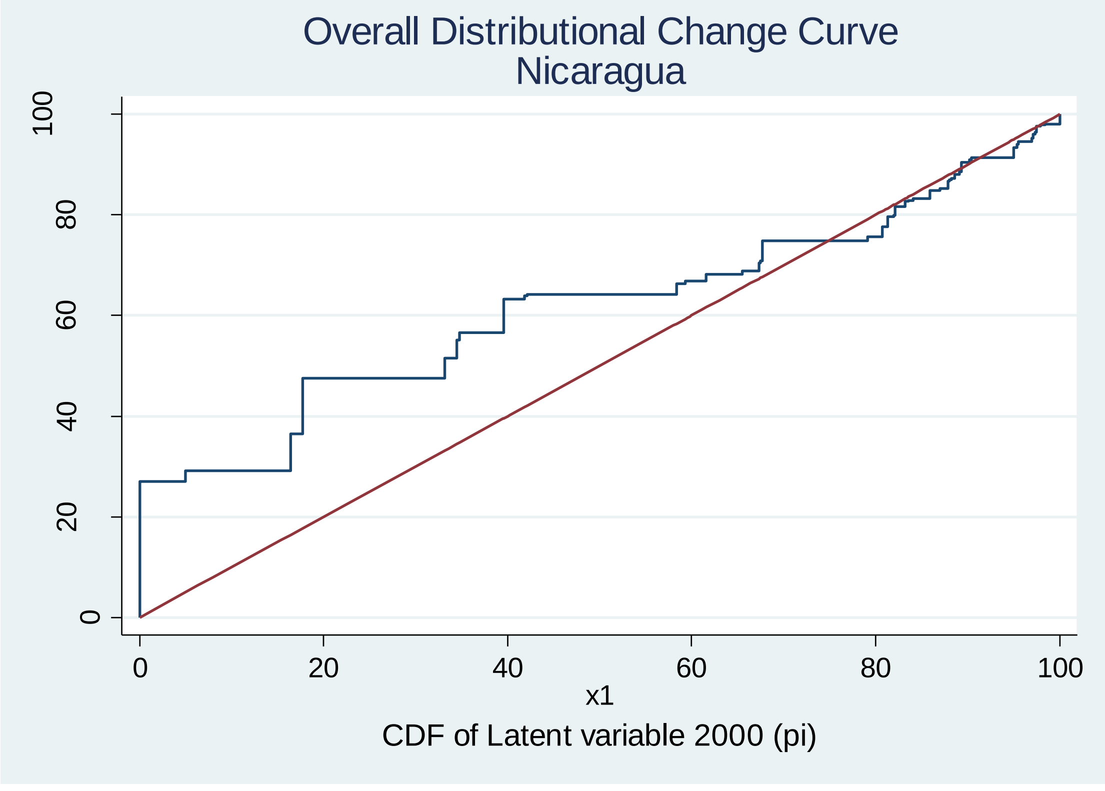
Overall Distributional Change Curve
El Salvador
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Guatemala
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

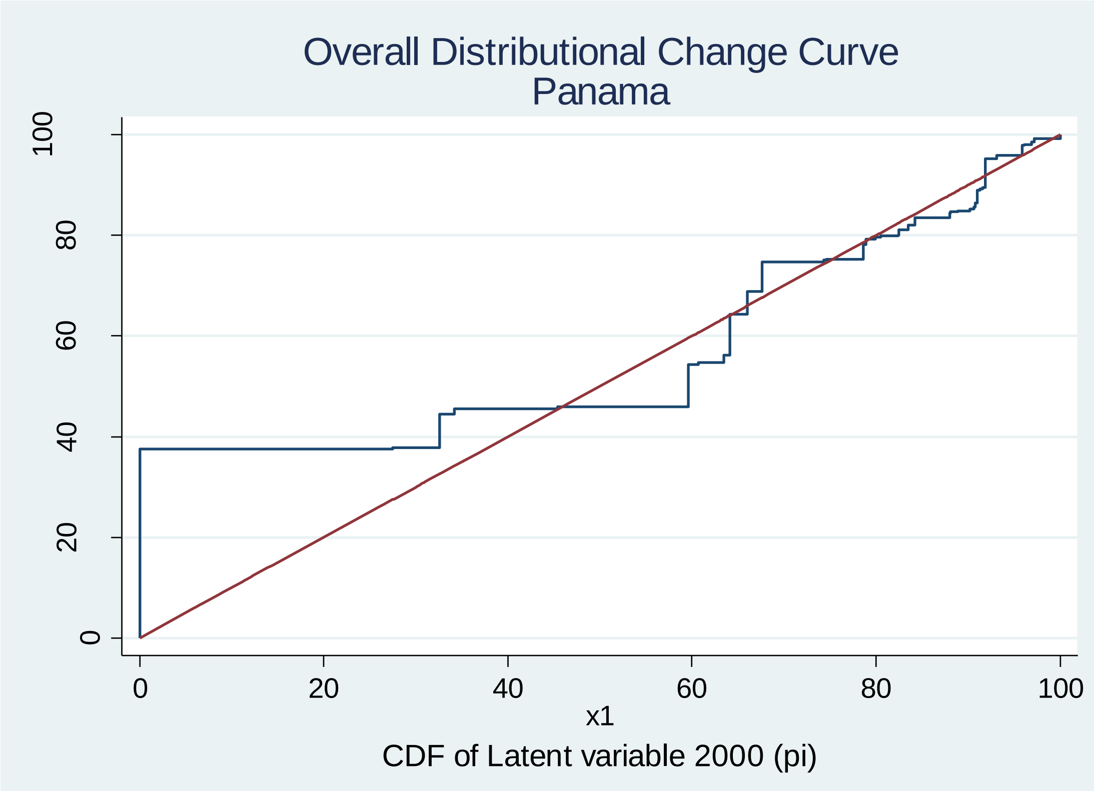

Overall Distributional Change Curve
Honduras
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

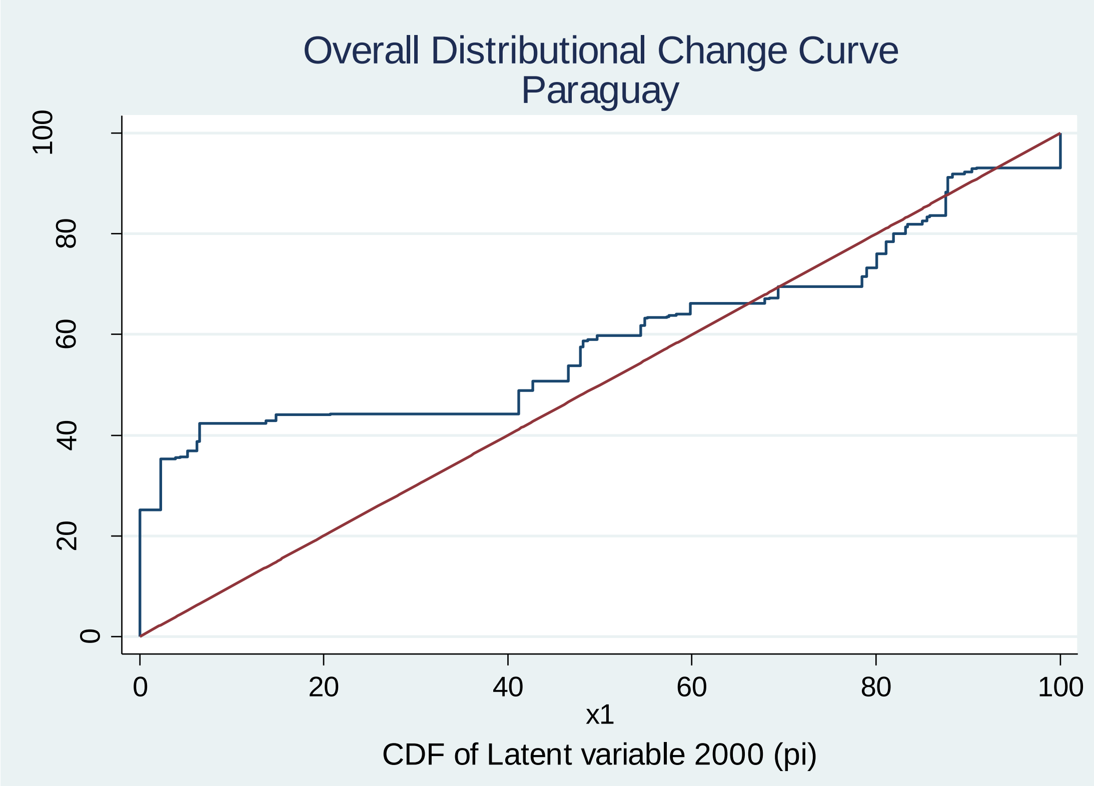


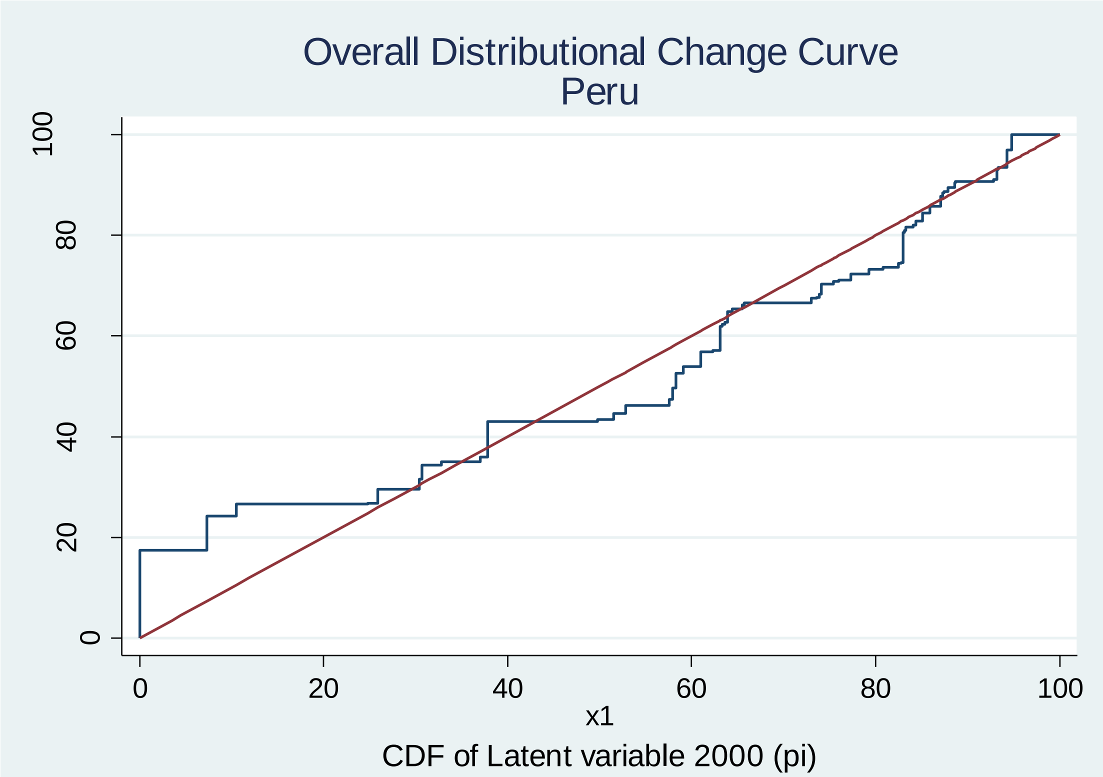
Overall Distributional Change Curve
Mexico
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)

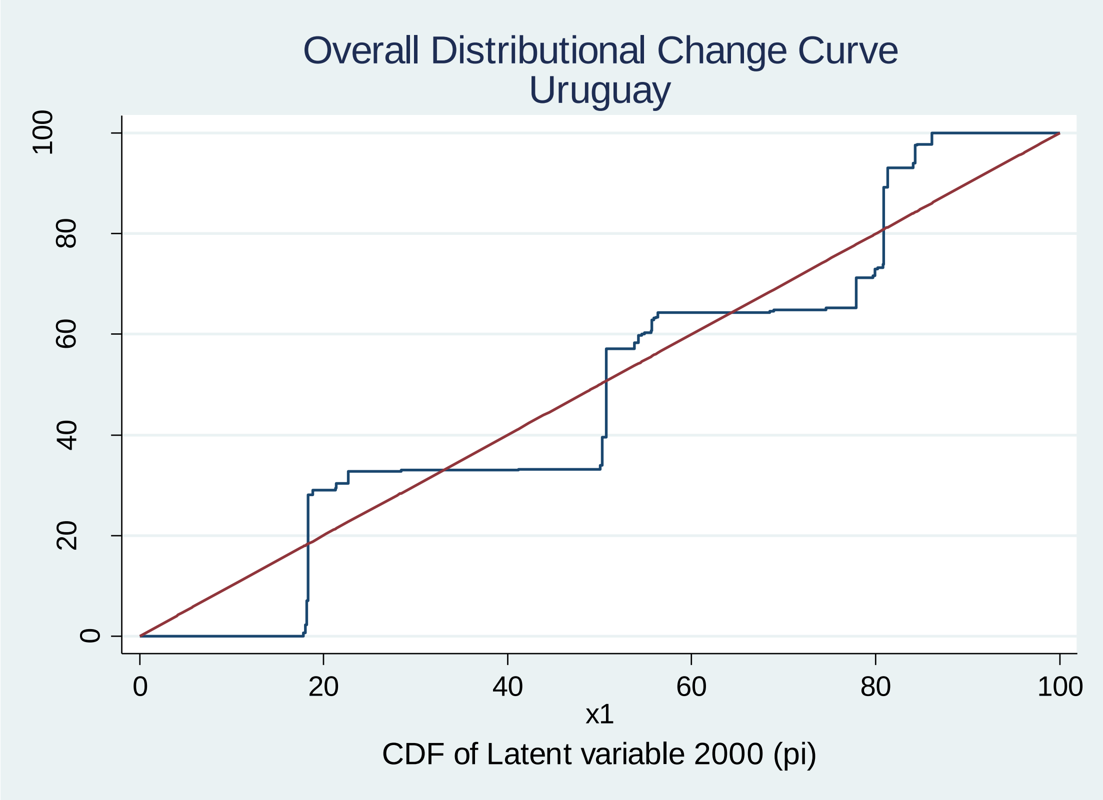


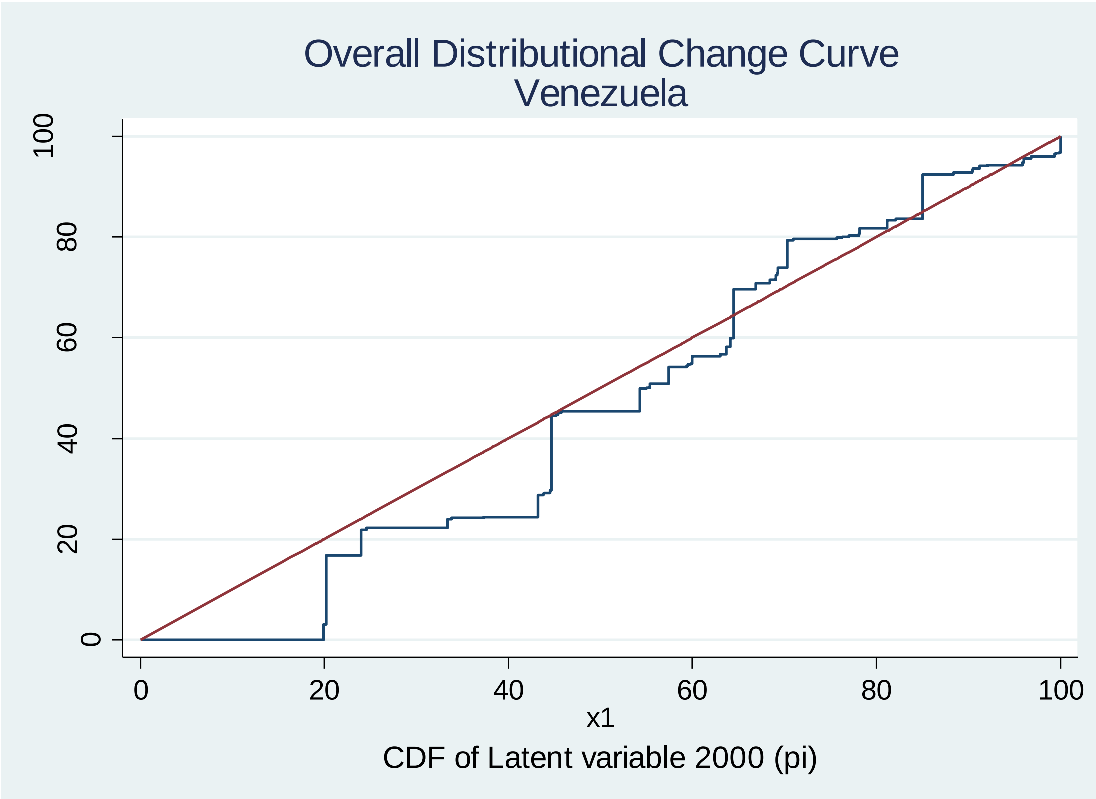
Overall Distributional Change Curve
Nicaragua
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Panama
100
80
60
40
20
0
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Paraguay
100
80
60
40
20
0
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Peru
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Uruguay
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)


Overall Distributional Change Curve
Venezuela
0
20
40
60
80
100
0
20
40
60
80
100
x1
CDF of Latent variable 2000 (pi)