1999 Conference on Electrical Insulation and Dielectric Phenomena

3-D and 1-D dynamics of slender liquid jets: Linear analysis with electric field and accuracy of 1-D models near the breakup

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Abstract

In a previous paper [Phys. Fluids, vol. 6, 2676 (1994)], the authors derived four 1-D models (Lee, Cosserat, averaged, and parabolic models) for slender axisymmetric liquid jets from the Navier-Stokes equations. The error of these 1-D models was calculated for small perturbations, in the absence of electric field. Here, we extend the linear error analysis to both perfectly insulating liquid jets in a tangential electric field and perfectly conducting liquid jets in a radial electric field. The accuracy of these models for studying the breakup, when nonlinear effects are no longer negligible, is also tested in the absence of electric field. A comparison of numerical 3-D solutions with results from 1-D models is made. A formulation of the energy conservation in 1-D models allows identifying and correcting a numerical instability of the averaged model near the breakup. It also explains why the Cosserat model overestimates the breakup time for moderate or large viscosity. Good agreement between 1-D and 3-D numerical results is found.

Introduction

A liquid jet emerging with large velocity from a circular nozzle of radius R always breaks up into droplets, owing to the destabilizing effect of surface tension for long wavelengths. Since the capillary force is the driving one, the surface tension γ of the liquid-air interface and the radius of the jet R are used to make variables nondimensional. We consider jets whose mean velocity is large compared to the capillary velocity, but small enough so that the surrounding gas has a negligible effect. In such a case, the liquid jet can be studied from a reference system moving with the same velocity as the liquid. The influence of the emergence velocity is then negligible [1]. From this point of view, a perturbation periodic in time in the nozzle is seen as a spatially periodic perturbation in an infinitely long liquid jet.

In the absence of electrical forces, the breakup time as well as the size of the droplets depend on the competition among capillary, viscous and inertia forces present in the Navier–Stokes equations and the associated boundary conditions. The only number that determines the neutral stability boundary of the liquid jet is the nondimensional wave number k of the perturbation, since an axisymmetric perturbation increases the surface of the cylindrical jet for $k > 1$. Therefore, the jet is unstable for $0 < k < 1$ [2]. In this work, only axysimmetric perturbations will be considered. Viscosity has no effect on the stability, but it has a marked effect on the dynamics. The nondimensional number that accounts for the strength of the viscous stress relative to the capillary pressure is known as the Ohnesorge number, $C = \mu/(\rho\gamma R)^{1/2}$, where ρ and μ are the density and viscosity of the liquid. The larger is C, the larger is the breakup time of the jet.

The difficulty of solving the Navier–Stokes equations with free-surface boundary conditions in three dimensions (3-D), even in the axisymmetric case, has lead to a great effort in simplifying the formulation. Several authors have proposed one-dimensional (1-D) models based on the large-slenderness approximation, i.e. the wavelength of the perturbation divided by R is large (see [3] and [4] for references). In terms of k, the liquid jet is slender if k is small. The variables of these 1-D models only depend on the axial variable z and the time t, so that the analytical and computational effort is greatly reduced. The authors have derived and generalized two well known 1-D models (Lee [5] and Cosserat [6] models) and other two new ones (averaged and parabolic models [3]) from the Navier-Stokes equations. Before using these models with confidence, it is indispensable to test them against available 3-D results.

The linear analysis of the infinite jet subjected to small initial perturbations is useful to determine the size of the resulting droplets. Typically, this size is inversely proportional to the most unstable wave number $k_{\max}$. The linear analysis also gives an estimate of the breakup time, which is inversely proportional to the growth rate $\alpha_{\max}$ of the most unstable perturbation. This estimate is very good in prac-

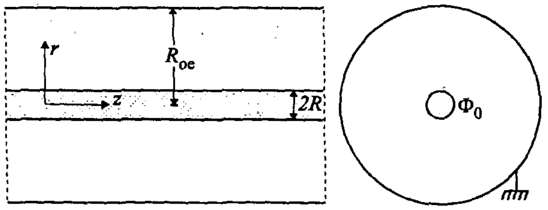

Figure 1: Scheme of an infinite, conducting liquid jet in a radial electric field.

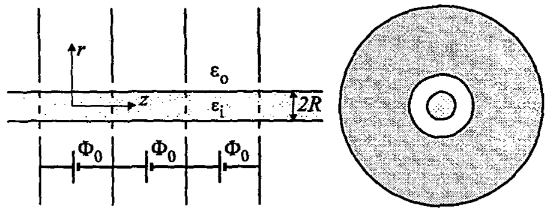

Figure 2: Scheme of an infinite, insulating liquid jet in a longitudinal electric field.

tice, since the amplitude of the perturbation is small during almost all the time until the breakup. The comparison of 3-D and 1-D linear results is very satisfactory for all models in the absence of electric field, especially for the averaged and parabolic models [3].

Here, we will extend the comparison of 3-D and 1-D linear results to two configurations with electric field: a perfectly conducting liquid jet in a radial electric field and a perfectly insulating liquid jet in a longitudinal electric field. In both cases, the dispersion relation is modified by adding an electric-pressure term to the capillary-pressure term [7, 8].

Even more important is to test the validity of 1-D models near the breakup, when nonlinear effects take place. In that regime, it is not clear that the slenderness hypothesis holds. Here, a comparison with 3-D solutions in the absence of electric field is made.

Conducting jet with radial electric field

Figure 1 shows how a cylindrical outer electrode is put in order to induce a dc radial electric field on the jet. In this case, the nondimensional number that evaluates the electric pressure relative to the capillary pressure is $\chi = \varepsilon_o \Phi_0^2/[\gamma R \ln^2(R_{oe}/R)]$. The effect of R_{oe}/R is negligible when it is large, which is usually the case. The 1-D linear results have been tested using the exact (3-D) solution given by Melcher [9] and Saville [10]. We will concentrate in the most unstable modes of the jet, which are the most interesting ones. For a fixed value of k, the error is not expected to increase with χ, since the electric term is the same for both 1-D and 3-D equations. However, the most unstable mode moves towards shorter wavelengths (k_{max} increases) as χ increases. This implies larger errors in α_{max} and k_{max} for the 1-D models. To see how well they behave, we have computed the relative error in α_{max} against C for $\chi = 3.2$. This is an unfavorable situation, difficult to reach in experiments [9]. The only model with large error for small viscosity is the more simple viscous Lee model. Its relative error is $\delta\alpha_{max} \simeq 20\%$. The Cosserat model, which is typically worse for moderate or large viscosity, gives $\delta\alpha_{max} \simeq 6\%$. The averaged and parabolic models have errors under 4% for all C, which are typically within experimental error. For smaller χ, the error diminishes.

Insulating jet with tangential electric field

Figure 2 shows a possible configuration of electrodes so as to induce a dc electric field tangential to the interface of the jet. Dielectric liquids behave as perfectly insulating under an ac electric field whose frequency is much larger than the inverse of the charge relaxation time (the permittivity divided by the conductivity of the liquid) and the capillary time [11]. Now the nondimensional number that evaluates the electric pressure relative to the capillary pressure is $\chi = \varepsilon_o E_0^2 R/\gamma$, where E_0 is the electric field far from the liquid. The relative permittivity, ε_r also has an influence on the stability and dynamics of the jet. We have extended the 3-D inviscid linear analysis of Nayyar and Murty [12] to viscous liquids [8] in order to test 1-D linear results. The effect of the electric field is now stabilizing, contrary to the conducting case: the modes that grow faster move to larger wavelengths when χ increases. In terms of 1-D errors, this means that 1-D results improve when applying an electric field. For a typical oil ($\varepsilon_r = 3$) with $\chi = 1$, the error of the averaged and parabolic models are always under 0.01%. The Lee model underestimates α_{max} in 0.6% for C small, while the Cosserat model overestimates it in 0.08% for moderate viscosity.

Breakup: 1-D numerical method

The linear analysis cannot provide the shape of the interface ($r = F(z,t)$) when the jet breaks up. In fact, it cannot predict the appearance of small droplets, called *satellites*, when C is small [13]. This is a non-linear phenomenon. Ashgriz and Tsamopoulos [14] have computed numerically the breakup of an infinite liquid jet of almost inviscid ($C = 0.005$) and very viscous ($C = 10$) liquid jets, for several values of k. We use these data for testing the accuracy of 1-D models.

We have implemented a very efficient numerical method for solving the four 1-D models mentioned above. The partial derivative equations are of order up to four in z and one in time. The spatial discretization has been made through a Galerkin finite element method, based in the Hermite interpolation. For the resulting system of ordinary differential equations in time, we have chosen an implicit predictor-corrector method with variable time step. The predictor is an Adams-Brashforth scheme and the corrector is the trapezoid rule. Only for the first four first steps, we have used the more dissipative implicit Euler method as corrector, initiated with the explicit Euler method as predictor. This guarantees the smoothness of the initial conditions without increasing appreciably the temporal error. The resulting system of nonlinear algebraic equations is solved by the Newton method, which involves the inversion of a jacobian matrix. This task is carried out very efficiently by an LU method that takes advantage of the block-diagonal form of the matrix. The computational effort is small: each run takes about a minute in a personal computer.

Conservation of energy in 1-D models

When using the averaged model, a numerical instability arises near breakup. This is more evident for large viscosity. We have identified and corrected this shortcoming through a study of the conservation of energy of the 1-D models. The law of conservation of energy applied to fluids can be expressed as

$$(\mathcal{E}_c + \mathcal{E}_p)_t = -\dot{\mathcal{E}}_v, \qquad (1)$$

where $\mathcal{E}_k$ and $\mathcal{E}_p$ are the kinetic and potential energies, and $\dot{\mathcal{E}}_v$ is the power dissipated by the viscous stresses, called *dissipation function*. Here, we have taken into account that boundary terms are zero due to the application of boundary conditions.

Following Landau [15], we have calculated the 3-D dissipation function. Similarly, we have deduced $\dot{\mathcal{E}}_v$ from the equations of each 1-D model. The resulting expressions can be compared to the 3-D dissipation function, provided that the velocity is expanded in the same way as in the derivation of 1-D models. As in the 3-D case, the Lee and Cosserat model have always definitely positive dissipation functions [4]. However, the dissipation function of the Cosserat model has an extra positive term with respect to the 3-D expression of $\dot{\mathcal{E}}_v$. This explains why the Cosserat model systematically overestimates the breakup time for moderate or large viscosity.

The expression of $\dot{\mathcal{E}}_v$ of the averaged model is positive

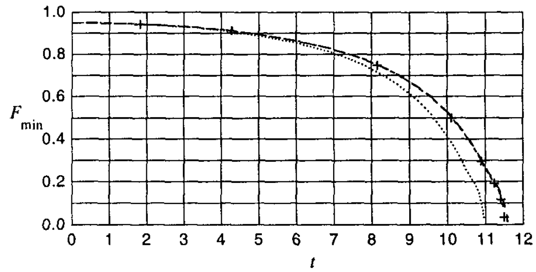

Figure 3: Radius of the neck versus nondimensional time, for $k = 0.9$ and $C = 0.005$; according to 3-D (Ashgriz and Mashayek, +), Lee (·····), Cosserat and averaged (– – –), and parabolic (-----) models.

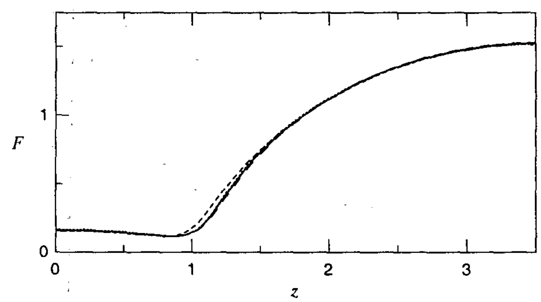

Figure 4: Shape of the interface near the breakup, for the same conditions as in figure 3. Solid line corresponds to 3-D results of Ashgriz and Mashayek.

for small or moderate amplitudes, but it is not a perfect square. Eventually, it becomes slightly negative in a portion of the jet before breakup. This is due to high-order terms not retained in the derivation of the model, which becomes not so small in the less slender zone of the jet near the breakup. However, this problem can be easily solved by retaining some high-order terms in the model that make the dissipation function definitely positive. In practice, test computations show that this has a negligible effect on the behaviour of the model until the numerical instability occurs.

Breakup: 1-D versus 3-D predictions

In order to make the errors of 1-D models evident, we have taken the less favorable wave number ($k = 0.9$) in the 3-D computations of Ashgriz and Mashayek [14]. For typical values of k, smaller than the latter, the error of 1-D models is smaller. In figure 3 we show the evolution of the neck radius $F_{\min}$ for a low-viscosity ($C = 0.005$) liquid jet. The error of the Lee model is significant, as was also the case for small perturbations [3], while the other models give a very accurate estimation of the breakup time.

Figure 4 shows how well the 1-D models can predict

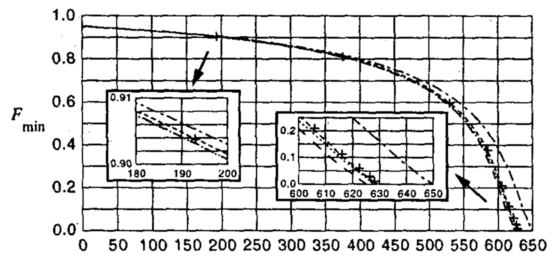

Figure 5: Radius of the neck versus nondimensional time, for $k = 0.9$ and $C = 10$; according to 3-D (Ashgriz and Mashayek, +), Lee (······), Cosserat (–·–·–), averaged (– – –), and parabolic (-----) models.

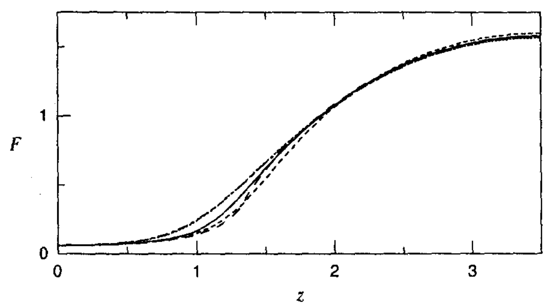

Figure 6: Shape of the interface near the breakup, for the same conditions as in figure 5.

the shape of the interface near the breakup (owing to symmetry, only half the period is represented). In particular, they predict accurately the existence and the size of the satellite drop (on the left).

Figure 5 shows $F_{\min}$ versus time for the same k, but now for large viscosity ($C = 10$). The error is slightly greater, as it happened in the linear analysis [3], but small compared with typical experimental errors. Only the Cosserat model, as discussed above, clearly overestimates the breakup time.

The error in the shape of the interface is also small for large C, as shown in figure 6, although it is larger than the corresponding one for low viscosity.

Conclusions

In this paper, we have tested the validity of 1-D models in predicting the evolution of perfectly conducting liquid jets under a radial electric field and perfectly insulating liquid jets under a longitudinal electric field. The linear analysis is encouraging, since the errors of 1-D models are small enough in most situations, even in the presence of strong electric fields.

In the absence of electric field, a comparison between 3-D and 1-D predictions of the breaking of liquid jets shows that the latter can predict accurately the breakup time, as well as the shape of the interface and the volume of the resulting droplets. Future work will implement the breaking of liquid jets with electric field.

Acknowledgments

This work has been supported by the Spanish DGES, under contract PB96-1375. The authors are very grateful to Dr. Ashgriz and Dr. Mashayek for providing some of the data shown in figures 3 to 6.

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